GEORG-AUGUST-UNIVERSITÄT GÖTTINGEN

The application of PROMETHEE with Prospect Theory - Opportunities and Challenges

1. Integration of Prospect Theory into PROMETHEE
2. Feedback from decision makers in a case study concerning sustainable bioenergy
3. Extensions: sensitivity analysis and integration of scenario planning
4. Summary

2nd International MCDA Workshop on PROMETHEE: Research and Case Studies, 23.01.2015, Vrije Universiteit Brussel - Université libre de Bruxelles, Belgium

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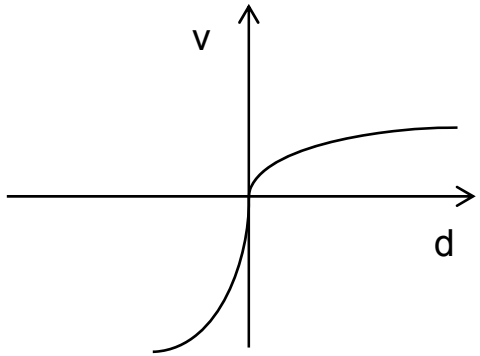
Prospect Theory

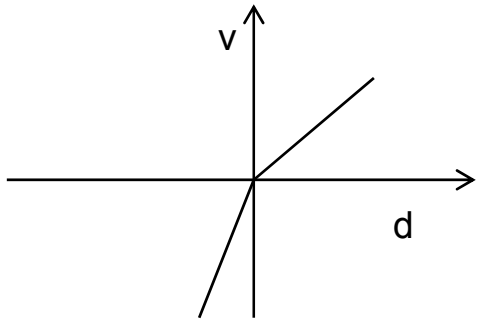
Findings of Prospects Theory:

- Reference dependency
- Division into gains and losses
- Humans show loss aversion
- Diminishing sensitivity
- Existence of so-called decision weights

S-shape value function



Piecewise linear value function

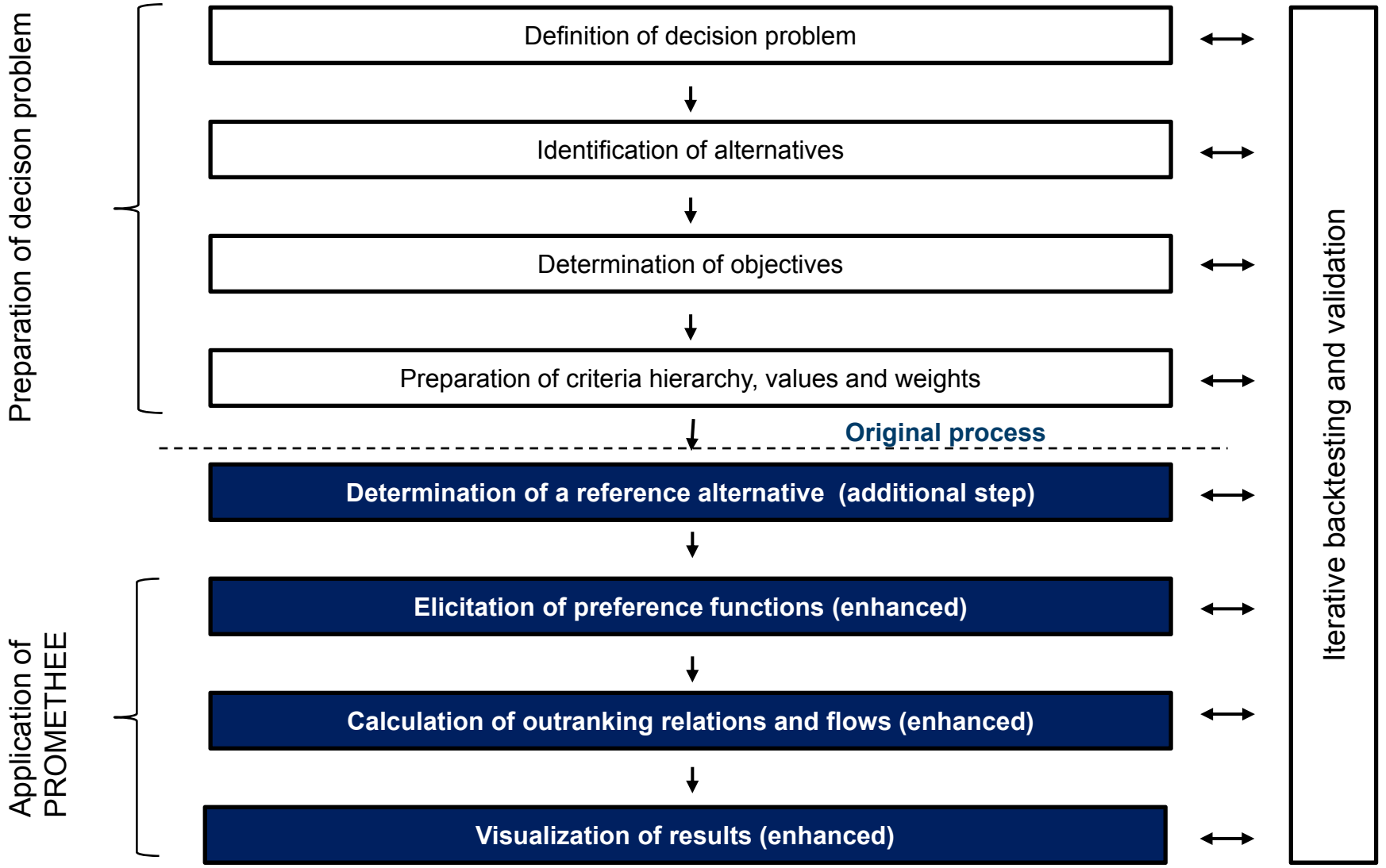


Source: Kahneman, D.; Tversky, A. (1979); Korhonen et al. (1990)

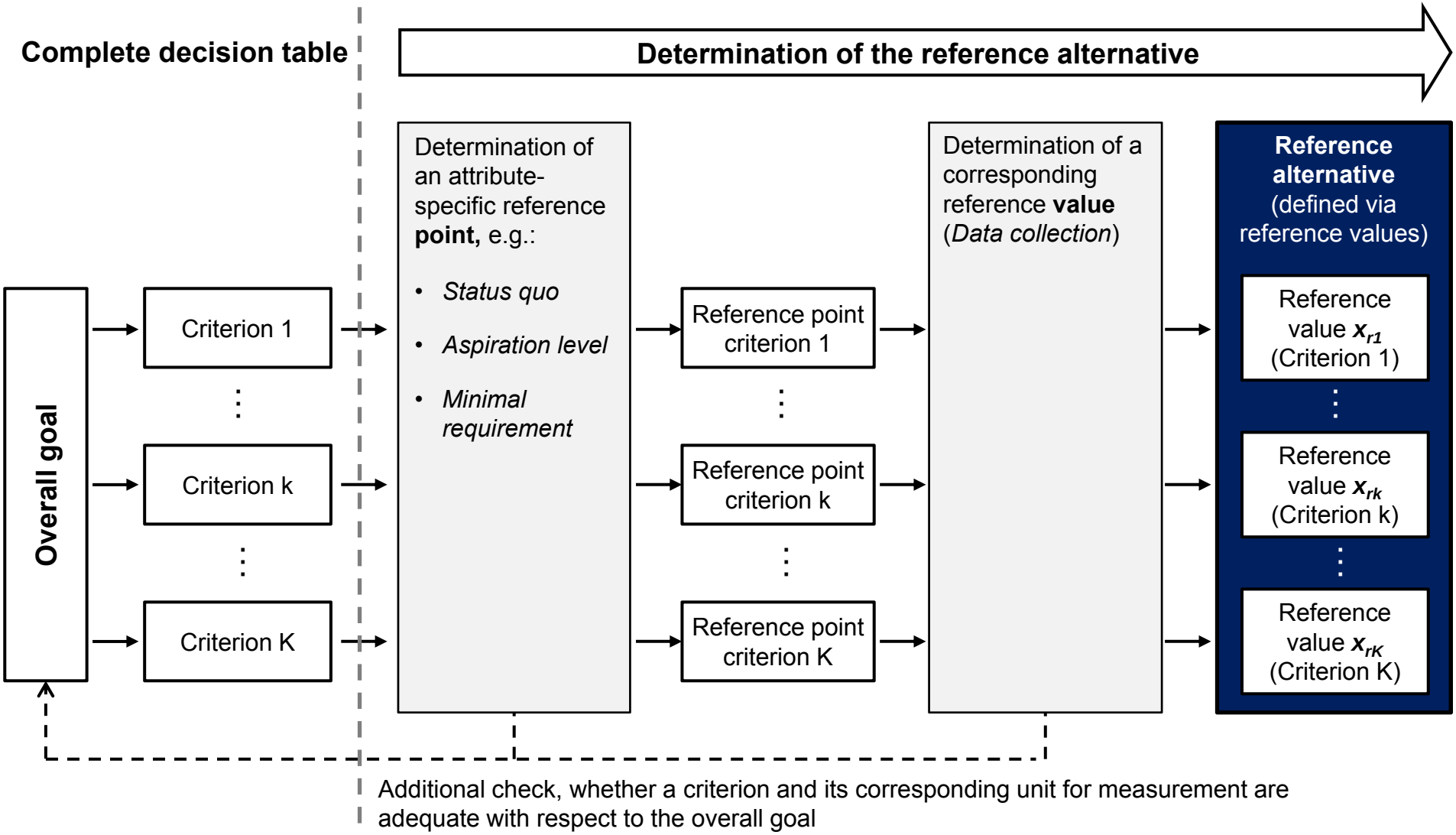
Existing research on the consideration of Prospect Theory within MCDA

Source	Content
Korhonen et al. (1990)	Interactive methods; Decision behaviour as described in prospect theory
Salminen, Wallenius (1993)	Interactive methods; Decision behaviour as described in prospect theory
Bleichrodt et al. (2009)	Attribute-specific definition of reference; Adjustment of MAUT about elements from prospect theory
Gomes, Lima (1991)	New method TODIM; Combination of elements from european and american school
Gomes, Gonzalez (2012)	Integration of cumulative prospect theory into TODIM
Bozkurt (2007)	Integration of prospect theory into PROMETHEE; Changing reference alternatives
Wang, Sun (2008)	Division of outcomes into gains and losses through integration of trapezoidal-shaped membership functions from Fuzzy theory into preference function into PROMETHEE

Process of PROMETHEE with Prospect Theory



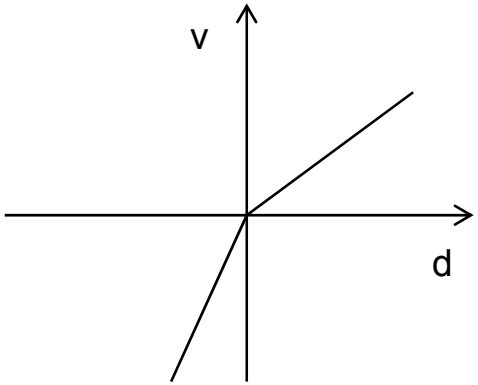
Determination of a reference alternative



Elicitation of preference functions

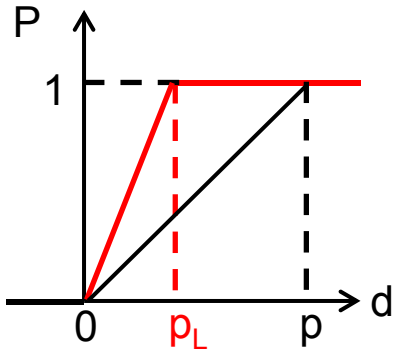
Transfer of parameter λ for loss aversion into PROMETHEE:

Prospect Theory
(piecewise linear)



$$v(d) = \begin{cases} d & d \geq 0 \\ \lambda \cdot d & d < 0 \end{cases}$$

Enhanced
PROMETHEE
(e.g. Type 3)



$$P_L(d) = \begin{cases} 0 & d \leq 0 \\ \frac{d \cdot \lambda}{p} & 0 < d \leq \frac{p}{\lambda} \\ 1 & d > \frac{p}{\lambda} \end{cases}$$

Derivation of threshold p_L

$$p \cdot m = 1 \cap p_L \cdot m \cdot \lambda = 1$$

$$p \cdot m = p_L \cdot m \cdot \lambda$$

$$p = p_L \cdot \lambda$$

$$p_L = \frac{p}{\lambda}$$

Kahneman, Tversky (1979) and Korhonen et al. (1990)

The determination of λ is difficult

Approach: Transfer of linguistic statements to quantitative factors using results of experiments

Within several experiments a range from 1.5 – 4 with a mean between 2 and 2.6 has been identified

Linguistic Scale	Quantitative Scale
Contrary effect (risk seeking)	0.5
No loss aversion	1
Very slightly loss averse	1.5
Slightly loss averse	2
Loss averse	2.5
Strongly loss averse	3
Very strongly loss averse	3.5
Losses almost unacceptable	4

Source: Tversky, Kahneman (1992) and Abdellaoui et al. (2008)

PROMETHEE

(1) Definition of a preference-function $p_k(d)$ for each criterion *i* based on the difference $d = g_i(a) - g_i(a')$ between criteria-values of alternatives *a* and *a'*

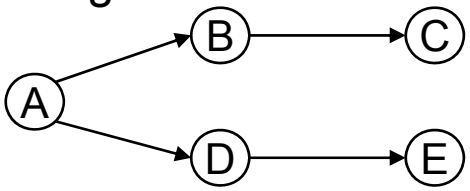
(2) Determination of Outranking-Relation using pairwise comparions:

$$\pi(a,a') = \sum_{i=1}^{K} w_i \cdot P_i(gi(a) - gi(a'))$$

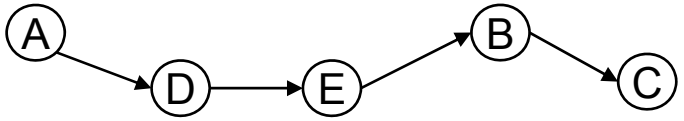
(3) Calculation of outflow ϕ^+ and inflow ϕ^- :

$$\Phi^+(a) = \frac{1}{n-1} \cdot \sum_{j=1}^{n} \pi(a,a') \qquad \Phi^-(a) = \frac{1}{n-1} \cdot \sum_{j=1}^{n} \pi(a',a)$$

(4) Determination of partial ranking:



(5) Determination of complete ranking (Based on Netflow: $\Phi(a) = \Phi^+(a) - \Phi^-(a)$) :



Brans et al. (1986)

Calculation of outranking relations and flows with Prospect Theory

Formulas for calculation of outranking relations:

$$\pi(a,a') = \sum_{i=1}^{K} w_i \cdot P_i\,(g_i(a) - g_i(a'))$$

Pairwise comparisons between normal alternatives

$$\pi(a,a_r) = \sum_{i=1}^{K} w_i \cdot P_i\,(g_i(a) - g_i(a_r))$$

Potential gains

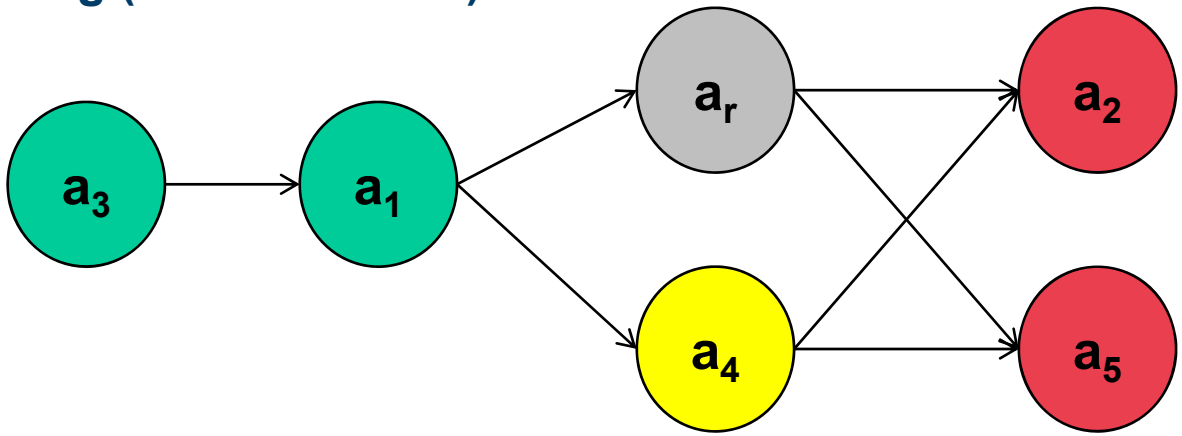
$$\pi(a_r,a) = \sum_{i=1}^{K} w_i \cdot \mathbf{P_{Li}}\,(g_i(a_r) - g_i(a))$$

Potential losses

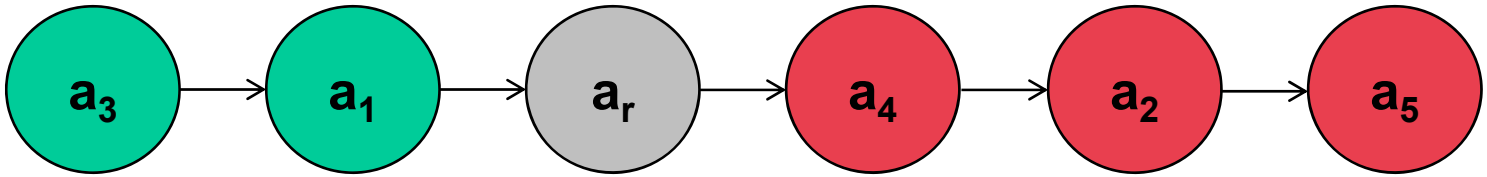
→ **The underlying procedure of the determination of out- and inflows remains unchanged**

Visualization of results (example)

Partial ranking (PROMETHEE I):



Complete ranking (PROMETHEE II):



$a_1,\ldots,a_5$ = real Alternatives (selectable)
a_r = Reference alternative (ficticious)

Case study: Evaluation of bioenergy concepts

Objective:

Identification of a sustainable concept for an energetic use of biomass on a regional scale

Alternatives:

1. Large-scale biogas plant (LBP)
2. Bionenergy village (BEV)
3. Small-scale biogas plant (SBP)

Data is provided by the project: **“Sustainable use of bioenergy: bridging climate protection, nature conservation and society”** funded by the “Ministry of Science and Culture of Lower Saxony” with a duration from 2009 – 2014.

Data: Eigner-Thiel et al. 2013

Case study – Procedure

Determination of a reference alternative and loss aversion parameters based on an already developed decision table:

- Interviews with three experts
- Determination of a reference point and reference value for each criterion (39 criteria)

Selection of criteria and corresponding data from the extended decision table:

Criterion	Unit	Min/ Max	LBP	BEV	SBP	a_r	λ
Global warming potential	CO_2-Eq./ha	Min	-4,937	-12,724	-13,734	0	4
Fertilizer nitrogen - biodiversity	kg N/ ha	Min	148	150	147	60	0.5
Participation	Points	Max	2	5	1	6	1.5

Case study – Results

Outranking-relations and flows:

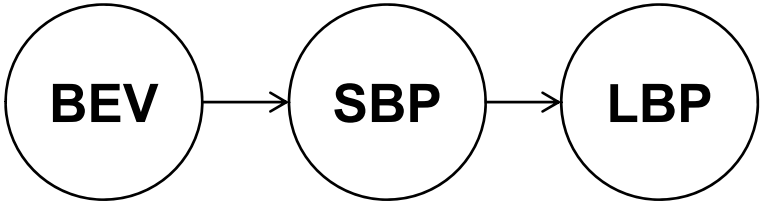
	LBP	BEV	SBP	a_r	Φ^+
LBP	0	0,137	0,139	0,240	**0,172**
BEV	0,703	0	0,504	0,341	**0,516**
SBP	0,432	0,218	0	0,262	**0,304**
a_r	0,596	0,270	0,399	0	**0,422**
Φ^-	**0,577**	**0,208**	**0,347**	**0,281**	

Normal pairwise comparisons (no frame):
Calculation using P (d)

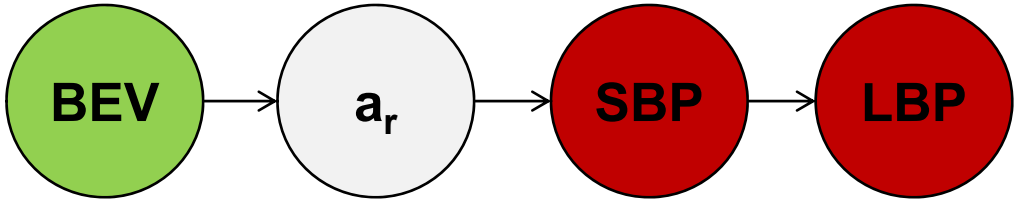
Potential Gains:
Calculation using P (d)

Potential Losses:
Calculation using P_L (d)

Original rankings:



Modified rankings:



Observations and feedback from decision makers – Determination of the reference alternative

Opportunities and advantages:

- Defining the reference values draws the attention steadily on the overall goal
- Some adjustment of criteria and/or corresponding units for measurement occurred
- Additional information, especially from the rankings, can be gained

Challenges and disadvantages:

- Formulating reference values for qualitative criteria is difficult
- Sometimes reference values are chosen very ambitious

Observations and feedback from decision makers – Determination of loss aversion

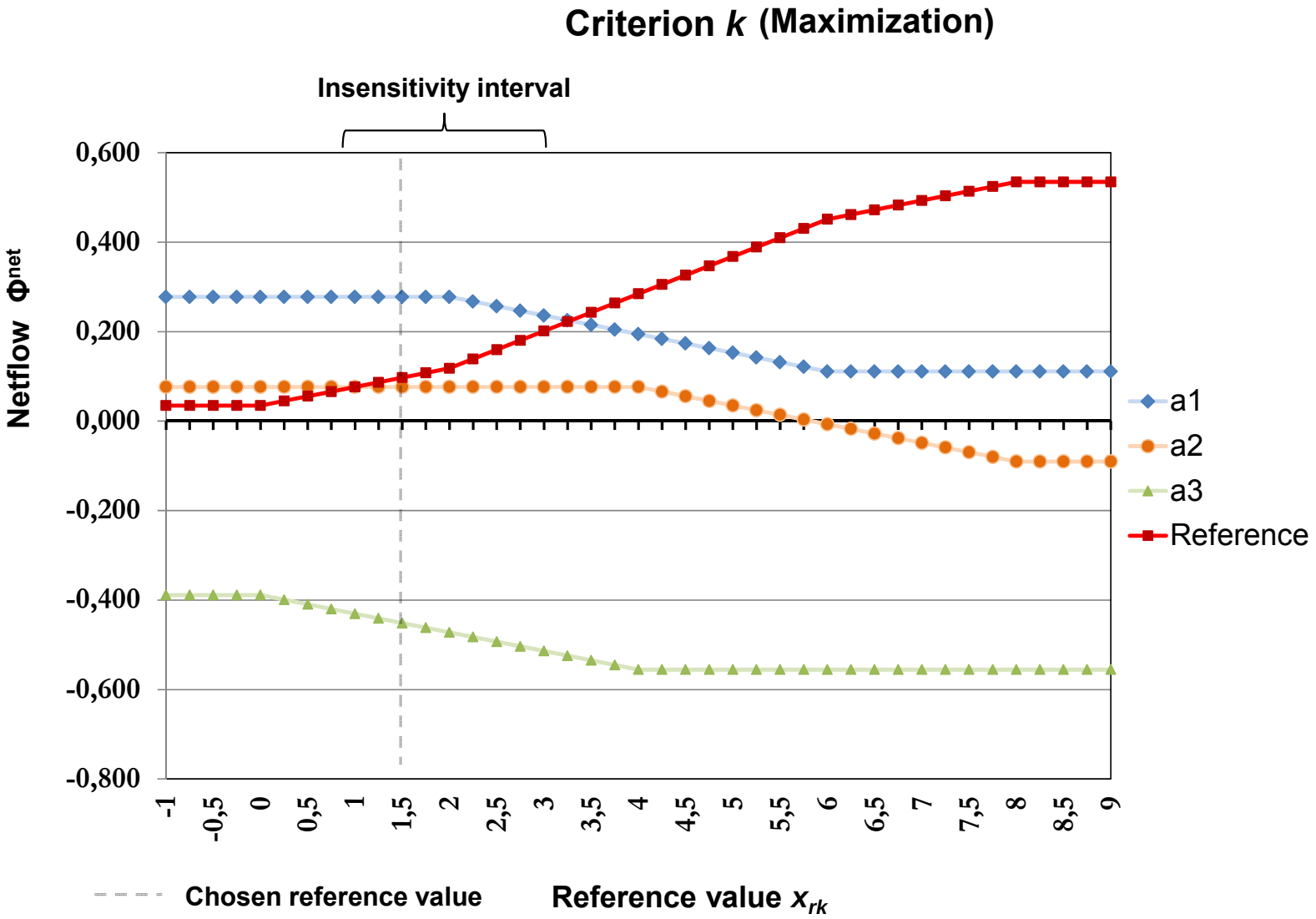
Opportunities and advantages:

- The experts were able to express for each criterion if loss aversion exist or not.
- A λ-value different to one occurs (existence of loss aversion) for most criteria.
- The concept of using a lingusitic scale was well understood and appreciated.
- All experts wanted to express also the contrary effect to loss aversion.

Challenges and disadvantages:

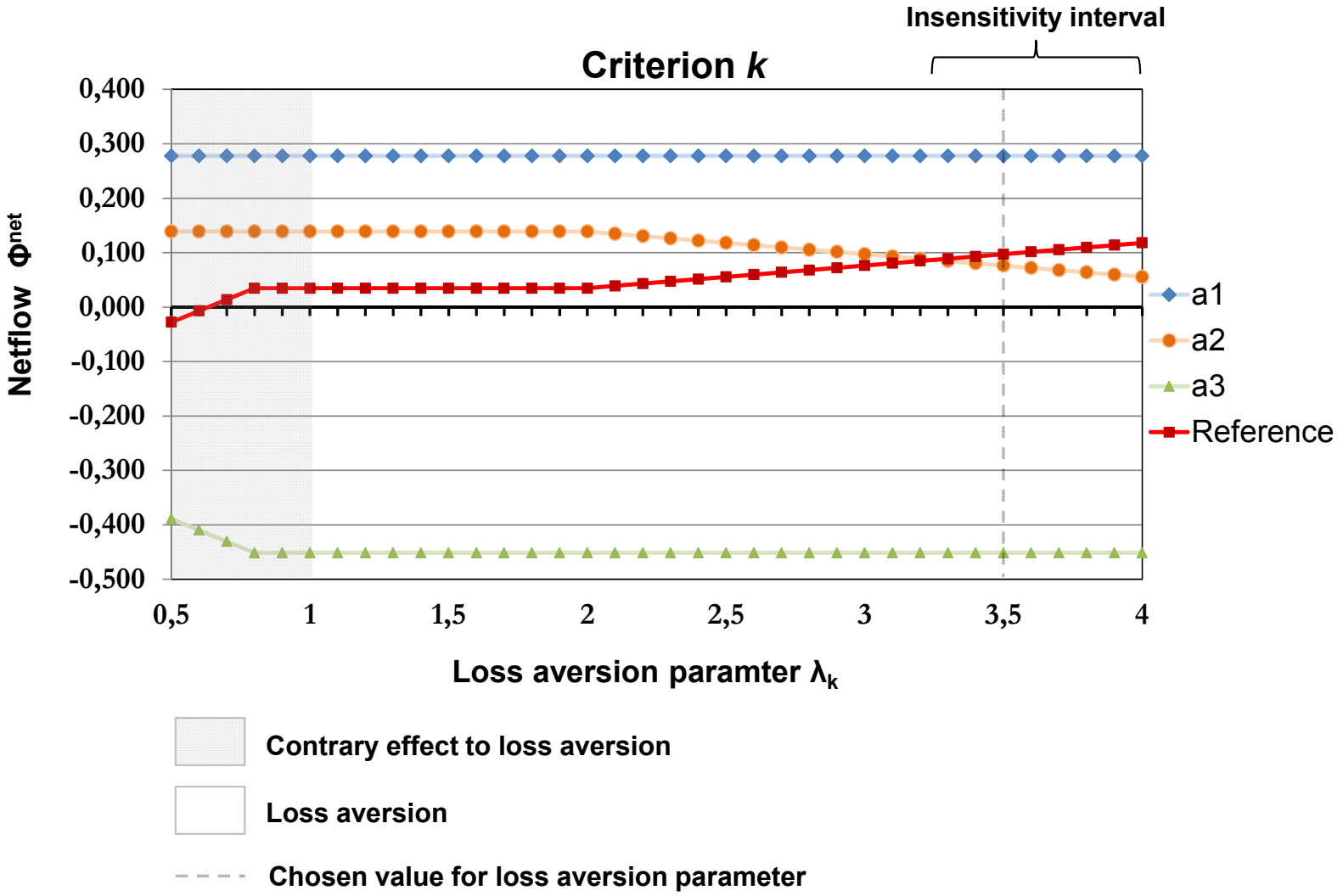
- Cognitively more challenging compared to defining the reference alternative.
- The underlying quantitative scale can differ between humans.

Sensitivity analysis for reference values - Analysed range in orientation on reference points or existing values



x_{1k}	4
x_{2k}	6
x_{3k}	2
x_{rk}	1.5
Function Type 3	
p_k	2
p_{Lk}	2
λ_k	1

Sensitivity analysis for loss aversion parameter λ - Analysed range in orientation on the underlying quantitative scale



x_{1k}	2
x_{2k}	4
x_{3k}	6
x_{rk}	3.5
Function Type 5	
p_k	2
p_{Lk}	0,57
q_k	1
q_{Lk}	0,29
λ_k	3,5

The consideration of external uncertainty by scenario planning

- No consideration of probabilities
- Evaluation via robustness instead of inter-scenario aggregation of values
- Separate application of PROMETHEE for each scenario offers several advantages:
  - Scenario-specific weights, loss aversion parameters and/ or reference values
  - European school
  - Less cognitvely challenging for decision makers

Stewart et al. 2013; Montibeller et al. (2006)

Summary

- Integration of Prospect theory into PROMETHEE offers the opportunity for the decision maker to express loss aversion and to consider reference dependency.
- Gaining additional information through the determination of adequate reference values.
- The opportunity to express loss aversion was appreciated by the experts and occurred with respect to the most criteria.
- Scenario planning is a good approach to address external uncertainties
- Further applications are needed for validation.

Literature

Bleichrodt, H.; Schmidt, Ul.; Zank, H. (2009): Additive Utility in Prospect Theory, Management Science, Vol. 55, H. 5, S. 863 - 873

Bozkurt, A. (2007): Multi-Criteria Decision Making with Interdependent Criteria Using Prospect Theory, Middle East Technical University, Ankara 2007

Brans, J.P.; Vincke, P.; Mareschal, B. (1986): How to Select and Rank Projects: The PROMETHEE Method, European Journal of Operational Research, 24, 228-238

Gomes, L.F.A.M.; Lima, M.M.P.P. (1991): TODIM: Basics and Application to Multicriteria Ranking of Projects with Environmental Impacts, in: Foundations of Computing and Decision Sciences, Vol. 16, H. 3-4, S. 114 – 127

Gomes, L.F.A.M.; Gonzalez, X.I. (2012): Behavioral Multi-Criteria Decision Analysis: Further Elaborations on the TODIM Method, in. Foundations of Computing and Decision Sciences, Vol. 37, H. 1, S. 3 - 8

Kahneman, D.; Tversky, A. (1979): Prospect Theory: An Analysis of Decision Under Risk, Econometrica, 47, 263-292

Korhonen, P.; Moskowitz, H.; Wallenius, J. (1990): Choice Behaviour in Interactive Multiple-Criteria Decision Making, Annals of Operations Research, 23, 161-179

Montibeller, G.; Gummer, H.; Tumidei, D. (2006): Combining Scenario Planning and Multi-Criteria Decision Analysis in Practice, in: Journal of Multi-Criteria Decision Analysis, Vol. 14, S. 5 - 20

Oberschmidt, J. (2010): Multikriterielle Bewertung von Technologien zur Bereitstellung von Strom und Wärme, Fraunhofer Verlag, ISI-Schriftenreihe Innovationspotenziale, Karlsruhe 2010

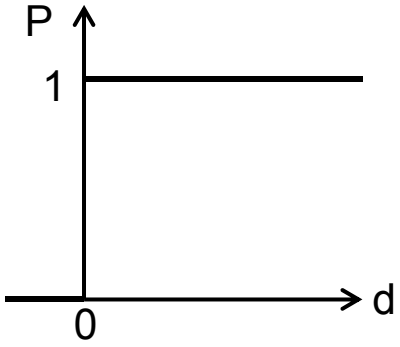
Salminen, P.; Wallenius, J. (1993): Testing Prospect Theory in a Deterministic Multiple Criteria Decision-Making Environment, in: Decision Sciences, Vol. 24, H. 2; S. 279 – 292

Stewart, T.J.; French, S.; Rios, J. (2013): Integrating multicriteria decision analysis and scenario planning – Review and extension, Omega Vol. 41, S. 679 - 688

Wang, J.Q.; Sun, T. (2008): Fuzzy Multiple Criteria Decision Making Method Based on Prospect Theory, in: Tagungsband 2008 International Conference on Information Management, Innovation Management and Industrial Engineering, S. 228 - 291

All six preference functions of PROMETHEE with loss aversion (1/2)

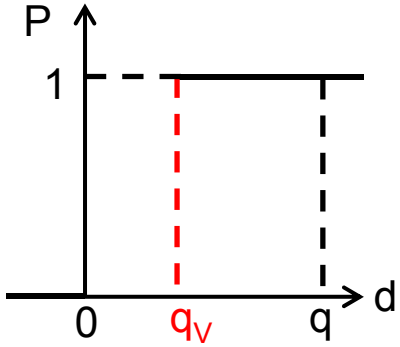
Type1*:* Usual criterion



$$P(d) = \begin{cases} 0 & d \le 0 \\ 1 & d > 0 \end{cases}$$

Loss function identical

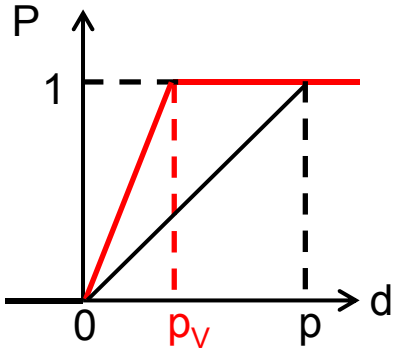
Type 2: Quasi-criterion



$$P(d) = \begin{cases} 0 & d \le q \\ 1 & d > q \end{cases}$$

$$P_V(d) = \begin{cases} 0 & d \le \frac{q}{\lambda} \\ 1 & d > \frac{q}{\lambda} \end{cases}$$

Type 3: Criterion with linear preference

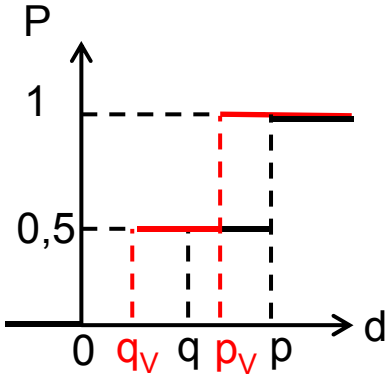


$$P(d) = \begin{cases} 0 & d \le 0 \\ \frac{d}{p} & 0 \le d \le p \\ 1 & d > p \end{cases}$$

$$P_V(d) = \begin{cases} 0 & d \le 0 \\ \frac{d \cdot \lambda}{p} & 0 < d \le \frac{p}{\lambda} \\ 1 & d > \frac{p}{\lambda} \end{cases}$$

All six preference functions of PROMETHEE with loss aversion (2/2)

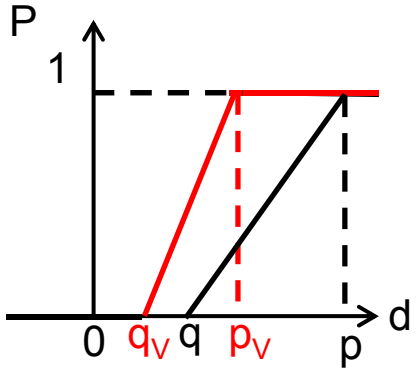
Type 4: Level criterion



$$P(d) = \begin{cases} 0 & d \le q \\ \frac{1}{2} & q < d \le p \\ 1 & d > p \end{cases}$$

$$P_V(d) = \begin{cases} 0 & d \le \frac{q}{\lambda} \\ 0{,}5 & \frac{q}{\lambda} < d \le \frac{p}{\lambda} \\ 1 & d > \frac{p}{\lambda} \end{cases}$$

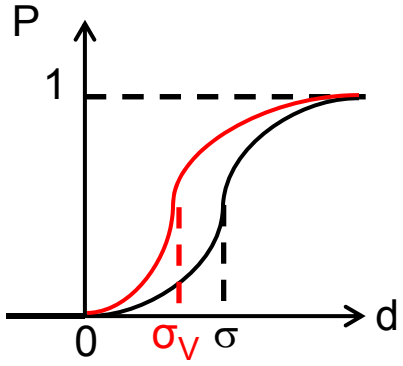
Typ 5: Criterion with linear Preference and indifferencearea



$$P(d) = \begin{cases} 0 & d \le q \\ \frac{d-q}{p-q} & q < d \le p \\ 1 & d > p \end{cases}$$

$$P_V(d) = \begin{cases} 0 & d \le \frac{q}{\lambda} \\ \frac{d \cdot \lambda - q}{p-q} & \frac{q}{\lambda} < d \le \frac{p}{\lambda} \\ 1 & d > \frac{p}{\lambda} \end{cases}$$

Typ 6: Gaussian criterion



$$P(d) = \begin{cases} 0 & d \le 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}} & d > 0 \end{cases}$$

$$P(d) = \begin{cases} 0 & d \le 0 \\ 1 - e^{-\frac{\lambda \cdot d^2}{2\sigma^2}} & d > 0 \end{cases}$$