An empirical distribution-based **approximation** of PROMETHEE II's net flow scores

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the stake

$$T(n) = \mathcal{O}\left(n^2\right)$$

n actions

promethee II

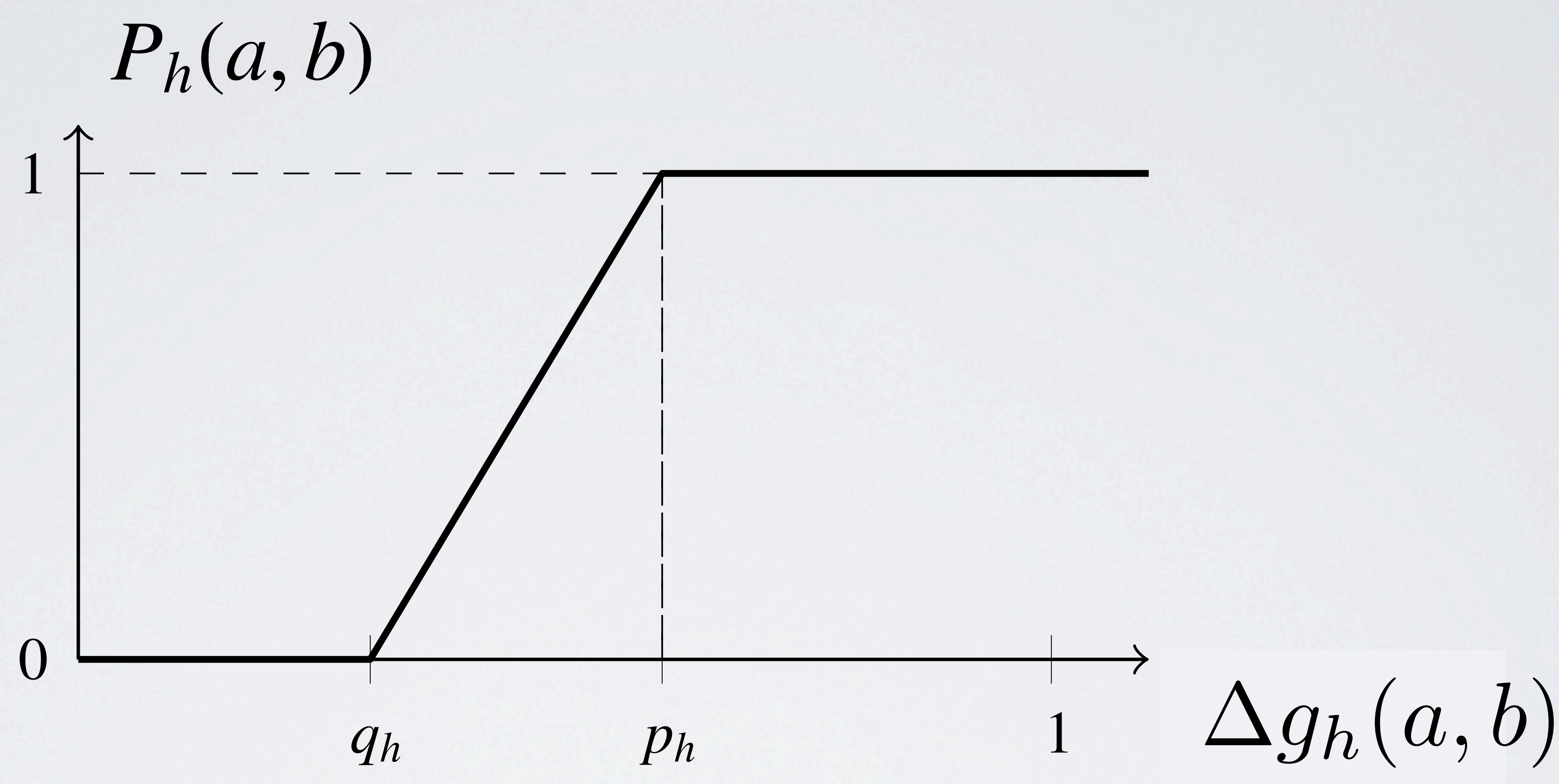
$$A = \{a_1 \ldots a_n\}$$

n actions: a_i $i \in I = \{1 \ldots n\}$

m criteria: g_h $h \in H = \{1 \ldots m\}$

$$g_h(a_i) \in [0, 1] \; , \; \forall (i, h) \in I \times H$$

promethee II



$$P_h(a,b) = \begin{cases} 0 & \text{, if } \Delta g_h(a,b) \leq q_h \\ \frac{\Delta g_h(a,b) - q_h}{p_h - q_h} & \text{, if } q_h < \Delta g_h(a,b) \leq q_h \\ 1 & \text{, if } \Delta g_h(a,b) > p_h \end{cases}$$

promethee II

$$\Delta P_h(a,b) = P_h(a,b) - P_h(b,a)$$

$$\phi_h(a) = \frac{1}{n-1} \sum_{b \in A} \Delta P_h(a,b)$$

unicriterion net flow score

$$\phi(a) = \sum_{h \in H} w_h \phi_h(a)$$

net flow score

promethee II

$$\Delta P_h(a,b) = P_h(a,b) - P_h(b,a)$$

$$\phi_h(a) = \frac{1}{n-1}\sum_{b\in A}\Delta P_h(a,b)$$ unicriterion net flow score

$$\phi(a) = \sum_{h\in H} w_h \phi_h(a)$$ net flow score

promethee II's continuous extension

$$\phi_h(a) = \frac{1}{n-1} \sum_{b \in A} \Delta P_h(a,b)$$

unicriterion net flow score

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y, \xi\right) f_h(\xi)\, d\xi$$

small abuse of notations

promethee II's continuous extension

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y, \xi\right) f_h(\xi)\, d\xi$$

approximation models

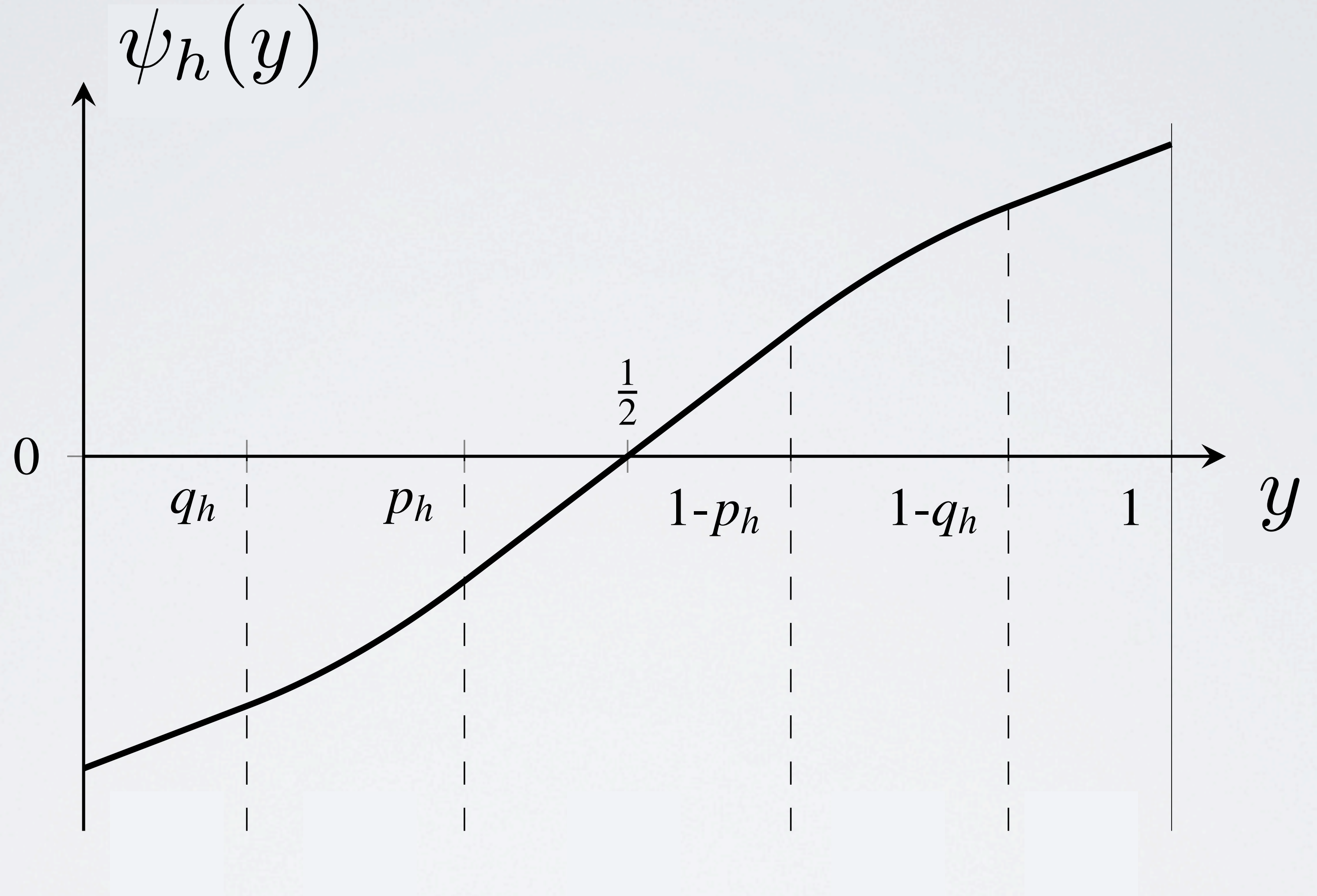
$$\psi_h(y) = \int_0^1 \Delta P_h(y, \xi)\, f_h(\xi)\, d\xi$$

$$g_h \sim U_{[0,1]}$$

$$f_h(\xi) = 1$$

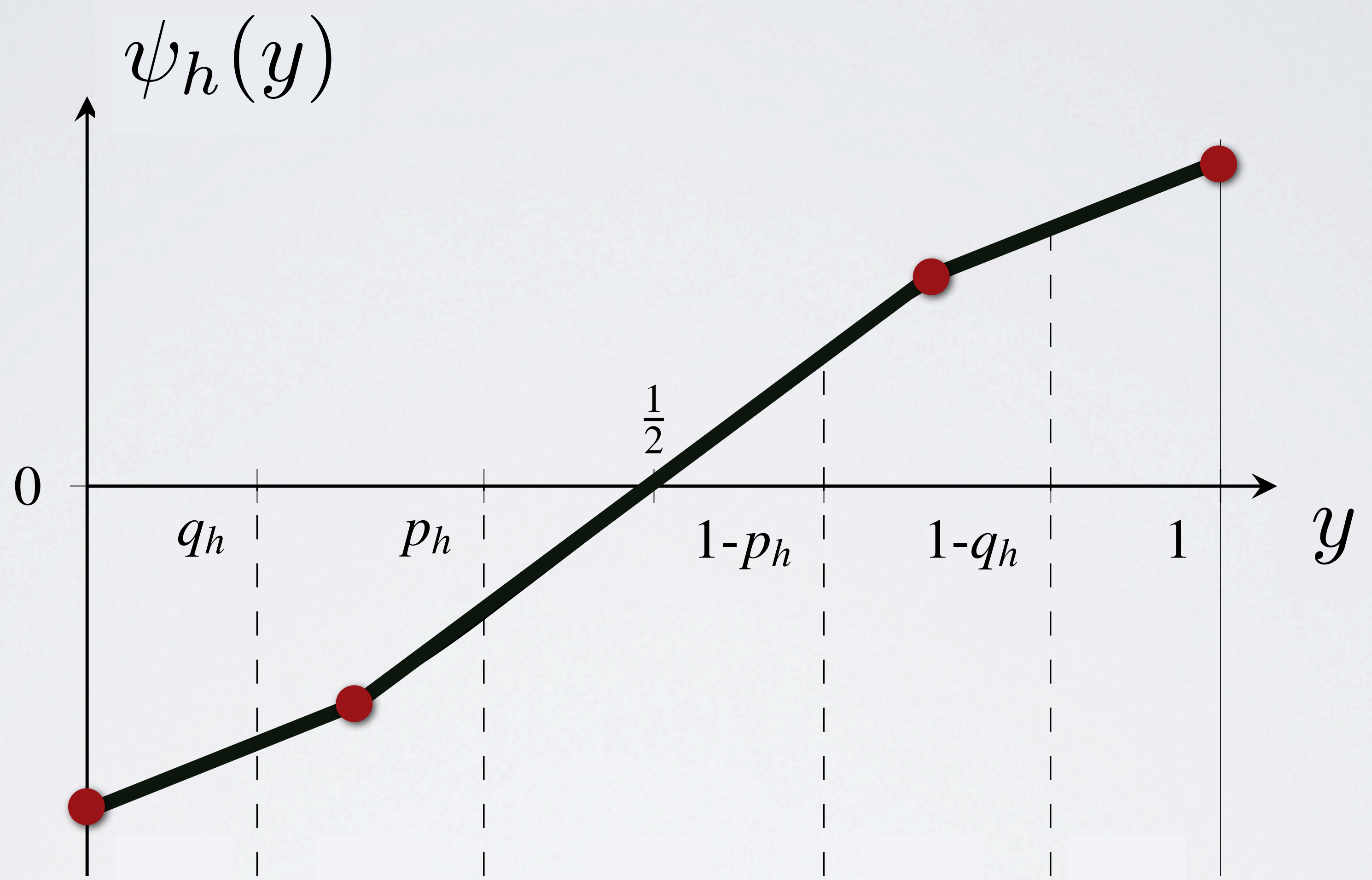
PLA

piecewise linear approximation



$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y, \xi\right)\, d\xi$$

piecewise linear approximation



approximation models

$$\psi_h(y) = \int_0^1 \Delta P_h(y, \xi)\, f_h(\xi)\, d\xi$$

$g_h \sim U_{[0,1]}$

$f_h(\xi) = 1$

PLA

piecewise linear approximation

$g_h \sim ?$

$f_h(\xi) = ?$

EDA

empirical distribution-based approximation

$$\psi_h(y) = \int_0^1 \Delta P_h(y, \xi)\, f_h(\xi)\, d\xi$$

integration by parts

empirical distribution of evaluations (needs sorting)

CDF by numerical integration

Details

empirical distribution-based approximation

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y,\xi\right) f_h(\xi)\, d\xi$$

$$\begin{aligned}\psi_h(y) \approx \;& \Delta P_h(y,1)\hat{F}_h(y) \\ &+ \tfrac{2}{p_h - q_h}\left[\mathcal{F}_h(y_q^-) - \mathcal{F}_h(y_p^-) + \mathcal{F}_h(y_p^+) - \mathcal{F}_h(y_q^+)\right]\end{aligned}$$

$$\mathcal{F}_h(y) \approx \int_0^y \hat{F}_h(\xi)\, d\xi$$ approximation by Riemann sum

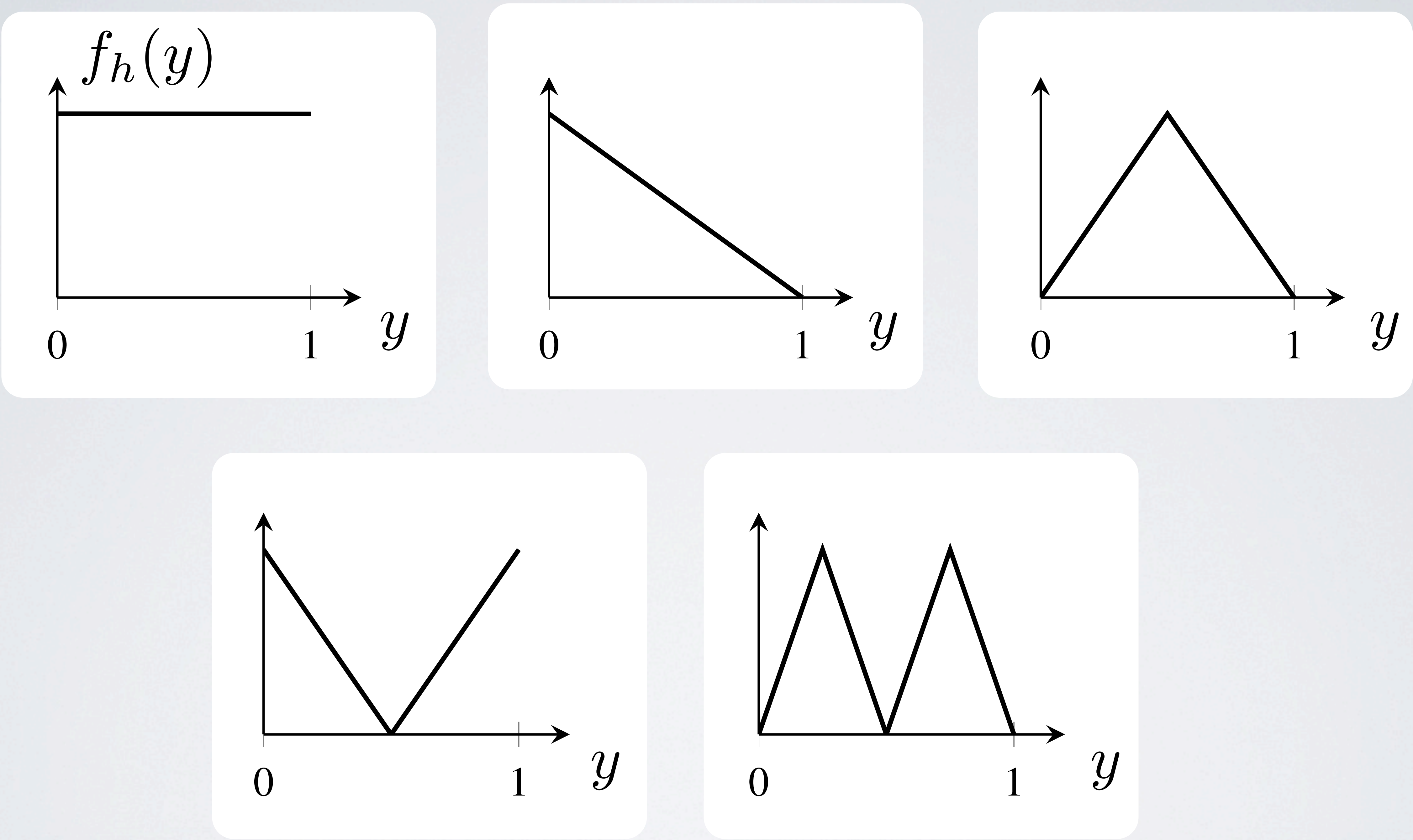
experimental exploration

simulations - setup

Parameter		Value(s)
Number of actions	n	10, 100, **1000**, 10000
Number of criteria	m	5, **7**, 10
Ex post approximation models		**`P3R`**
Ex ante approximation models		**`PLA`**, `EDA`
Runs per instance config.	N_{trials}	100

no correlation between criteria

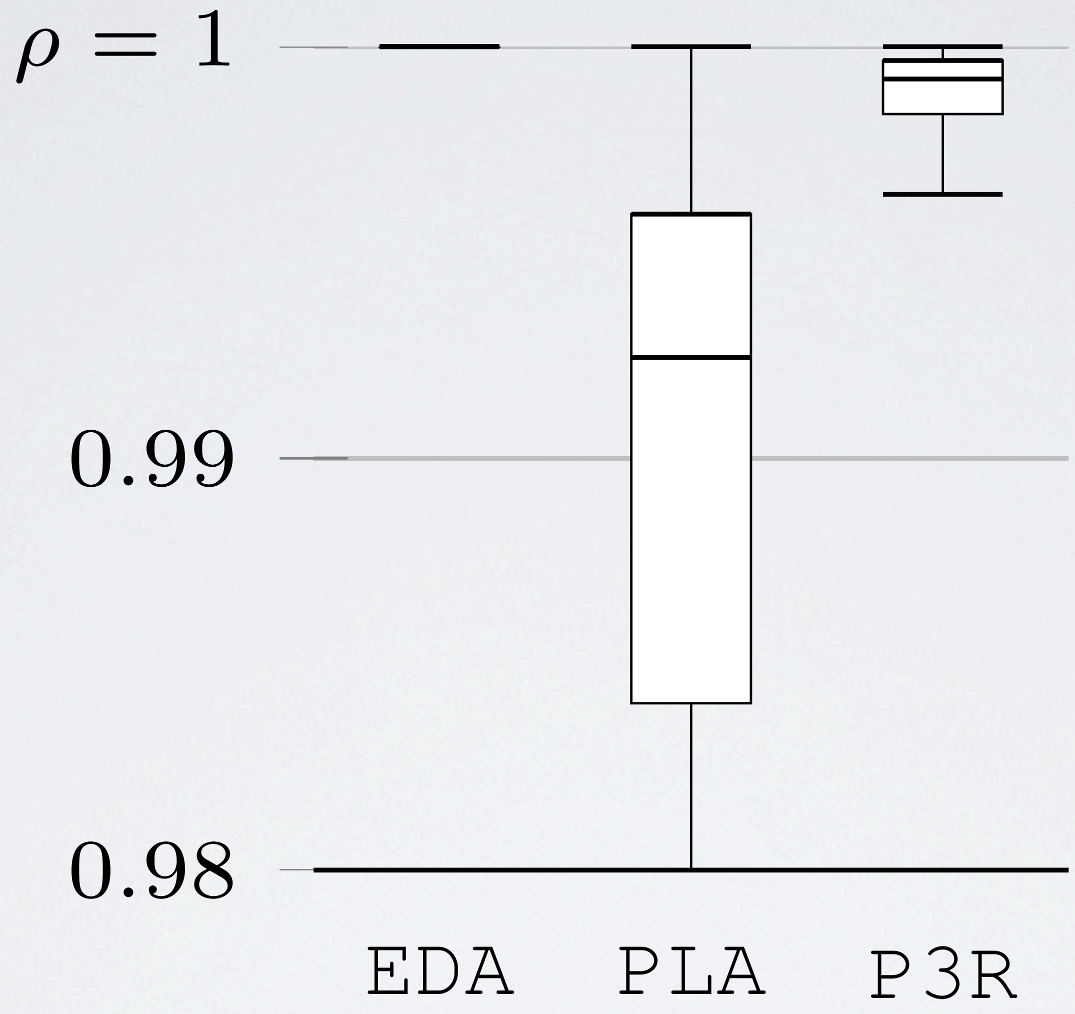
simulations - setup



no correlation between criteria

results

simulations - results



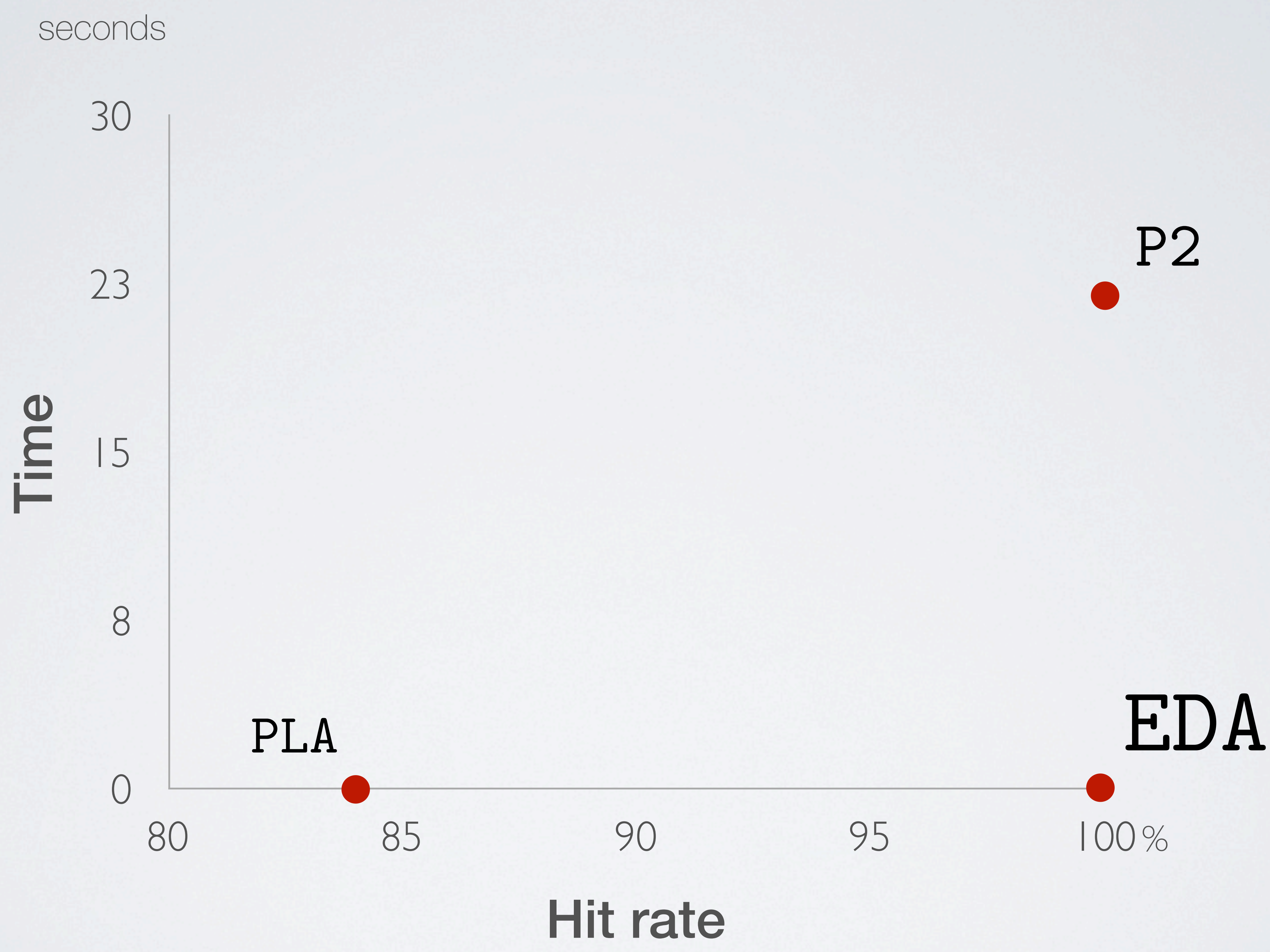
Pearson's correlation coefficient   1000 actions / 7 criteria   100 runs   mixed distribution

simulations - results

	Hit rate	Time	Complexity
PROMETHEE II	100% 100%	21” 100%	n^2
EDA	99,9% 99%	0,077” 0,3%	$n \log n$ n
PLA	84% 58%	0,003” 0,01%	n

10.000 actions / 7 criteria 100 runs mixed distribution

conclusion



$$\psi_h\,(y) = \int_0^1 \Delta P_h\,(y,\xi)\, f_h(\xi)\, d\xi$$

integration by parts

$$\int_a^b u(\xi) v'(\xi)\, d\xi = \Big[u(\xi)\, v(\xi)\Big]_a^b - \int_a^b u'(\xi) v(\xi)\, d\xi$$

$$\begin{cases} u(\xi) & = \Delta P_h\,(y,\xi) \\ v'(\xi) & = f_h(\xi) \end{cases}$$

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y,\xi\right) f_h(\xi)\, d\xi$$

$$\psi_h(y) = \Big[\Delta P_h(y,\xi)\, F_h(\xi)\Big]_0^y - \int_0^y \tfrac{d\Delta P_h}{d\xi}\left(y,\xi\right) F_h(\xi)\, d\xi$$

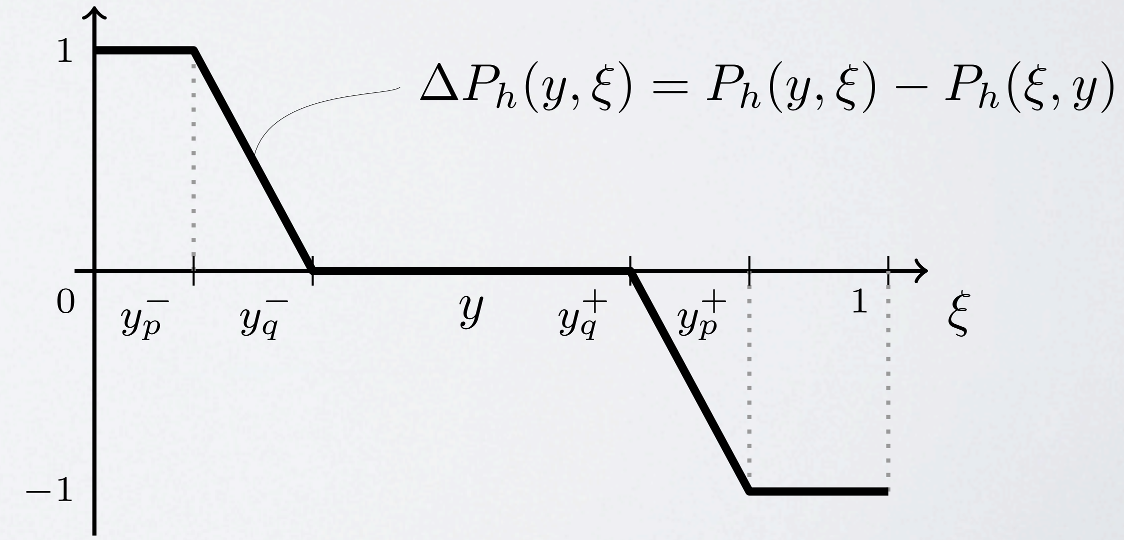
empirical distribution-based approximation

$$\psi_h(y) = \int_0^1 \Delta P_h(y,\xi)\, f_h(\xi)\, d\xi$$

$$\psi_h(y) = \Big[\Delta P_h(y,\xi)\, F_h(\xi)\Big]_0^y - \int_0^y \frac{d\Delta P_h}{d\xi}(y,\xi)\, F_h(\xi)\, d\xi$$

$$F_h(0) = 0$$

$$\frac{d\Delta P_h}{d\xi}(y,\xi) = \begin{cases} -\frac{2}{p_h - q_h} \\ 0 \end{cases}$$



empirical distribution-based approximation

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y, \xi\right) f_h(\xi)\, d\xi$$

$$\psi_h(y) = \Delta P_h(y,1) F_h(y) + \tfrac{2}{p_h - q_h} \left(\int_{y_p^-}^{y_q^-} F_h(\xi)\, d\xi + \int_{y_q^+}^{y_p^+} F_h(\xi)\, d\xi \right)$$

$$\psi_h\left(y\right) = \int_0^1 \Delta P_h\left(y, \xi\right) f_h(\xi)\, d\xi$$

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empirical cumulated distribution function

$$F_h(y) \approx \hat{F}_h(y) = \tfrac{1}{n} N_h(y)$$

$$\psi_h\,(y) = \int_0^1 \Delta P_h\,(y,\xi)\, f_h(\xi)\, d\xi$$

$$\psi_h(y) = \Delta P_h(y,1) F_h(y) + \tfrac{2}{p_h - q_h} \left(\int_{y_p^-}^{y_q^-} F_h(\xi)\, d\xi + \int_{y_q^+}^{y_p^+} F_h(\xi)\, d\xi \right)$$

numerical integration function (Riemann sum)

$$\mathcal{F}_h(y) = \sum_{i=1}^{n-1} \hat{F}_h\left(a_{\sigma_h(i)}\right) \left[\, f_h\left(a_{\sigma_h(i+1)}\right) - f_h\left(a_{\sigma_h(i)}\right) \right] \approx \int_0^y F_h(\xi)\, d\xi$$