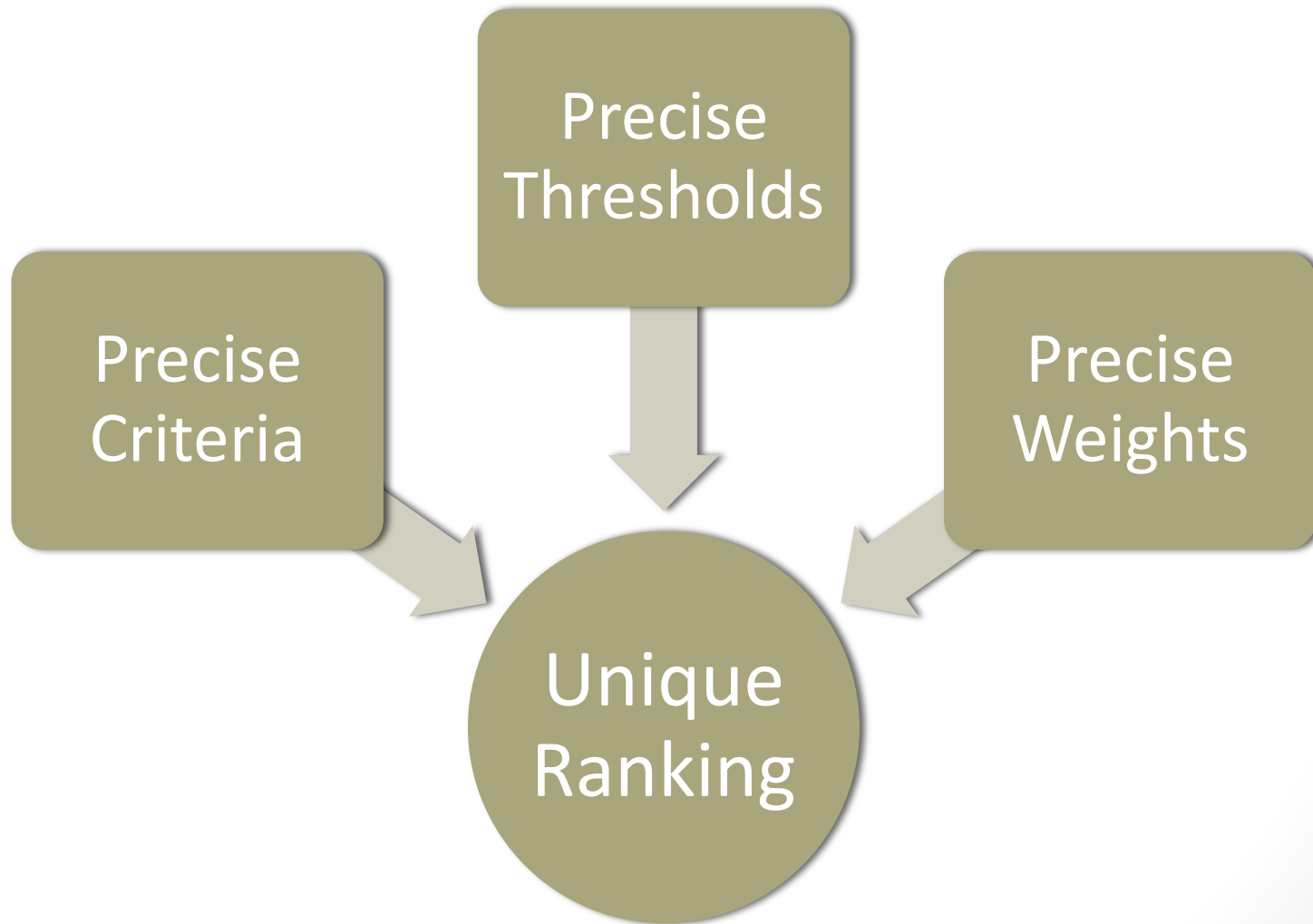# SMAA-GAIA: A Complementary Tool of the SMAA PROMETHEE Method

Hubinont J.P., DE SMET Y., LIDOUH K.,
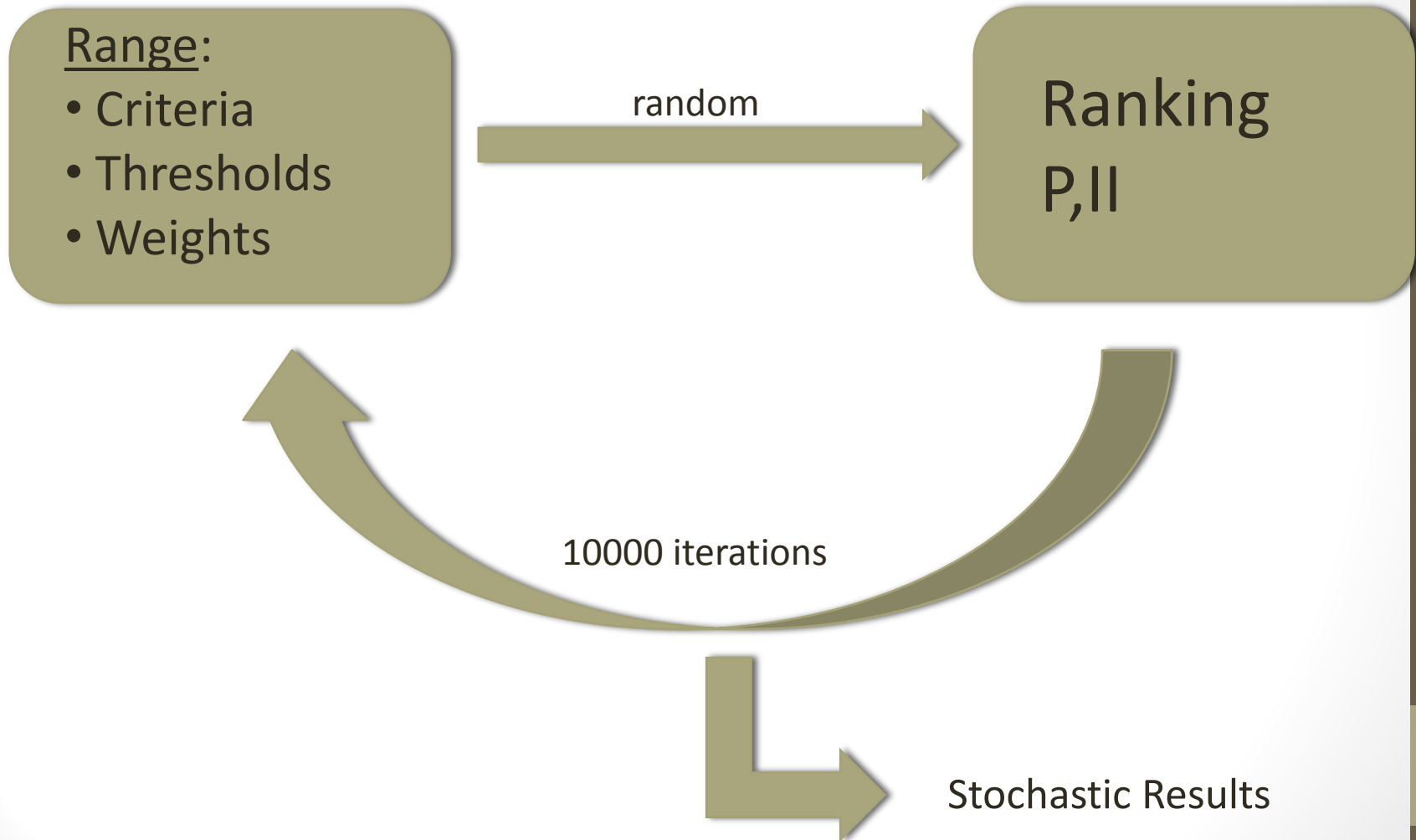
Université Libre de Bruxelles, Ecole Polytechnique, CoDE, SMG unit.

2nd International MCDA Workshop on PROMETHEE : Research and Case Studies
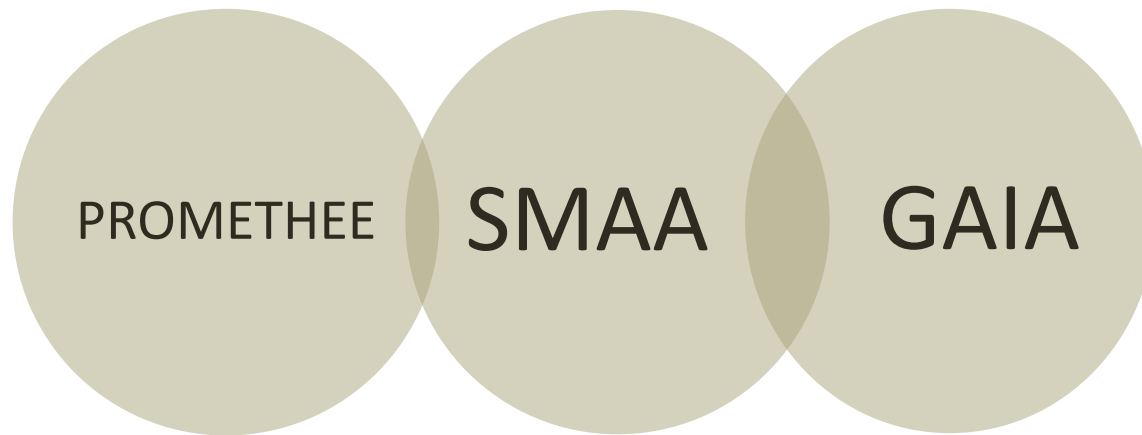
PROMETHEE II

Precise Thresholds

Precise Criteria

Precise Weights

Unique Ranking

SMAA-PROMETHEE[1]

Range:
- Criteria
- Thresholds
- Weights

random →

Ranking
P,II

10000 iterations

Stochastic Results

Stakes

PROMETHEE SMAA GAIA

* SMAA : Stochastic Multicriteria Acceptability Analysis

PROMETHEE II

Table 1: Performance table: the cars - PROMETHEE II

	Cars	Price [Euro]	Acceleration from 0 to 100 kilometer/hour	Max speed [km/h]	Consumption [l/100 km]
PE	PEUGEOT 208 1.6 8 V	16295	10.57	183.35	3.09
CI	CITROEN C3	15800	13.39	162.85	3.92
FI	FIAT 500 0.9	15230	11.08	172.53	4.15
SK	SKODA Fabia 1.2	15250	14.09	170.21	3.89
LA	LANCIA Ypsilon 5p	16170	11.77	183.87	3.84
q_j		773	2.52	30	0.22
p_j		1778	4.02	40	0.5
w_j		0.52	0.14	0.06	0.27

Car	Net Flow	Rank
PE	0.1684	1
CI	-0.0587	4
FI	-0.0538	3
SK	0.0314	2
LA	-0.0873	5

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Results of SMAA-PROMETHEE[1]

	b_j^1	b_j^2	b_j^3	b_j^4	b_j^5
(a) Rank acceptability indices (in percentage)					
PE	1	5	12	23	60
CI	9	31	32	20	8
FI	12	36	26	18	8
SK	76	16	6	2	0
LA	2	12	24	38	25

Cars	w_P	w_A	w_M	w_C
(b) Central weight vectors				
PE	0.46	0.14	0.06	0.33
CI	0.49	0.15	0.06	0.29
FI	0.54	0.13	0.05	0.26
SK	0.52	0.14	0.06	0.26
LA	0.49	0.13	0.06	0.30

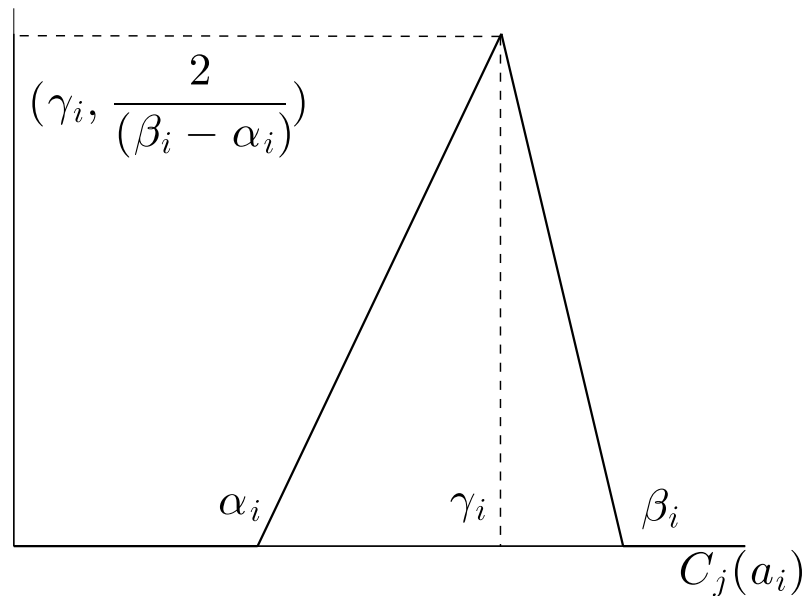| w_P	w_A	w_M	w_C
(c) Center of mass			
0.52	0.14	0.06	0.27

*b_j^k = probability that alternative j is preferred to k other alternatives.

[1] S.Corrente , J.R. Figueira, and S. Greco. The smaa-promethee method. *European Journal of Operational Research,* 239(2):514-522,2014
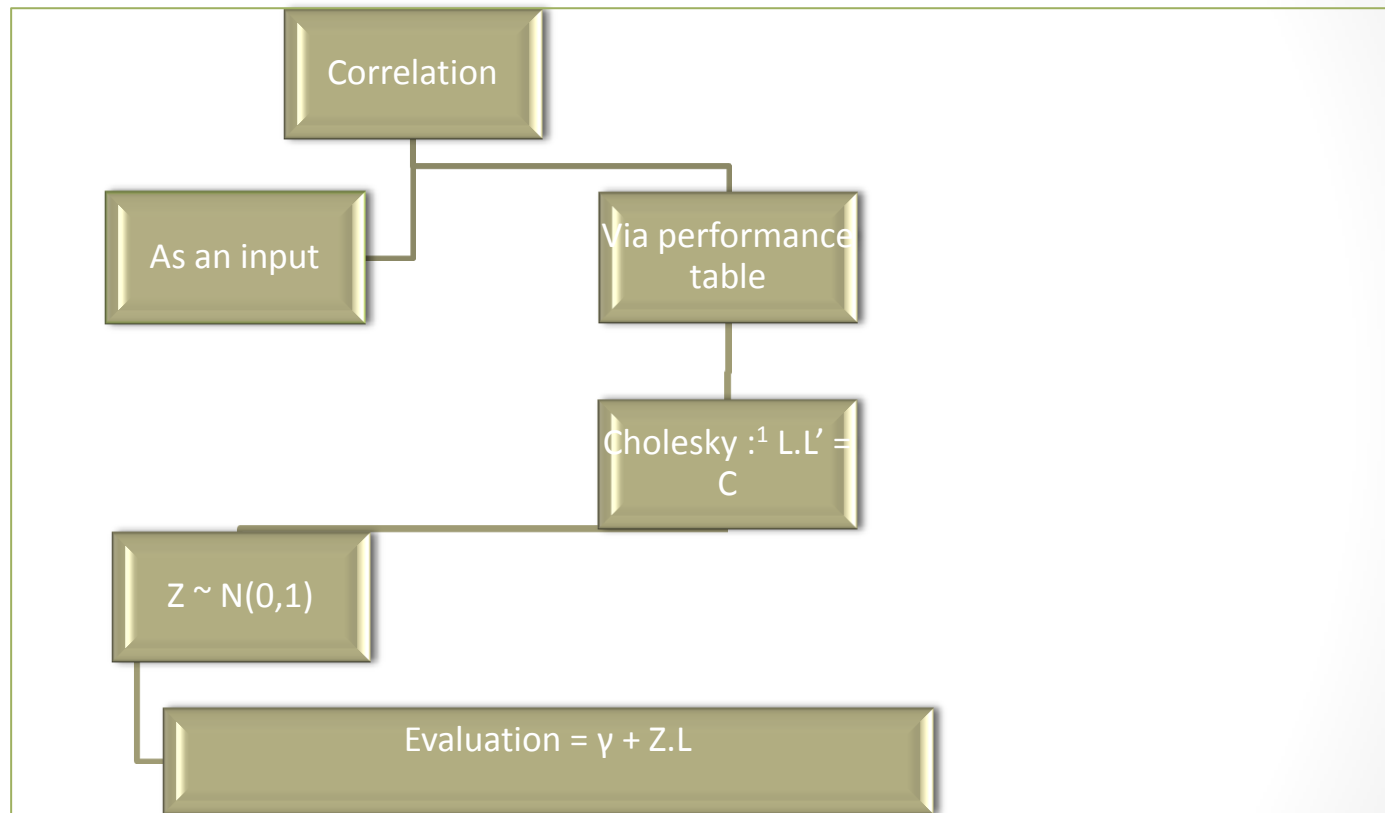
Generating Random Evaluation(1)

•Uniform Repartition:

•Triangular Repartition:

$(\gamma_i, \dfrac{2}{(\beta_i - \alpha_i)})$

$\alpha_i \qquad \gamma_i \qquad \beta_i$

$C_j(a_i)$

Generating random Evaluations(2)

[1]M. Haugh. The Monte Carlo framework, example from finance and generating correlated random variables, 2004 :
http://www.columbia.edu/~mh2078/MCS04/MCS_framework_FEegs.pdf, aout 2014.

SMAA-PROMETHEE, exemple

	Cars	Price [Euro]	Acceleration from 0 to 100 kilometer/hour	Max speed [km/h]	Consumption [l/100 km]
	PE	[16000, 16295, 18000]	[9.5, 10.57, 11.5]	[175, 183.35, 190]	[2.8, 3.09, 4.5]
	CI	[15000, 15800, 16500]	[12.7, 13.39, 14.2]	[155, 162.85, 170]	[3.4, 3.92, 4.6]
	FI	[14500, 15230, 15800]	[10, 11.08, 12]	[165, 172.53, 180]	[3, 4.15, 5]
	SK	[14100, 15250, 15650]	[13.2, 14.09, 15.2]	[160, 170.21, 181]	[2.5, 3.89, 4.3]
	LA	[15500, 16170, 17100]	[10.6, 11.77, 12.8]	[175, 183.87, 191]	[3.2, 3.84, 4.4]
q_j		[500, 773, 1000]	[2, 2.52, 3]	[30, 30, 30]	[0, 0.22, 0.5]
p_j		[1500, 1778, 2000]	[3, 4.02, 5]	[40, 40, 40]	[0.5, 0.5, 0.5]
w_j		[0, 0.52, 1]	[0, 0.14, 1]	[0, 0.06, 1]	[0, 0.27, 1]

Cars	Worst * Ranking	Best* Ranking
PE	5	2
CI	5	2
FI	5	1
SK	3	1
LA	5	2

Cars	First Rank Probability
PE	4.9%
CI	2.8%
FI	8.0%
SK	80.7%
LA	2.6%

* With 95% of probability

SMAA-GAIA

Evaluations : range of values

Visualization on the GAIA Plane?
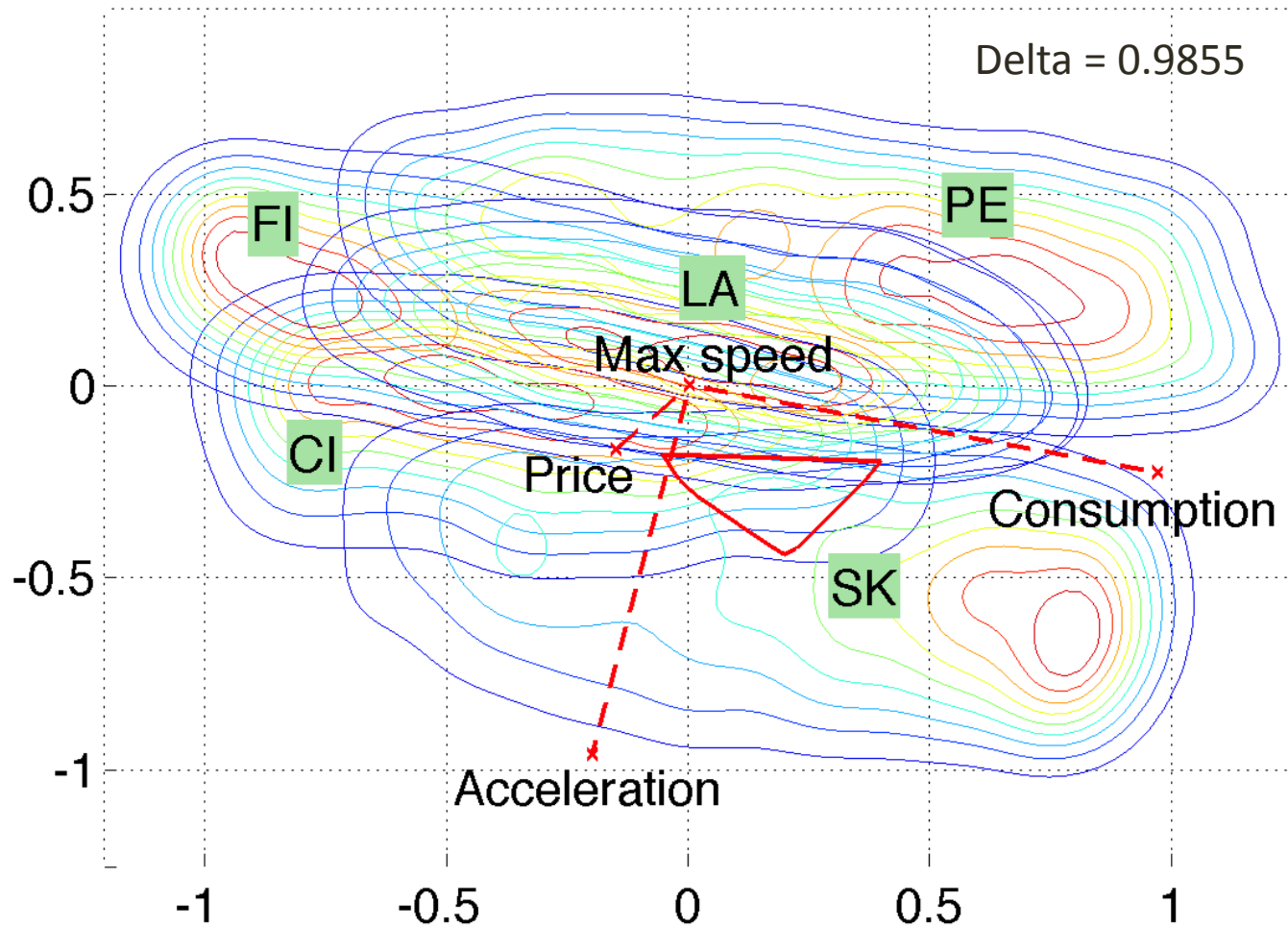
Net Flows: KDE*

Weights polygon
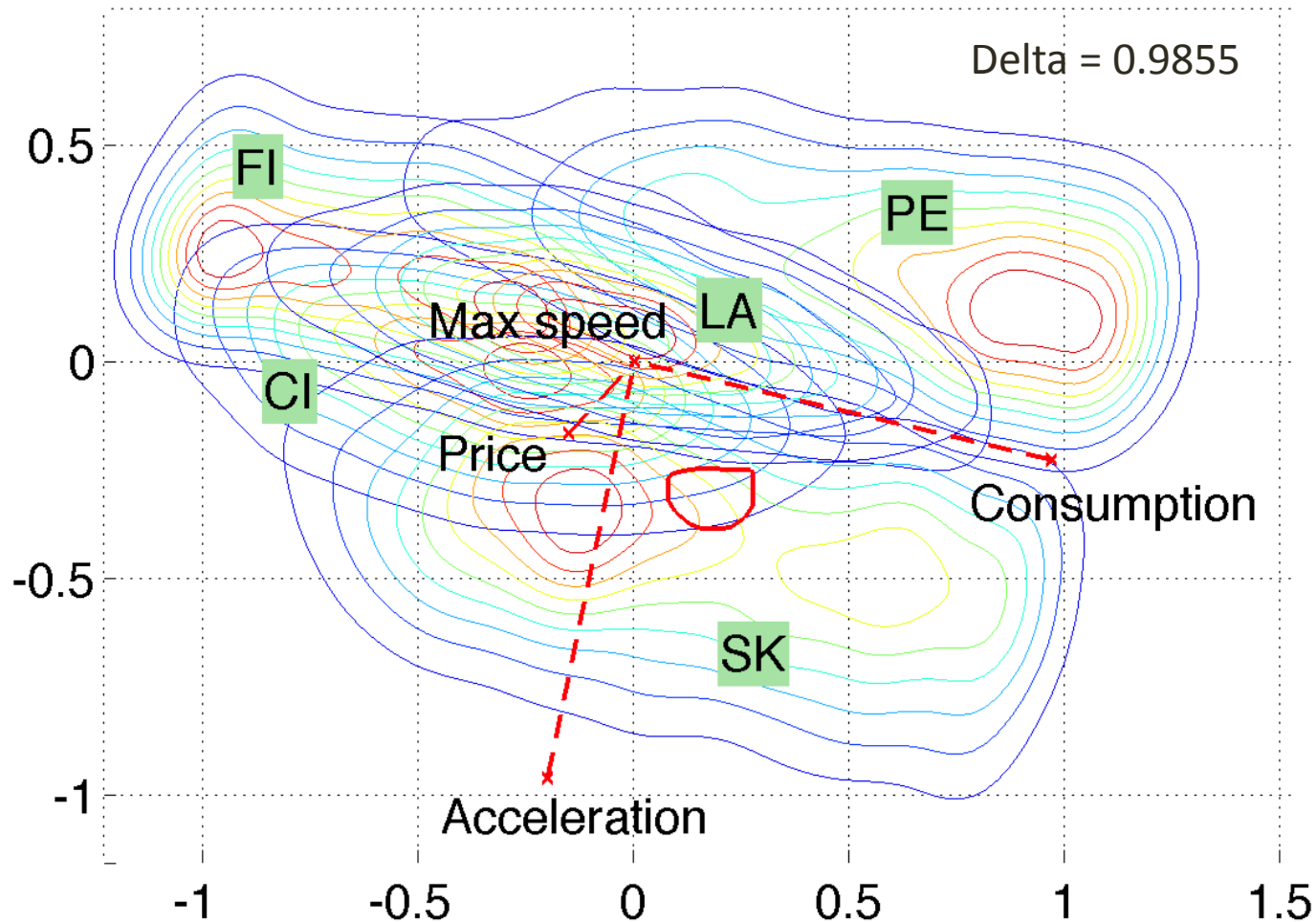
GAIA Plane built with the more probable values

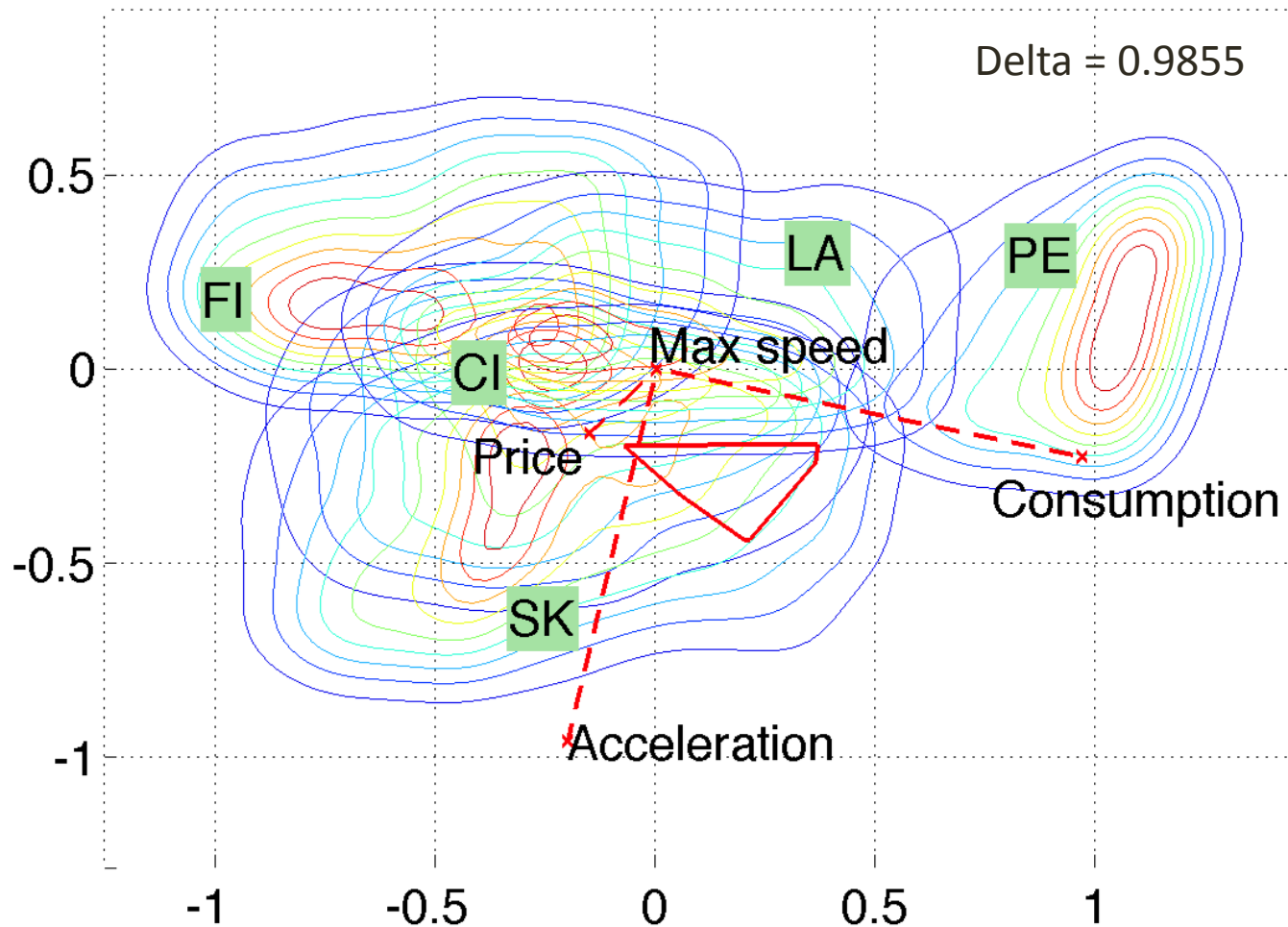*KDE : Kernel Density Estimation of the Stochastic net flows found with SMAA-PROMETHEE
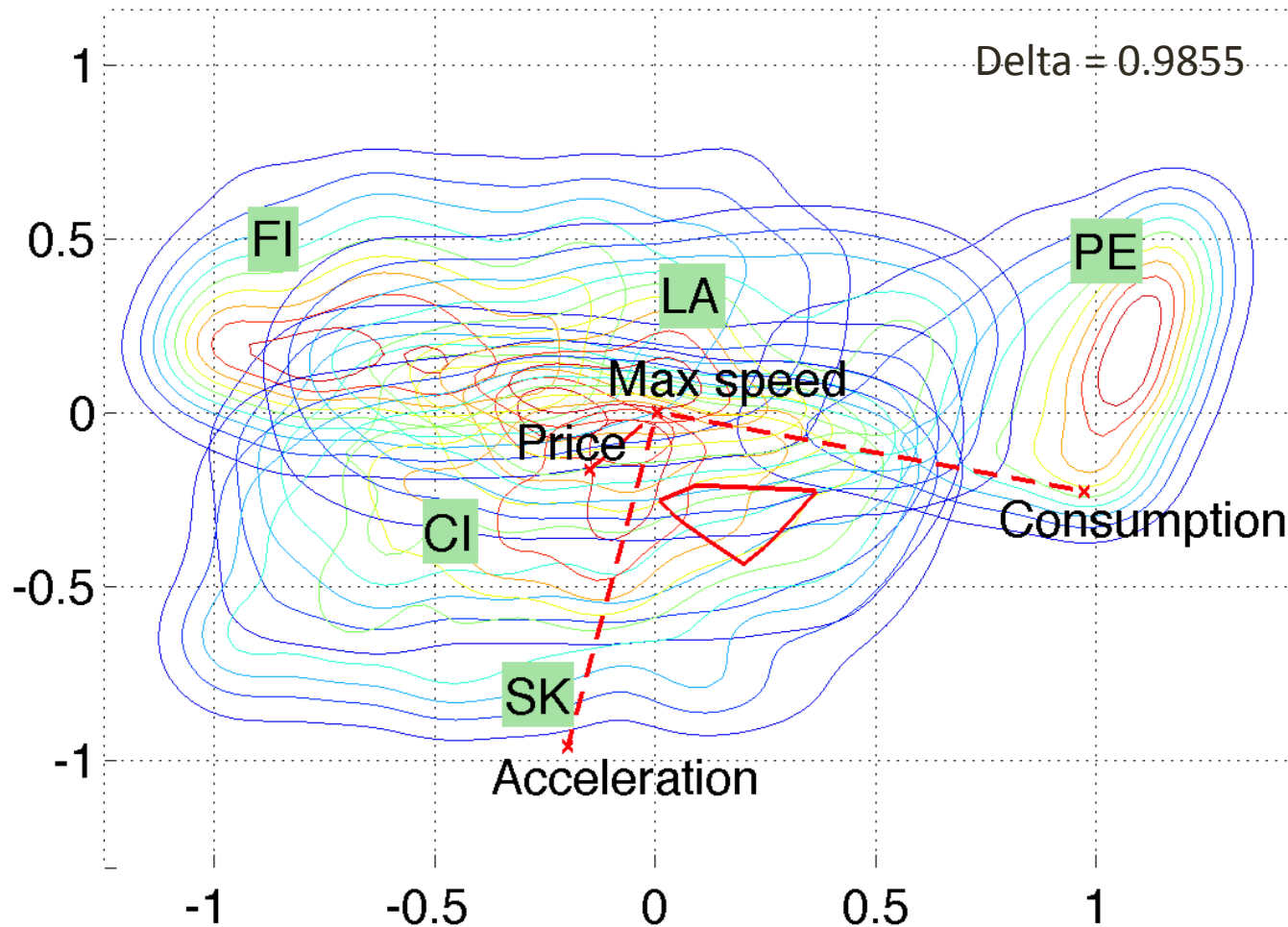
SMAA-GAIA : Uniform Repartition

SMAA-GAIA : Triangular Repartition

SMAA-GAIA : Covariance : Uniform

SMAA-GAIA : Covariance : Triangular

Conclusion

- SMAA-GAIA : Complementary tool to visualize the robustness of multicriteria rankings.

➔ DM more confident or DM warned about the sensitivity of the conclusion

- Limitation : GAIA Plane used as reference.

- Generation of random evaluations : triangular, uniform or potential correlation.

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Questions ?

Thank you for your attention !

Weight Space Navigation

• MCDA additives:

$$score(a_i) = \sum_{j=1} w_j \, score_k(a_i)$$

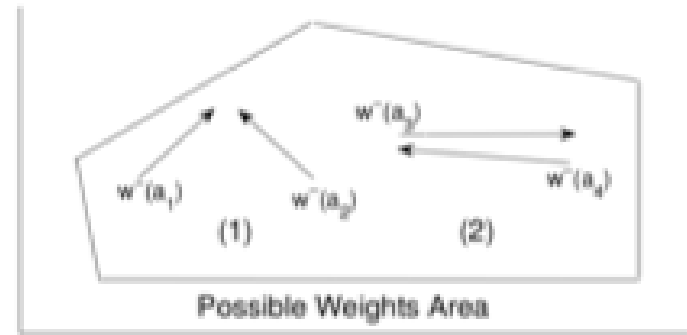Où :   • $\sum_{J=1}^{K} w_J = 1$

   • $w_j \pounds 1$

•Poids maximisant et minimisant le score:

$$w^-(a_i) = \min \sum_{j=1}^{k} w_j \, score_k(a_i)$$

$$w^+(a_i) = \max \sum_{j=1}^{k} w_j \, score_k(a_i)$$

• Projection sur le plan des poids:

ACP dans l'espace des poids

Weight Space Navigation

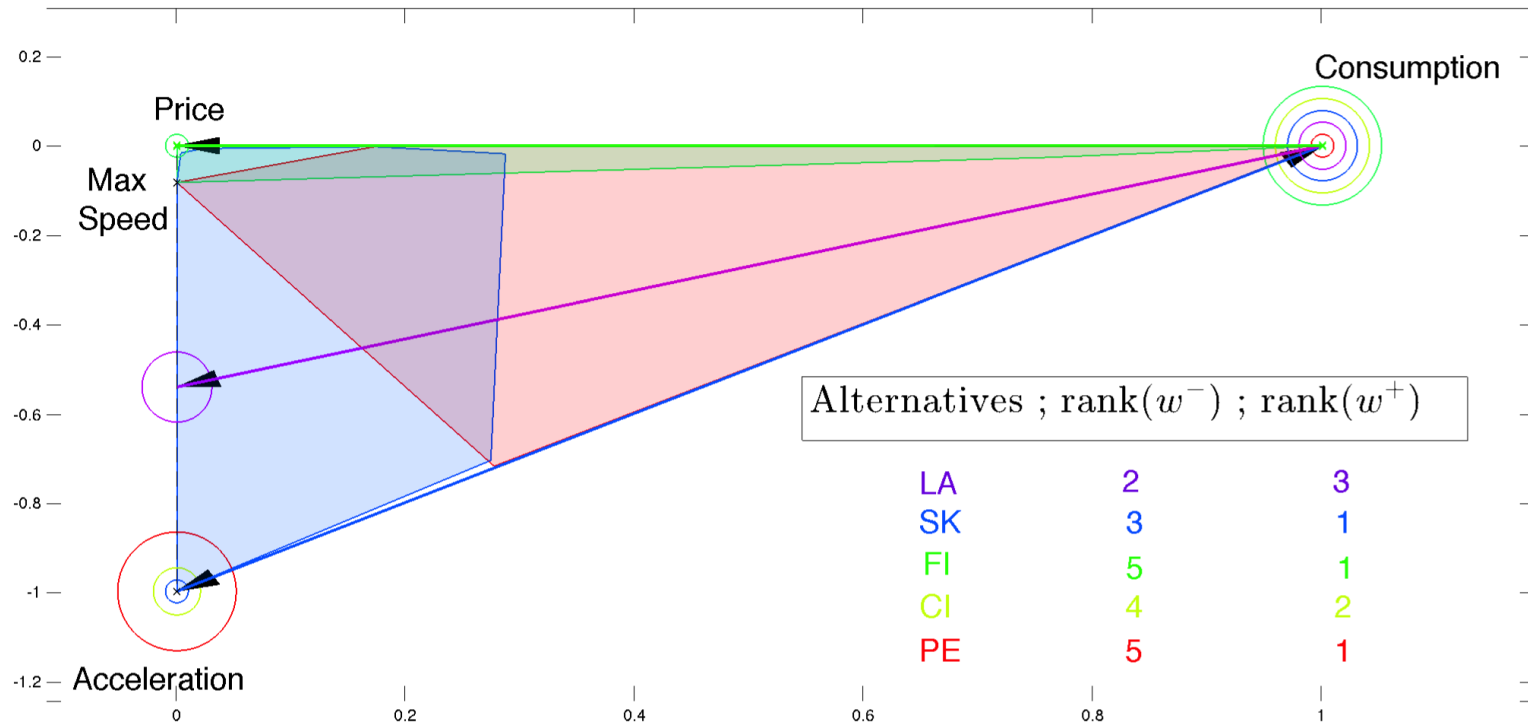- Zone où chaque alternative peut atteindre le premier rang:

$$score(a_i) - score(a_j) > 0, \ (j = 1, ..., n)(j \ne 1)$$

Où:
$$\cdot \sum_{J=1}^{K} w_J = 1$$
$$\cdot w_j \le 1$$
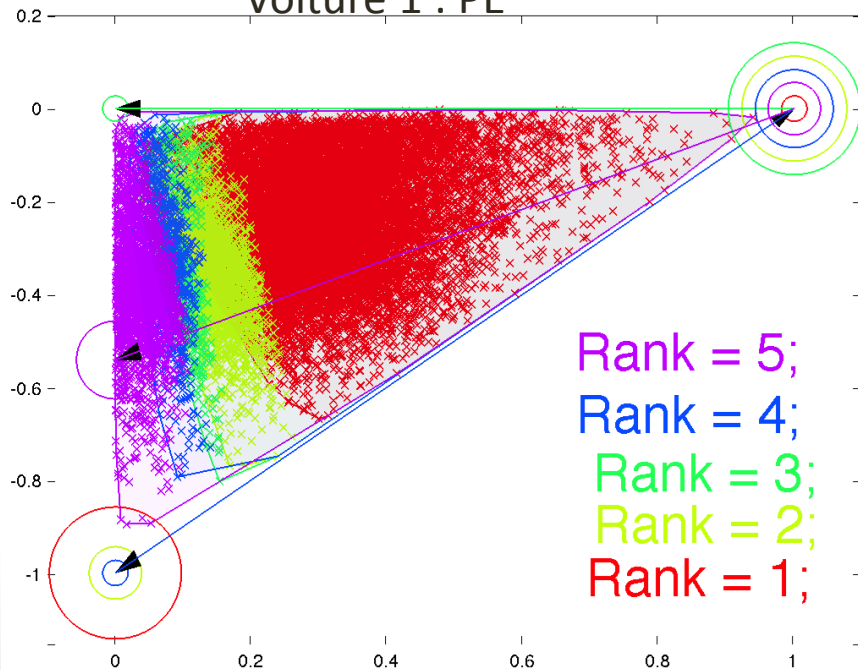
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Weight Space Navigation
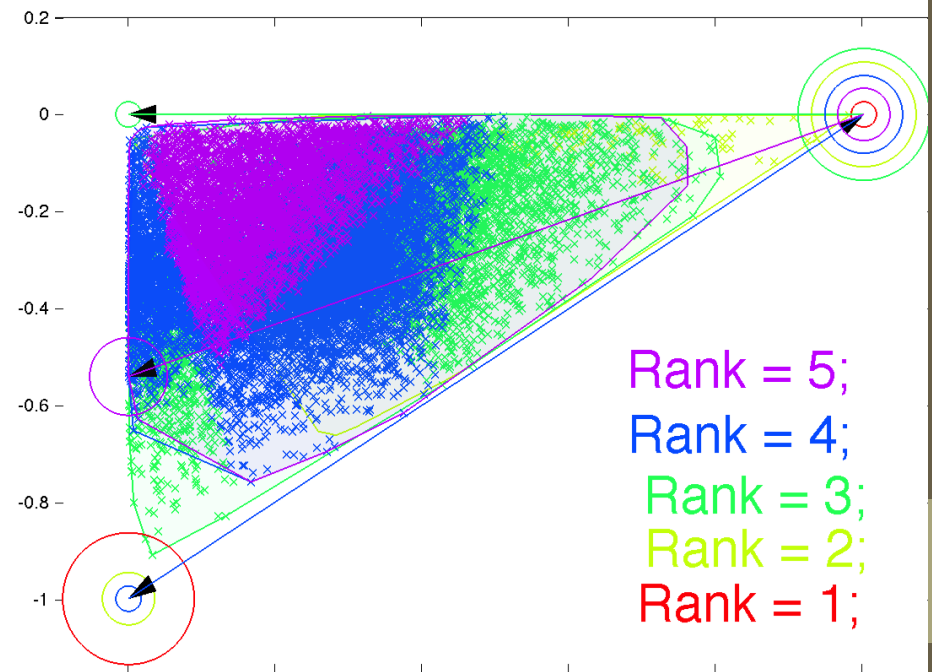
Application à PROMETHEE II: l'exemple des voitures.

Weight Space Navigation

•Pour chaque alternative, leur rang en fonction de la position dans le plan des poids par la methode de Monte Carlo:

Voiture 1 : PE

Rank = 5;
Rank = 4;
Rank = 3;
Rank = 2;
Rank = 1;

Voiture 5 : LA

Rank = 5;
Rank = 4;
Rank = 3;
Rank = 2;
Rank = 1;

Différence entre score et rang

Flux nets associés aux poids qui minimisent et maximisent le flux net de la voiture 5 : LA .

Alternatives	Φ_{min}	rank	Φ_{max}	rank
PE	1	1	-0.1083	5
FI	-0.2411	4	0.0250	2
CI	-0.3750	5	-0.0408	4
SK	-0.2143	3	0.1242	1
LA	-0.1696	2	0	3

Conclusion

- SMAA-PROMETHEE : première approche
- SMAA-GAIA : outil supplémentaire à GAIA
- Weight Space Navigation : outil supplémentaire aux Methodes additives.
- Très chouette service ! ☺

Bibliography

[1] Salvatore Corrente, José Rui Figueira, Salvatore Greco, The SMAA-PROMETHEE method, European Journal of Operational Research, Volume 239, Issue 2, 1 December 2014, Pages 514-522, ISSN 0377-2217,