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PROMETHEE

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Approximating the Results of the PROMETHEE II Method through Comparisons with Global Profiles

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Introduction

- Several problems can be difficult to solve using outranking methods due to their size
- Example: Spatial decision problems where the number of alternatives is too big

PROMETHEE II

$$\forall k \in \{1,2,\ldots,q\}, \qquad \forall a_i, a_j \in A \quad : \quad d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$$

$$P_k : \mathbb{R} \to [0,1] : d_k(a_i, a_j) \mapsto P_k(d_k(a_i, a_j))$$

$$P(a_i, a_j) = \sum_{k=1}^{q} \omega_k \cdot P_k(d_k(a_i, a_j))$$

$$\varphi_k(a_i) = \frac{1}{n-1} \sum_{a_j \in A} \left[P_k(a_i, a_j) - P_k(a_j, a_i)\right]$$

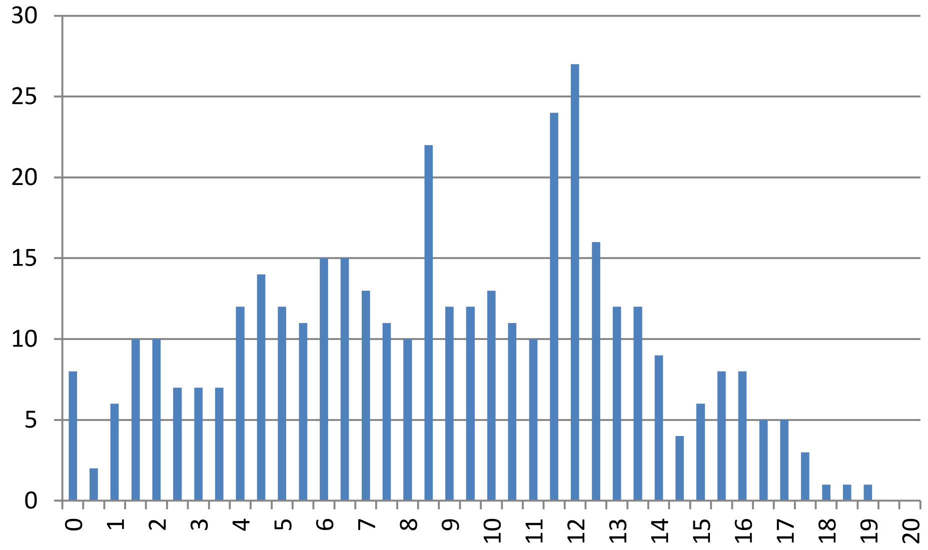
$$\varphi(a_i) = \frac{1}{n-1} \sum_{a_j \in A} \left[P(a_i, a_j) - P(a_j, a_i)\right] = \sum_{k=1}^{q} \varphi_k(a_i) \cdot \omega_k$$

Global profiles

- PROMETHEE works by comparing each alternative to all the others
- Several works proposed to circumvent this fact
- In this work: we propose to define profiles that will globally represent the rest of the dataset

1. Match the initial distribution

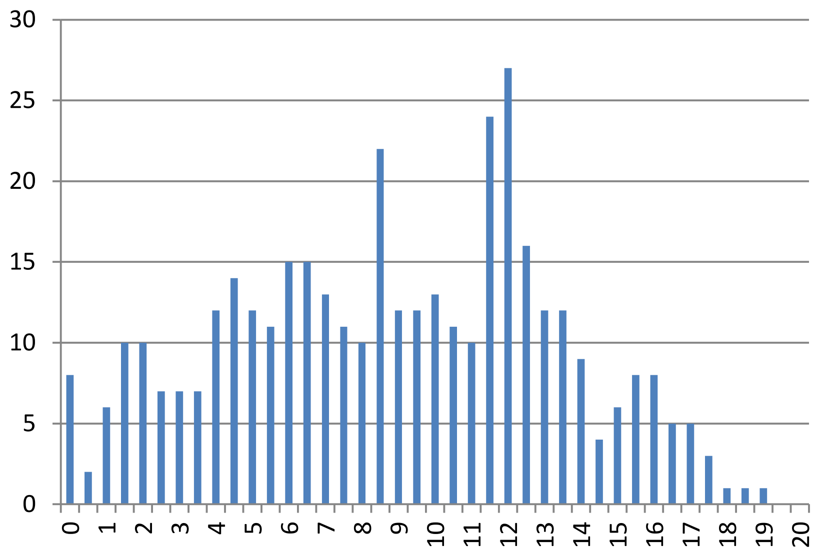
- Evaluations for one criterion



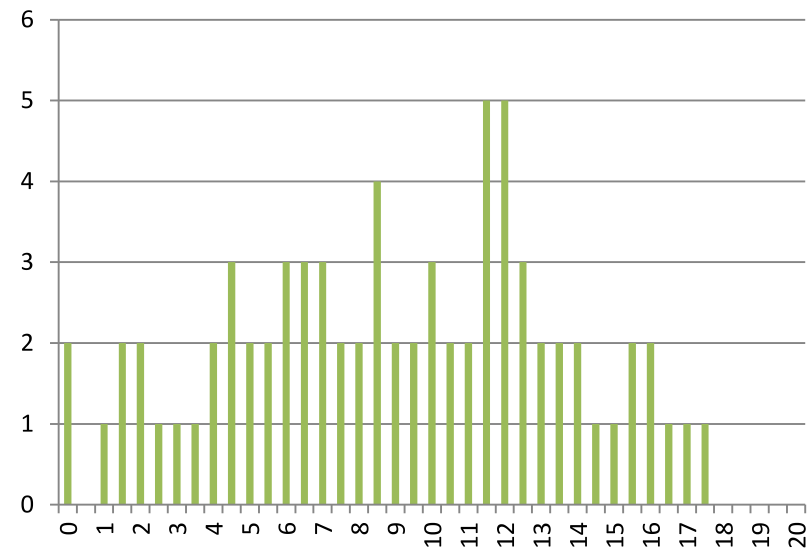
- 400 values, 159600 comparisons

1. Match the initial distribution

- Define classes and use their central values for comparisons



400 values
159600 pairwise comparisons



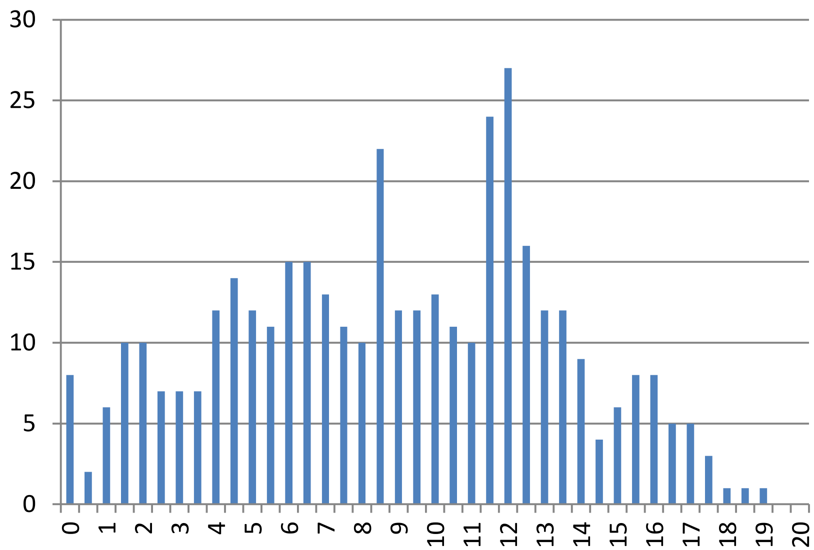
75 values
30000 pairwise comparisons
16400 profile comparisons

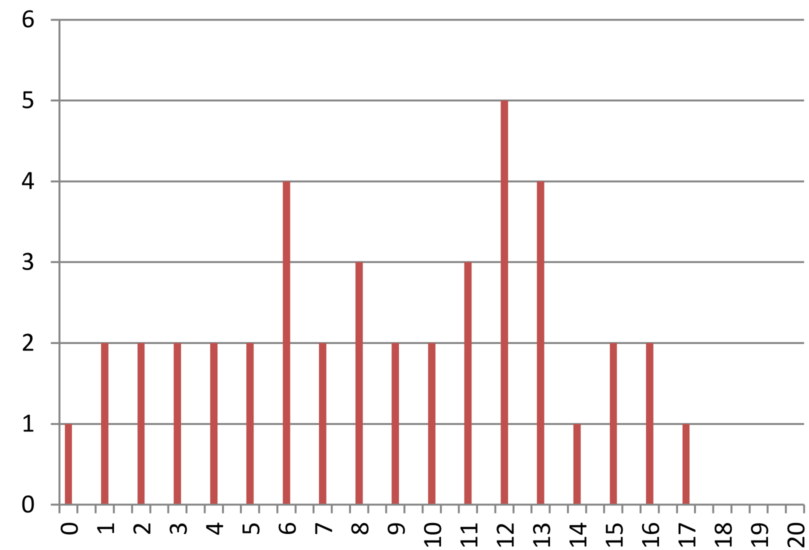
$$\varphi_k(a_i) = \frac{1}{n-1} \sum_{a_j \in A} \left[P_k(a_i, a_j) - P_k(a_j, a_i) \right]$$

$$\varphi_k^*(a_i) = \frac{1}{n_k^*} \sum_{a_j^* \in A_k} n_{jk} \left[P_k(a_i, a_j^*) - P_k(a_j^*, a_i) \right]$$

1. Match the initial distribution

- Define classes and use their central values for comparisons





400 values
159600 pairwise comparisons

42 values
16800 pairwise comparisons
5550 profile comparisons

$$\varphi_k(a_i) = \frac{1}{n-1} \sum_{a_j \in A} \left[P_k(a_i, a_j) - P_k(a_j, a_i)\right] \quad \varphi_k^*(a_i) = \frac{1}{n_k^*} \sum_{a_j^* \in A_k} n_{jk}\left[P_k(a_i, a_j^*) - P_k(a_j^*, a_i)\right]$$

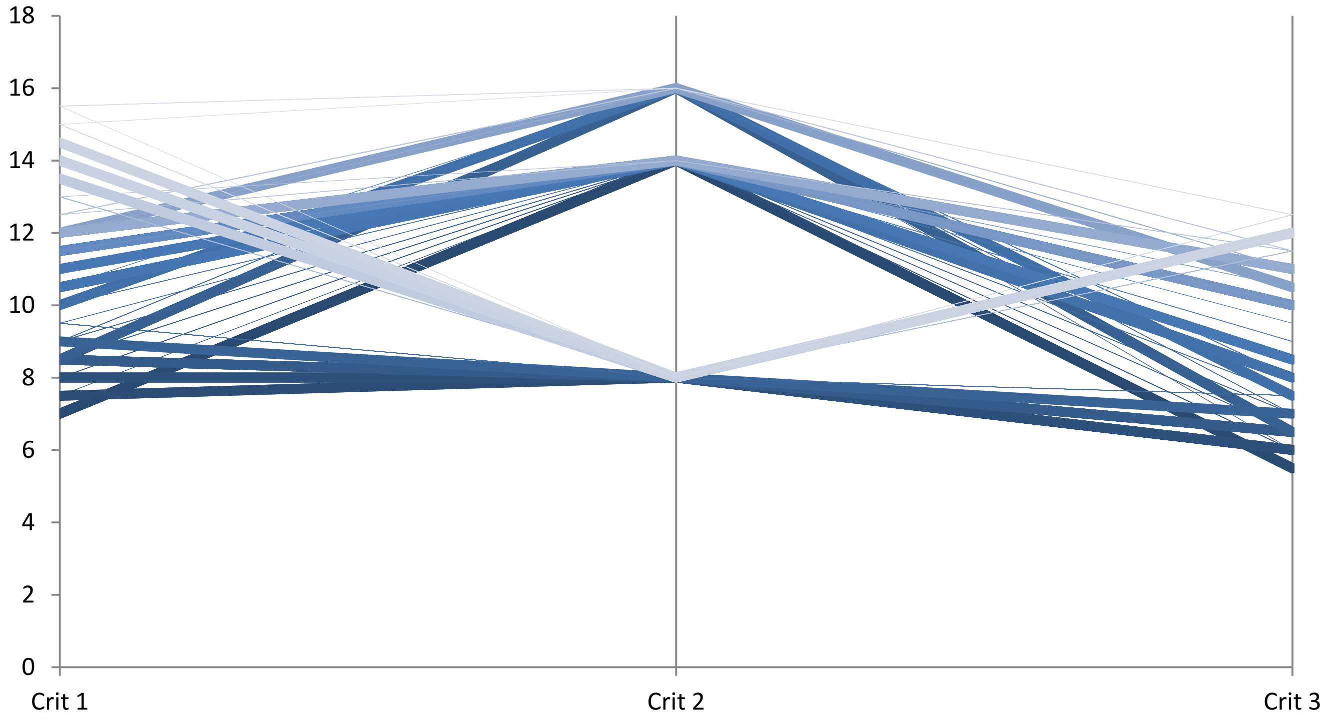
1. Match the initial distribution

- Several techniques to define the classes to be used
  - Ask the decision maker
  - Pearson, Sturges' rule
  - …

- Be wary of the drawback of having too few classes
  - Results may become less accurate

2. Separate analysis per criterion

- Different distributions

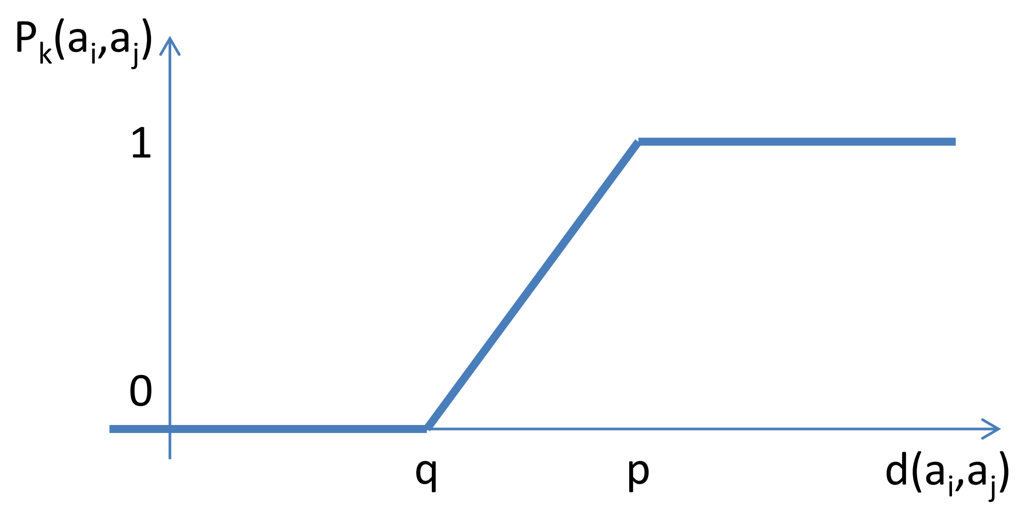


2. Separate analysis per criterion

- Different analysis for each criterion
- Different times for each processing
- Having the input of the decision maker or of an expert might help

3. Use preference functions

- The preference functions (if available) can help us define the classes of values
- By using the indifference area, we can determine an adequate size that would rarely change the results



4. Random sampling

- This approach relies on having the entire dataset available

- If the number of alternatives is so high that the analysis would take too long…
  - Apply the analysis on a randomised subset of the problem

… what are we even trying to do?

- However is such problems, a ranking is less likely to be useful

- Ordered classification might be preferred:
  - FlowSort
  - …

Conclusions

- Defining smaller sets of values for each criterion greatly helps in reducing computation times
  - Depend on the number of alternatives
- The approximated results are often close to the actual ones
  - Unicriterion net flows
- Additional simulations are needed to assess the quality on different examples
  - Numbers of classes, global profiles
  - Preference functions

Conclusions

- This approach supposes that we are in the same conditions as for the PROMETHEE II method
  - No uncertainty
  - No missing values

- If this is not the case, another method would be advised