PLR Partitions: A Conceptual Model of Maps

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Overview

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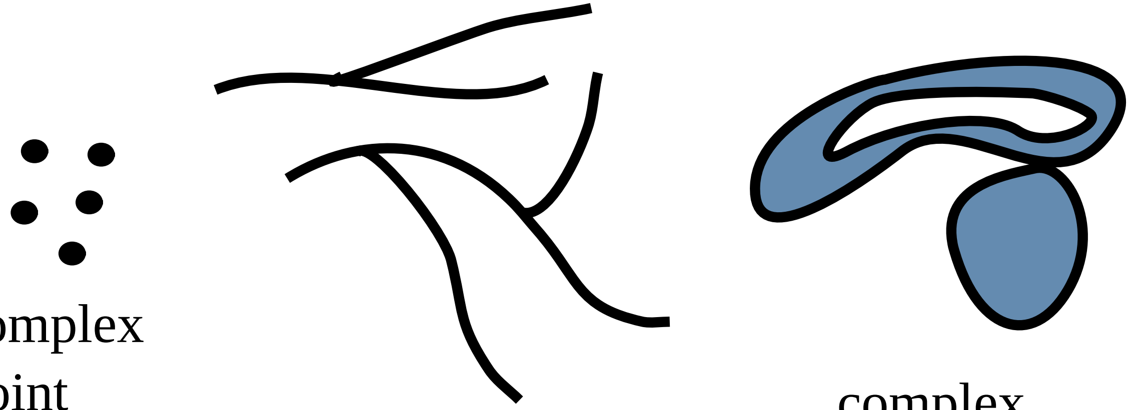
Introduction

Traditional Spatial Modelling



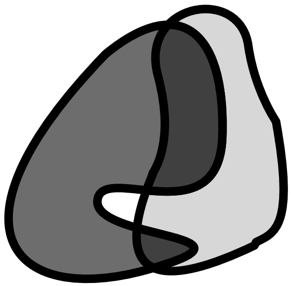
complex point

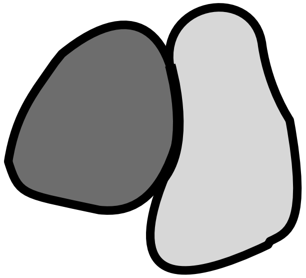
complex line

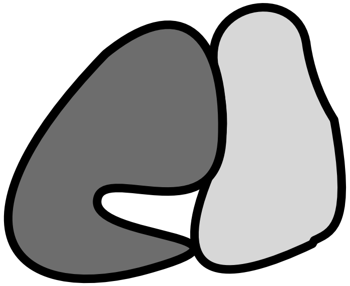
complex region

Problems With Traditional Spatial Modeling

- The Dimension Reduction Problem







Problems With Traditional Spatial Modeling

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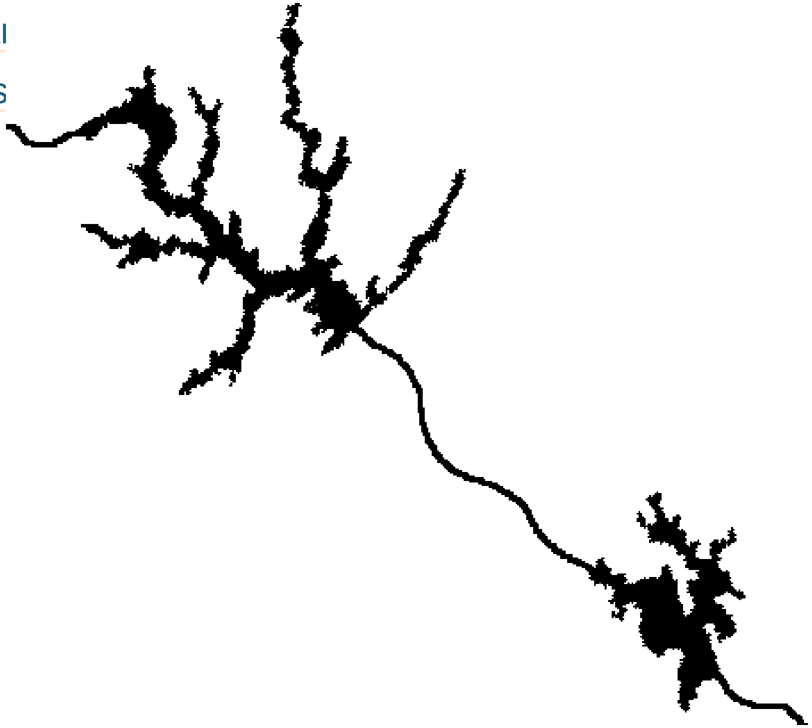
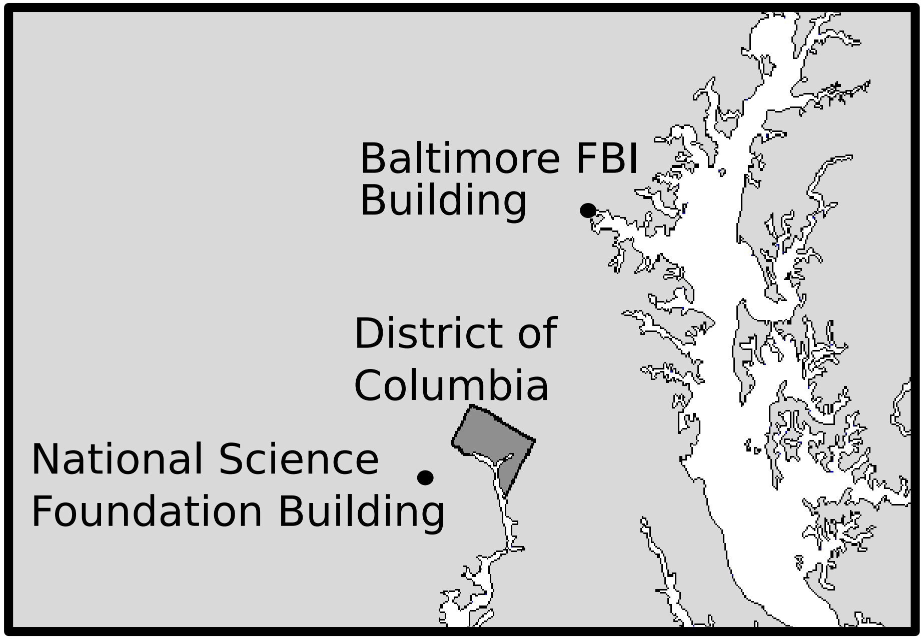
- The Dimension Representation Problem





Problems With Traditional Spatial Modeling

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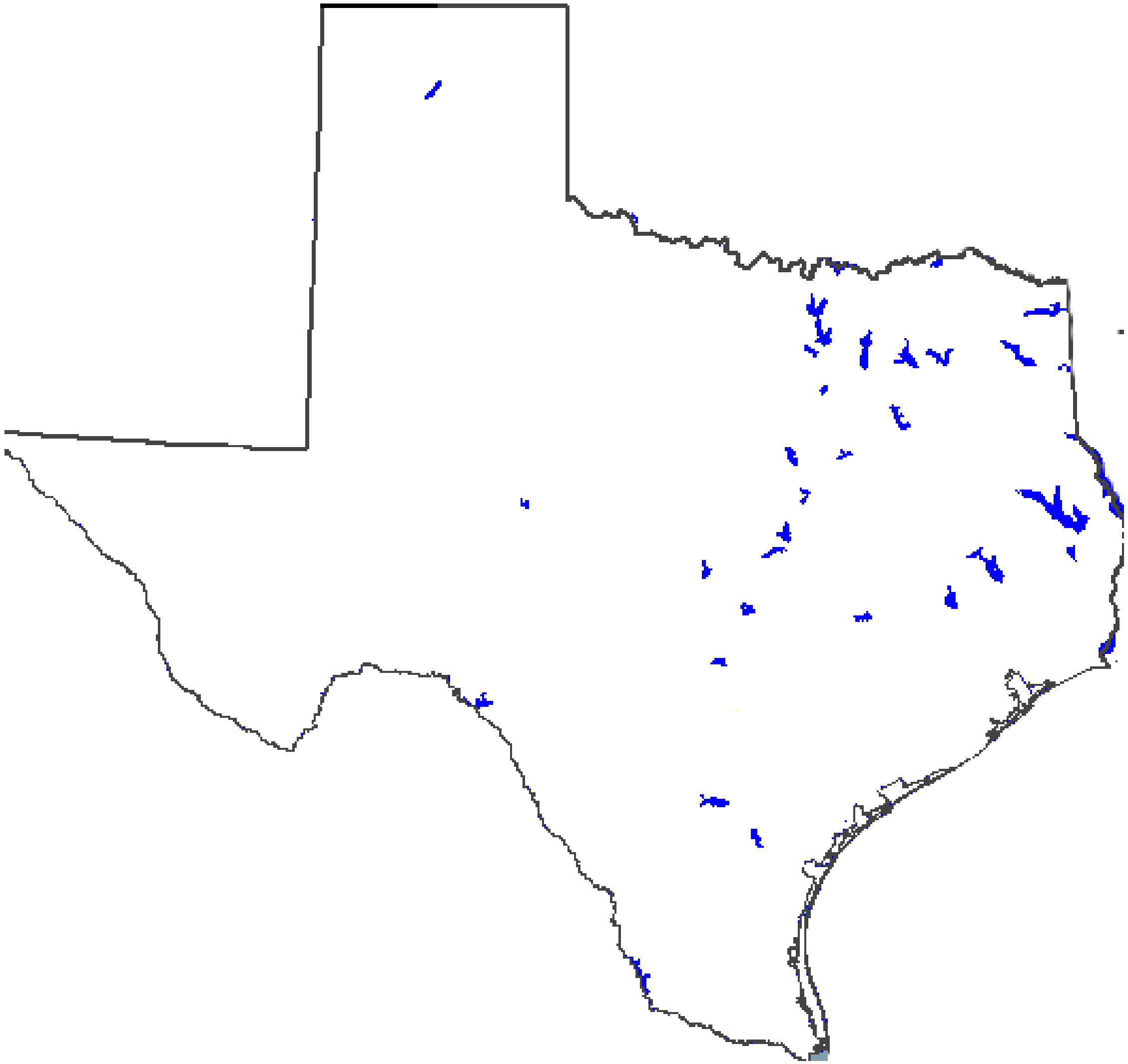
- The Feature Restriction Problem



Summary

- The Dimension Reduction Problem
- The Dimension Representation Problem
- The Feature Restriction Problem
- How do we solve this?

Related Work

Collections

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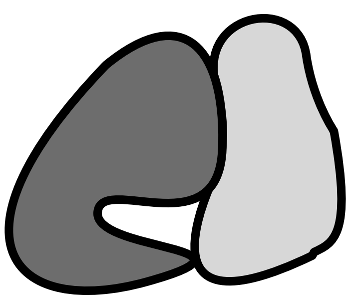
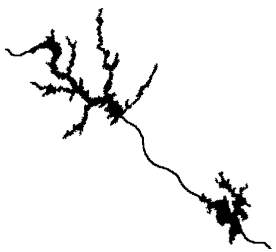
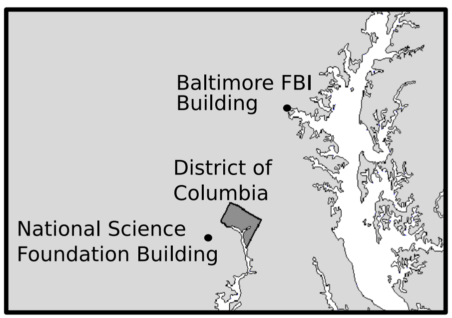
- Collect different spatial objects into a single *collection*
  - Solves the problems
  - Approach taken by Open Geospatial Consortium
  - No formal type definition for collections
  - Operations on collections are not formally defined
  - Efficiency of operations?

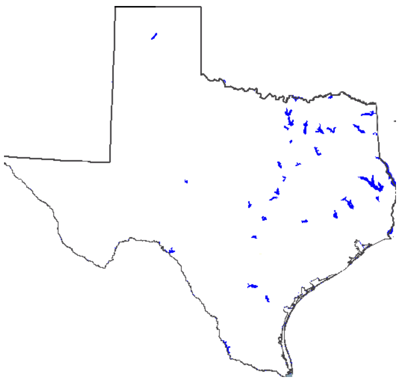
Maps

- Spatial situations described are special cases of *maps*
- Most approaches to modeling maps are collection approaches
- Some attempts have been made to create a type for maps:
  - Raster maps (not general enough for our purposes)
  - Database data type approaches (collections)
  - Graph approaches (lack a formal definition)
  - Spatial partitions (can only contain regions)









PLR Partitions

Our Goal

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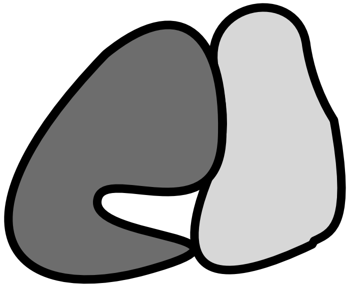
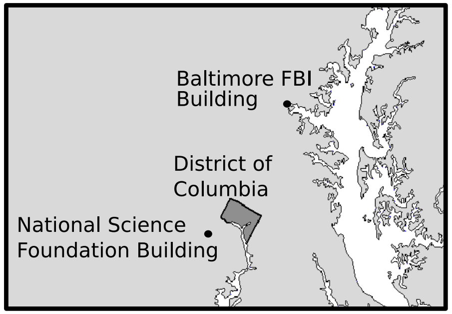
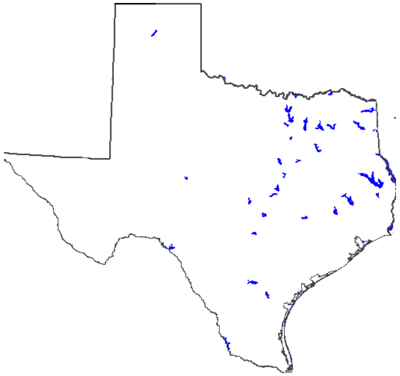
- Define a formal type definition for *PLR partitions* (point, line, and region partitions)
- Approach
  - Partition the plane into point sets
  - Associate each point set with a *label* that identifies it
  - Allow labels to carry general thematic information
  - Define constraints on the point sets







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- Map each point in the plane to a specific label $l \in 2^A$

Definition 1 *A* spatial mapping *of type A is a total function $\pi : \mathbb{R}^2 \rightarrow 2^A$.*

- A is the set of labels corresponding to spatial *features*
- Points not specifically labeled receive the label $\perp$
- For the map below: $A = \{Canada, USA, Mexico, \perp\}$

Spatial Mapping: Problems

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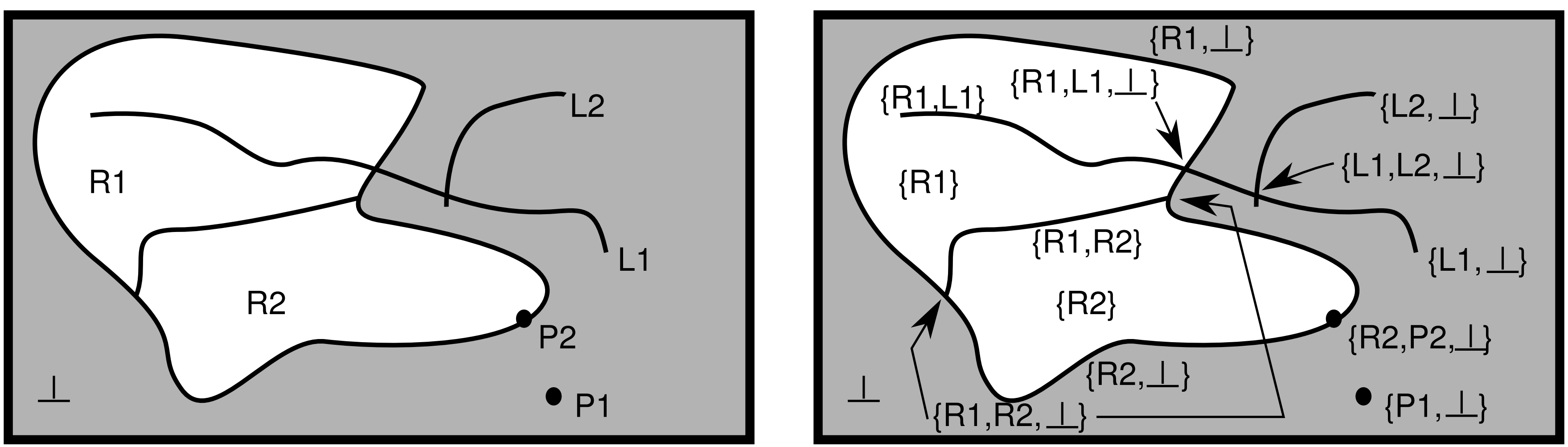
- Spatial mapping is insufficient to define a PLR partition
- Spatial features must be in some sense *regular*
- We model spatial features as *regular open point sets*
- Problem: how to extract the point sets belonging to a spatial feature?
  - Take the inverse spatial mapping of a label? No
  - We must be able to identify point sets belonging to a spatial feature
  - Must define spatial mappings that discriminate between types of spatial features



Identifying Spatial Features

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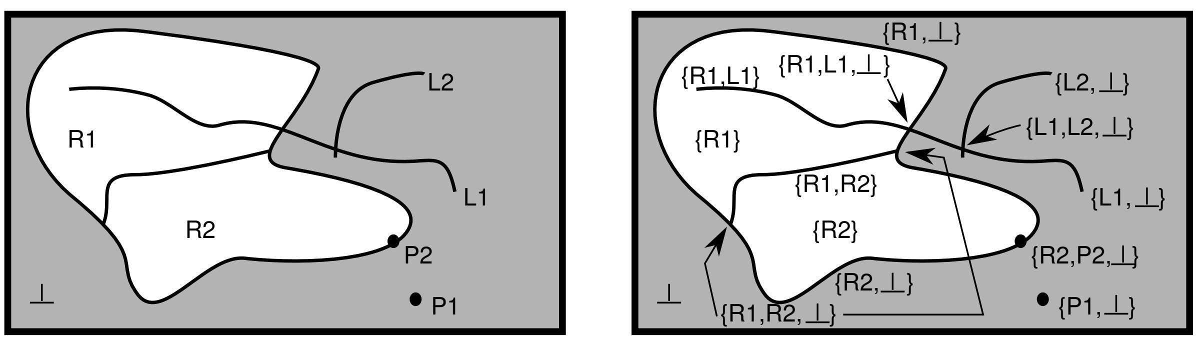
- How do we know a point belongs to a region?
  - Trivially, if the point has the same label as every point in its *neighborhood*
  - In regular open point sets, a point is surrounded by points that are in the set

$$isBasicRegion := \mathbb{R}^2 \rightarrow \mathbb{B}$$
$$isBasicRegion(p) = (\pi(p) = \bigcup_{q \in N(p)} \pi(q))$$

  - Generally, every point in its neighborhood is trivially a region with identical labels

$$isRegion := \mathbb{R}^2 \rightarrow \mathbb{B}$$
$$isRegion(p) = |\{\pi(q) | q \in N(p) \ \wedge \ isBasicRegion(q) \ \wedge \ \pi(q) \subseteq \pi(p)\}| = 1$$
$$\pi_r : \mathbb{R}^2 \rightarrow 2^A$$
$$\pi_r(p) = \{\pi[q] | q \in N(p) \ \wedge \ isBasicRegion(q) \ \wedge \ isRegion(p)\}$$



Identifying Spatial Features

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- A point's label is unique in its neighborhood

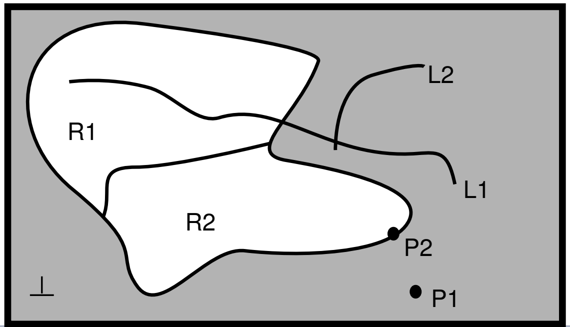
$$isPoint := \mathbb{R}^2 \rightarrow \mathbb{B}$$
$$isPoint(p) = |\pi(p) - \bigcup_{q \in N(p)} \pi(q)| > 0$$
$$\pi_p : \mathbb{R}^2 \rightarrow 2^A$$
$$\pi_p(p) = \pi(p) - \bigcup_{q \in N(p)} \pi(q)$$

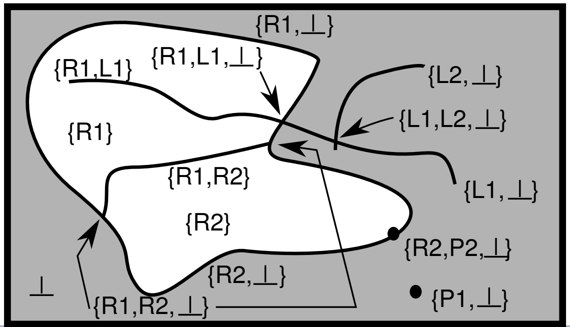
- To identify a line, we first remove the point and region labels:

$$S := \mathbb{R}^2 \rightarrow 2^A$$
$$S(p) = \pi(p) - \pi_r(p) - \pi_{rb}(p) - \pi_p(p)$$

- A line must extend in at least two directions from a point in its interior

$$isLine := \mathbb{R}^2 \rightarrow \mathbb{B}$$
$$isLine(p) = \exists q, s \in N(p) | q \neq s \ \wedge \ S(q) = S(s) \ \wedge \ S(q) \neq \varnothing \ \wedge \ \pi(q) \subseteq \pi(p)$$
$$\pi_l : \mathbb{R}^2 \rightarrow \{2^A\}$$
$$\pi_l(p) = \{S(q) | q, s \in N(p) \ \wedge \ isLine(p) \ \wedge \ S(q) = S(s) \ \wedge \ q \neq s \ \wedge \ S(q) \neq \varnothing\}$$





Identifying Spatial Features: Summary

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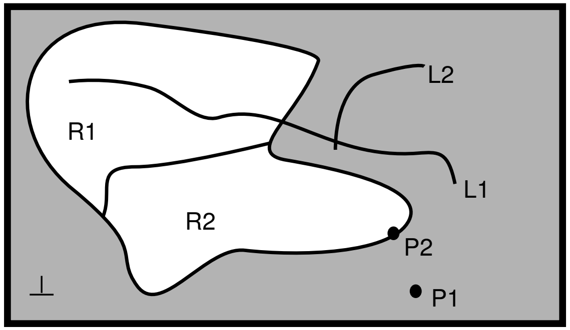
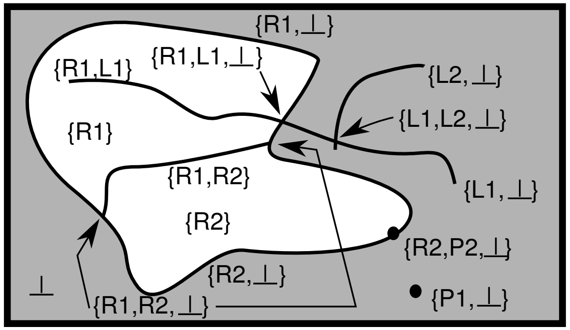
Definition 2 Let π be a spatial mapping of type A

$$
\begin{array}{lll}
\text{(i)} & \rho(\pi) := \pi_r^{-1}(2^A) & (regions) \\
\text{(ii)} & \omega_\rho(\pi) := \pi_{rb}^{-1}(2^A) & (region\ borders) \\
\text{(iii)} & \lambda(\pi) := \pi_l^{-1}(2^A) & (lines) \\
\text{(iv)} & \omega_\lambda(\pi) := \pi_{lb}^{-1}(2^A) & (line\ borders) \\
\text{(v)} & \varphi(\pi) := \pi_p^{-1}(2^A) & (points)
\end{array}
$$





Definition of PLR Partitions

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- A spatial mapping is a PLR partition if:
  - Its regions and lines are regular open sets
  - The borders between features carry the labels of all adjacent features

Definition 3 A *PLR partition* is a spatial mapping π of type A with:

$$
\begin{array}{ll}
\text{(i)} & \forall r \in \rho(\pi) : r = \overline{r}^{\circ} \\
\text{(ii)} & \forall l \in \lambda(\pi) : l = \overline{l}^{\circ} \\
\text{(iii)} & \forall b \in \omega_{\rho} : \pi_{rb}[b] = \{\pi_r[[r]] | r \in \rho(\pi) \ \wedge \ b \subseteq \partial r\} \\
\text{(iv)} & \forall b \in \omega_{\lambda} : \pi_{lb}[b] = \{\pi_l[[l]] | l \in \lambda(\pi) \ \wedge \ b \subseteq \partial l\}
\end{array}
$$

Conclusions and Future Work

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- Defined the PLR partition spatial data model
  - Solves the Dimension Reduction Problem
  - Solves the Dimension Representation Problem
  - Solves the Feature Restriction Problem
  - General enough to model thematic maps in general
- Future work: operations over PLR partitions
  - Because features are regular open point sets, we should be able to prove closure over spatial operations (i.e., map overlay)
  - Can specificaly define the semantics of PLR partition operations

Thanks!