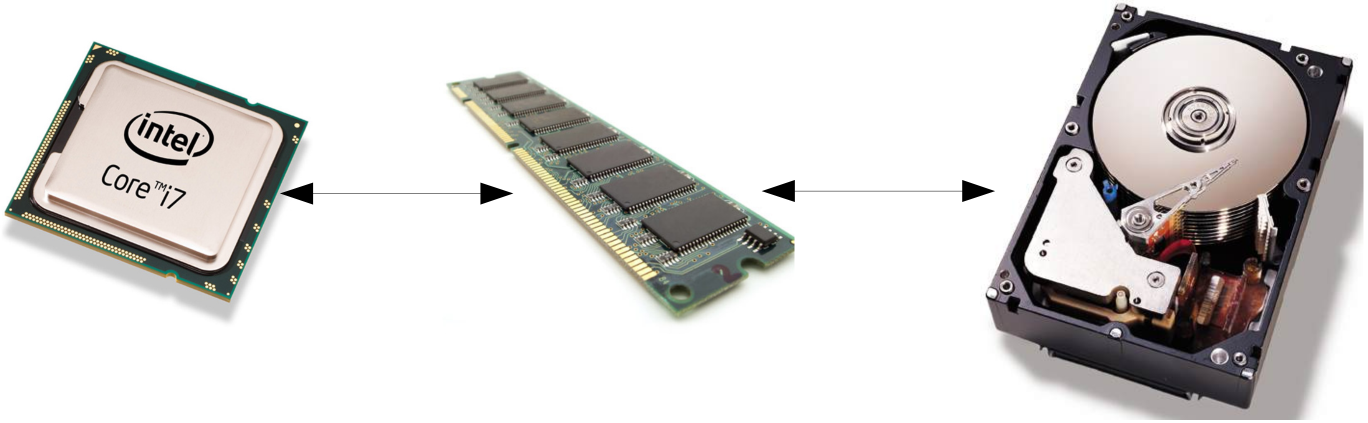
One-dimensional index structures

Motivation: The I/O model of computation

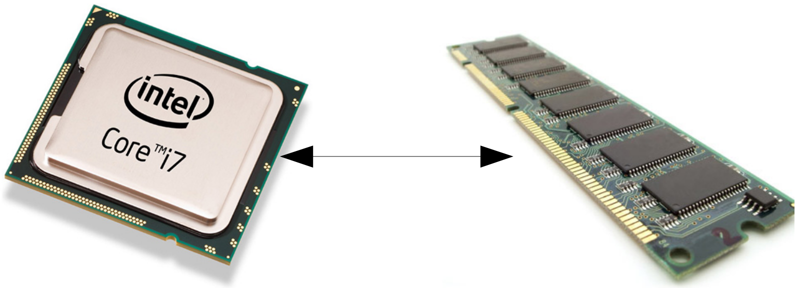


The I/O model

- Data is stored on disk, which is divided into blocks of bytes (typically 4 kilobytes)

  (each block can contain many data items)
- The CPU can only work on data items that are in memory, not on items on disk
- Therefore, data must first be transferred from disk to memory
- Data is transferred from disk to memory (and back) in whole blocks at the time
- The disk can hold D blocks, at most M blocks can be in memory at the same time (with $M << D$).

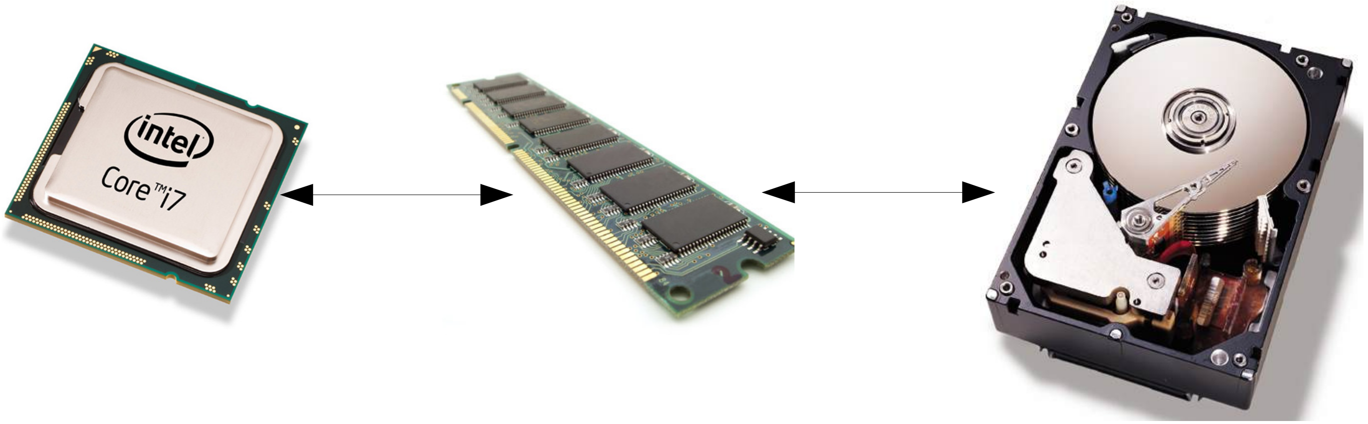
Motivation: The I/O model of computation



However: complexity of algorithms is traditionally analyzed in the RAM model of computation

- Data is stored in an (infinite) memory
- The CPU works on data items in memory
- Complexity is measured in terms of the number of memory accesses and CPU operations.

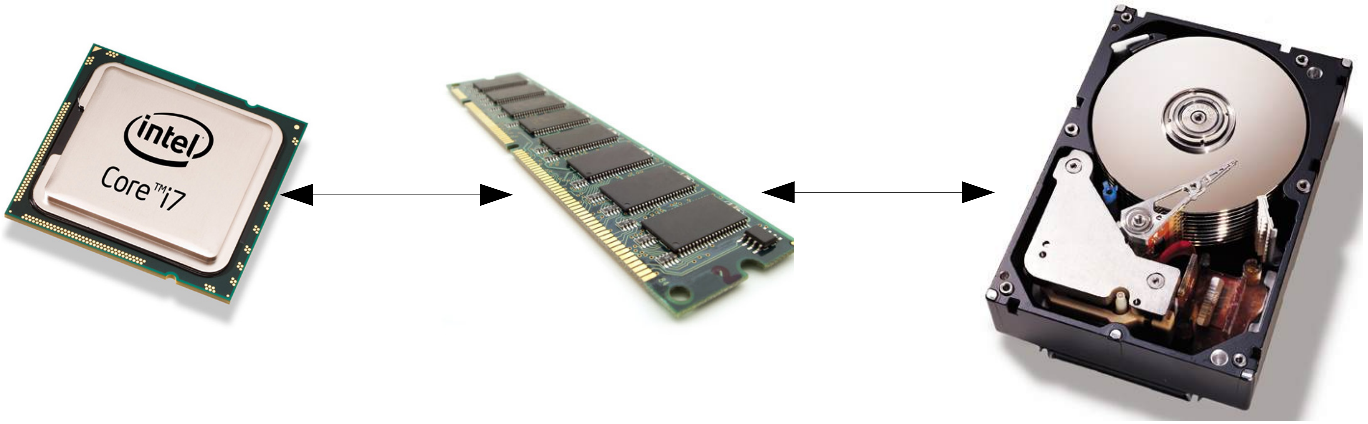
Motivation: The I/O model of computation



"The difference in speed between modern CPU and disk technologies is analogous to the difference in speed in sharpening a pencil using a sharpener on ones desk or by taking an airplane to the other side of the world and using a sharpener on someone elses desk."

(D. Comer)

Motivation: The I/O model of computation



- In-memory computation is fast (memory access latency $\approx 10^{-8}s$)
- Disk-access is slow (disk access latency: $\approx 10^{-3}s$)
- Hence: execution time is dominated by disk I/O

We will use the number of I/O operations required as cost metric

Motivation: searching in a database

A hypothetical database

- A relation $R(A, B, C, D)$. Each tuple comprises 32 bytes.
- Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

Searching for record with $C = 10$ in case R is arbitrary

- For every block X in R:
  - Load X from disk in memory
  - Check whether there is a tuple with $A = 10$ in X;
  - If so output record and terminate loop; otherwise continue
  - Release X from memory
- Worst case I/O Cost: the total number of blocks in R, or 10^6 I/O's.
- At 10^{-3} s per IO this takes 16.6 minutes. $\Rightarrow$ **Can we do better?**

Index structures

See corresponding slides

Searching in a database with a index (1/2)

The database

- A relation $R(A, B, C, D)$. Each tuple comprises 32 bytes.
- Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

The index

- There is a secondary index on attribute C.
- A $(\texttt{key value}, \texttt{ptr})$ pair in the index takes 16 bytes.
- **Question:** How many $(\texttt{key}, \texttt{ptr})$ pairs fit in a block?
- **Question:** How many blocks does the dense 1st level index take?
- **Question:** How many blocks does the sparse 2nd level index take?

Searching in a database with a index (1/2)

The database

- A relation $R(A, B, C, D)$. Each tuple comprises 32 bytes.
- Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

The index

- There is a secondary index on attribute C.
- A $(\texttt{key value}, \texttt{ptr})$ pair in the index takes 16 bytes.
- **Question:** How many $(\texttt{key}, \texttt{ptr})$ pairs fit in a block? 256
- **Question:** How many blocks does the dense 1st level index take? $5 \cdot 10^5$
- **Question:** How many blocks does the sparse 2nd level index take? 1954

Searching in a database with a index (2/2)

Searching for records with $C = 10$ using the index

- Algorithm:
  - Loop through all of the blocks X in sparse index, one, by one, and find the $(\texttt{key}, \texttt{ptr})$ pair in X with the largest `key` value satisfying `key` $<= 10$.
  - Follow `ptr` to dense index block, and use the information in this block to locate the block in R containing the record with $C = 10$ (if it exists).
- Worst case I/O Cost: loading of all blocks of sparse index + 1 block of dense index + 1 block of R, or $1954 + 1 + 1 = 1956$ I/Os.
- At 10^{-3} s per I/O this takes 2 seconds.

Since the sparse index is sorted, we could perform binary search on it if it is sequential.

- I/O Cost: binary search in sparse index + 1 block of dense index + 1 block of R, or $\log_2(1954) + 1 + 1 = 14$ I/Os $\rightarrow 0.014$ seconds.

Searching in a database with a BTree index (1/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index on attribute C.
- A `key value` takes 8 bytes, a `ptr` also 8 bytes.
- **Question:** What is the maximum order n of the BTree, taking into account that blocks are 4096 bytes large?

Searching in a database with a BTree index (2/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index on attribute C.
- A `key value` takes 8 bytes, a `ptr` also 8 bytes.
- **Question:** What is the maximum order n of the BTree, taking into account that blocks are 4096 bytes large?
- **Answer:** A BTree of order n stores $n+1$ pointers and n key values in each block. We are hence looking for the largest integer value of n satisfying:

  $$(n+1) \text{ ptrs } \times 8 \text{ bytes/ptr } + n \text{ keys } \times 8 \text{ bytes/ptr } \leq 4096 \text{ bytes}$$

  As such, $n = 255$: we store 256 pointers and 255 keys in a block.

Searching in a database with a BTree index (3/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Question:** What is the height of the BTree assuming that leaf blocks are full and internal blocks contain 255 pointers?

Searching in a database with a BTree index (4/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Question:** What is the height of the BTree assuming that leaf blocks are full and internal blocks contain 255 pointers?
- **Answer:** : Observe:
  - there are $\left\lceil \frac{128 \cdot 10^6}{255} \right\rceil$ leaf blocks (at level 1)

Searching in a database with a BTree index (4/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Question:** What is the height of the BTree assuming that leaf blocks are full and internal blocks contain 255 pointers?
- **Answer:** : Observe:
  - there are $\left\lceil \frac{128 \cdot 10^6}{255} \right\rceil$ leaf blocks (at level 1)
  - there are $\left\lceil \frac{128 \cdot 10^6}{(255)^2} \right\rceil$ blocks at level 2

Searching in a database with a BTree index (4/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Question:** What is the height of the BTree assuming that leaf blocks are full and internal blocks contain 255 pointers?
- **Answer:** : Observe:
  - there are $\left\lceil \frac{128 \cdot 10^6}{255} \right\rceil$ leaf blocks (at level 1)
  - there are $\left\lceil \frac{128 \cdot 10^6}{(255)^2} \right\rceil$ blocks at level 2
  - there are $\left\lceil \frac{128 \cdot 10^6}{(255)^3} \right\rceil$ blocks at level 3

Searching in a database with a BTree index (4/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Question:** What is the height of the BTree assuming that leaf blocks are full and internal blocks contain 255 pointers?
- **Answer:** : Observe:
  - So, there are $\left\lceil \frac{128 \cdot 10^6}{(255)^h} \right\rceil$ blocks at level h

  Since the root is at the level where there is only one block, we are looking for the smallest value of h such that $\left\lceil \frac{128 \cdot 10^6}{(255)^h} \right\rceil = 1$.

  So, $h = \lceil \log_{255} 128 \cdot 10^6 \rceil = 4$.

Searching in a database with a BTree index (5/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Observe:** The height of the BTree is the smallest when all blocks are full. It is the largest when all blocks are only half full (when each block has its minimum size).
- **Question:** What is the height of the BTree assuming that all blocks are only half full?

Searching in a database with a BTree index (5/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- **Observe:** The height of the BTree is the smallest when all blocks are full. It is the largest when all blocks are only half full (when each block has its minimum size).
- **Question:** What is the height of the BTree assuming that all blocks are only half full? **Answer**: Same reasoning as before:
  $= \lceil \log_{\mathbf{128}} 128 \cdot 10^6 \rceil = 4$

Searching in a database with a BTree index (6/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- Hence we can store at most 256 pointers; and 255 key values in a block.
- **Question:** What is the cost of searching for the record with $C = 10$ using this BTree, assuming the worst-case scenario that each block in the BTree is half full?

Searching in a database with a BTree index (6/6)

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.

The index

- There is a BTree index of order 255 on attribute C.
- Hence we can store at most 256 pointers; and 255 key values in a block.
- **Question:** What is the cost of searching for the record with $C = 10$ using this BTree, assuming the worst-case scenario that each block in the BTree is half full?

  Answer: height of the Bree in which blocks are half full + 1 I/O to access main file
  $= \lceil \log_{\mathbf{128}} 128 \cdot 10^6 \rceil + 1 = 5 \rightarrow$ at $10^{-3}s$ per I/O this takes 0.005 seconds.

Inserting in a BTree index

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation.

The index

- There is a BTree index of order 255 on attribute C.
- Hence we can store at most 256 pointers; and 255 key values in a block.
- **Question:** What is the cost of inserting a new record in this BTree, assuming the record is already in the main file, and assuming the worst-case scenario where each block in the BTree is full?

Inserting in a BTree index

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation.

The index

- There is a BTree index on attribute C.
- Hence we can store at most 256 pointers; and 255 key values in a block.
- **Question:** What is the cost of inserting a new record in this BTree, assuming the record is already in the main file, and assuming the worst-case scenario where each block in the BTree is full?

  Answer: in this scenario, we will need to split an existing block at each level, and create a new root.

Inserting in a BTree index

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation.

The index

- There is a BTree index on attribute C.
- Hence we can store at most 256 pointers; and 255 key values in a block.
- **Question:** What is the cost of inserting a new record in this BTree, assuming the record is already in the main file, and assuming the worst-case scenario where each block in the BTree is full?

  Answer: cost of a search + 2 I/O's per level of the BTree + new root $= \lceil \log_{255} 128 \cdot 10^6 \rceil + 2 \lceil \log_{255} 128 \cdot 10^6 \rceil + 1 = 3 \lceil \log_{255} 128 \cdot 10^6 \rceil + 1 = 13 \rightarrow 0.013s$