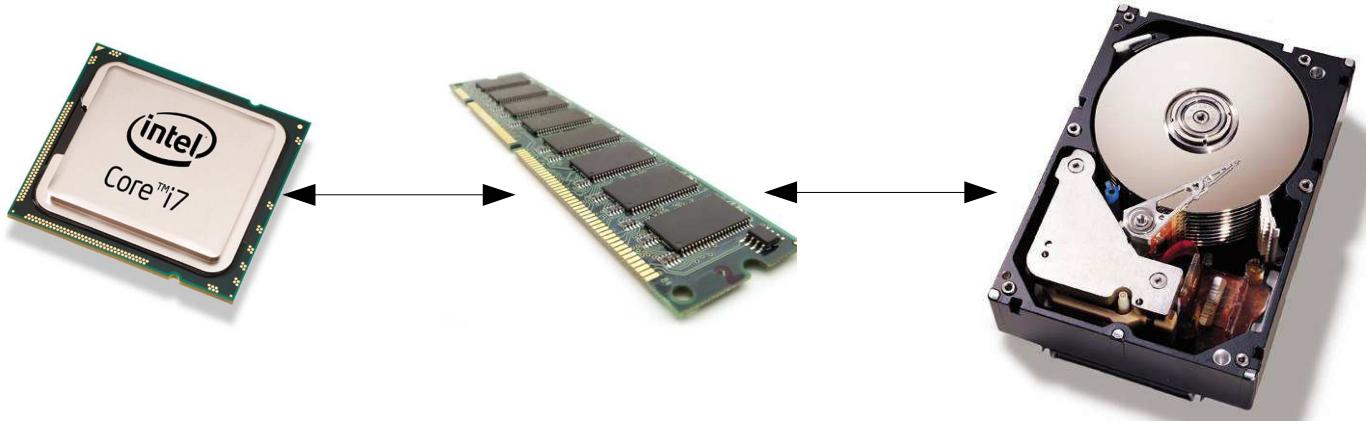
One-dimensional index structures

Motivation: The I/O model of computation



The I/O model

- Data is stored on disk, which is divided into blocks of bytes (typically 4 kilobytes)

  (each block can contain many data items)
- The CPU can only work on data items that are in memory, not on items on disk
- Therefore, data must first be transferred from disk to memory
- Data is transferred from disk to memory (and back) in whole blocks at the time
- The disk can hold D blocks, at most M blocks can be in memory at the same time (with $M << D$).

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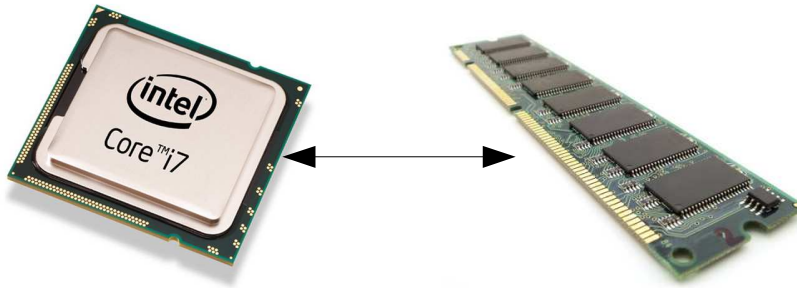
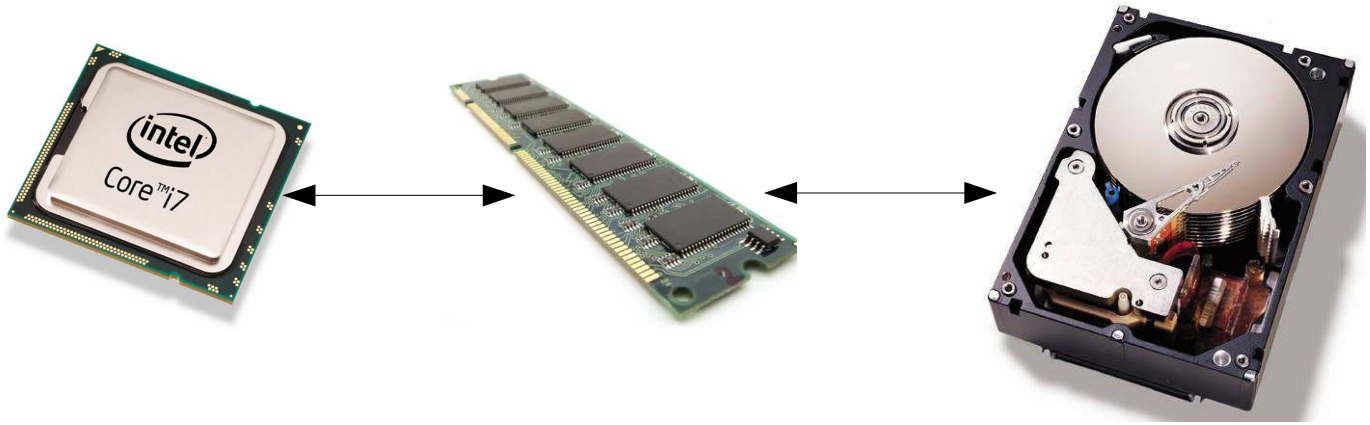


However: complexity of algorithms is traditionally analyzed in the RAM model of computation

- Data is stored in an (infinite) memory
- The CPU works on data items in memory
- Complexity is measured in terms of the number of memory accesses and CPU operations.

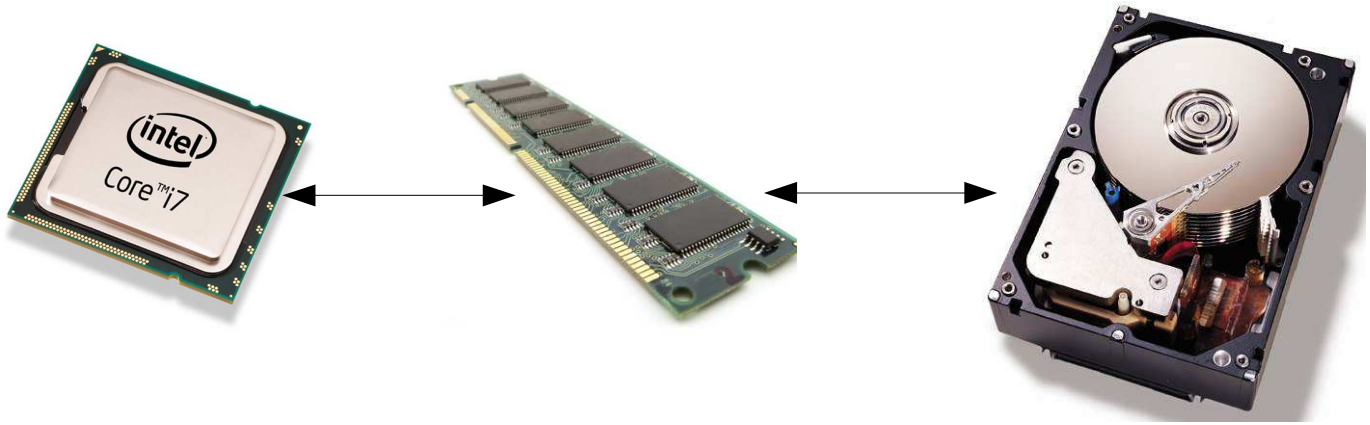
Motivation: The I/O model of computation



"The difference in speed between modern CPU and disk technologies is analogous to the difference in speed in sharpening a pencil using a sharpener on ones desk or by taking an airplane to the other side of the world and using a sharpener on someone elses desk."

(D. Comer)

Motivation: The I/O model of computation



- In-memory computation is fast (memory access $\approx 10^{-8}s$)
- Disk-access is slow (disk access: $\approx 10^{-3}s$)
- Hence: execution time is dominated by disk I/O

We will use the number of I/O operations required as cost metric

Motivation: searching in a database

A hypothetical database

- A relation $R(A, B, C, D)$. Each tuple comprises 32 bytes.
- Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

Searching for record with $C = 10$ in case R is arbitrary

- For every block X in R:
  - Load X from disk in memory
  - Check whether there is a tuple with $A = 10$ in X;
  - If so output record and terminate loop; otherwise continue
  - Release X from memory
- Worst case I/O Cost: the total number of blocks in R, or 10^6 I/O's.
- At 10^{-3} s per IO this takes 16.6 minutes. $\Rightarrow$ **Can we do better?**

Index structures

See corresponding slides

Searching in a database with a index (1/2)

The database

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- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

The index

- There is a secondary index on attribute C.
- A $(\texttt{key value}, \texttt{ptr})$ pair in the index takes 16 bytes.
- **Question:** How many $(\texttt{key}, \texttt{ptr})$ pairs fit in a block?
- **Question:** How many blocks does the dense 1st level index take?
- **Question:** How many blocks does the sparse 2nd level index take?

Searching in a database with a index (1/2)

The database

- A relation $R(A, B, C, D)$. Each tuple comprises 32 bytes.
- Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation. The block size $B = 4096$ bytes.
- Hence there are 128 tuples per block, or 10^6 blocks in total.

The index

- There is a secondary index on attribute C.
- A $(\texttt{key value}, \texttt{ptr})$ pair in the index takes 16 bytes.
- **Question:** How many $(\texttt{key}, \texttt{ptr})$ pairs fit in a block? 256
- **Question:** How many blocks does the dense 1st level index take? $5 \cdot 10^5$
- **Question:** How many blocks does the sparse 2nd level index take? 1954

Searching in a database with a index (2/2)

Searching for records with $C = 10$ using the index

- For every block X in the sparse index on C
  - Load X from disk in memory
  - Check whether there is a tuple with $\texttt{key} = 10$ in X
    - If yes follow pointer to dense index, and from there to R. Halt loop.
    - If no, continue loop.
  - Release X from memory
- Worst case I/O Cost: loading of sparse index + 1 block of dense index + 1 block of R, or $1954 + 1 + 1 = 1956$ I/Os.
- At 10^{-3} s per I/O this takes 2 seconds.

Since the sparse index is sorted, we could perform binary search on it.

- I/O Cost: binary search in sparse index + 1 block of dense index + 1 block of R, or $\log_2(1954) + 1 + 1 = 14$ I/Os $\rightarrow 0.014$ seconds.

Searching in a database with a BTree index

The database

- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
- There are $128 \cdot 10^6$ tuples in the relation.

The index

- There is a BTree index on attribute C.
- A (`key value` takes 8 bytes, a `ptr`) also 8 bytes.
- Hence we can store at most 256 pointers and ; and 255 key values.
- Worst case: assume that each block in the BTree is half full.
- **Question:** What is the cost of searching on attribute C in this BTree?

Searching in a database with a BTree index

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- **Question:** What is the cost of searching on attribute C in this BTree?

  Answer: height of the Bree + 1 I/O to access main file
  $= \log_{128} 128 \cdot 10^6 + 1 = 5 \rightarrow 0.005$ seconds.

Inserting in a BTree index

The database

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- There are $128 \cdot 10^6$ tuples in the relation.

The index

- There is a BTree index on attribute C.
- A (`key value` takes 8 bytes, a `ptr`) also 8 bytes.
- Hence we can store at most 256 pointers and ; and 255 key values.
- Worst case: assume that each block in the BTree is full.
- **Question:** What is the cost of inserting a new record in this BTree (assuming the record is already in the main file)?

Inserting in a BTree index

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- A relation $R(A, B, C, D)$. Attribute C is a (secondary) key for R.
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The index

- There is a BTree index on attribute C.
- A (`key value` takes 8 bytes, a `ptr`) also 8 bytes.
- Hence we can store at most 256 pointers and ; and 255 key values.
- Worst case: assume that each block in the BTree is full.
- **Question:** What is the cost of inserting a new record in this BTree (assuming the record is already in the main file)?

  Answer: cost of a search + 2 I/O's per level of the BTree
  $= \log_{128} 128 \cdot 10^6 + 2 \log_{128} 128 \cdot 10^6 + 1 = 3 \log_{128} 128 \cdot 10^6 + 1 = 13 \rightarrow 0.013s$