



Université
libre
de Bruxelles



Service de
Mathématiques
de la gestion

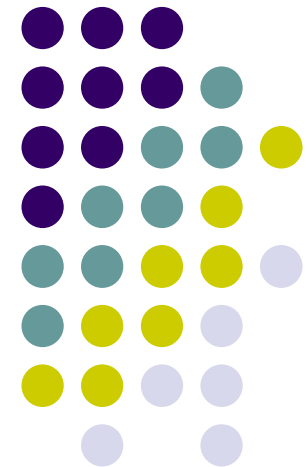
Extension of PROMETHEE methods to temporal evaluations

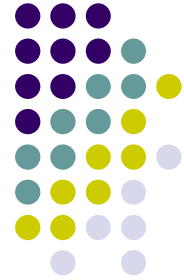
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Supervisor:

Prof. Yves De Smet





- **Temporal MCDA problem**
- **PROMETHEE II Method and Gaia Plane**
- **Temporal PROMETHEE II and Gaia Plane**
- **Dynamic preference threshold**
- **Dynamic alternatives**
- **Illustration of Temporal Gaia Plane**
- **Prospects**



- **Temporal MCDA problem**
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- Illustration of Temporal Gaia Plane
- Prospects

ULB 1- A temporal MCDA problem

In a junior football club:



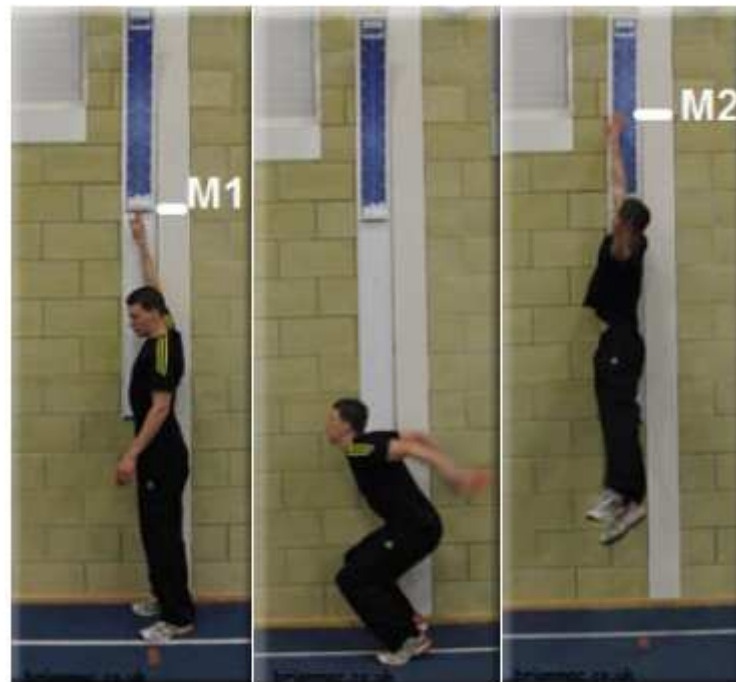
Assessment of **5 players** after **4 weeks** of regular monitoring

With respect to **5 criteria**



The criteria:

1- Speed test
5,5 s-->4s



3- Peak power
15 --> 17 W / kg



4- VO²max
30-->50 ml.kg⁻¹.min⁻¹



2- Lactic capacity
40 --> 50 Joules / kg





5- Team work
(qualit.)
12 --> 15 /20



Conventional MCDA methods are not effective because

- *Evaluations*
- *Preferences of Decision maker*

are NOT constants in time



Patients monitoring:

- ***Puls***
- ***Cholesterol***
- ***Blood pressure***
- ***....***

During weeks



Sustainable development:

- ***Social***
- ***Ecology***
- ***Economy***

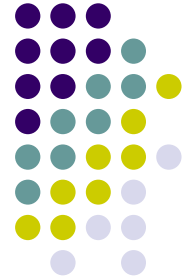
During years





How to get a global ranking after successive evaluations ?

Before that let's have a look over...



- 1- Temporal MCDA problem
- 2- PROMETHEE II Method and Gaia Plane**
- 3- Temporal PROMETHEE II and Gaia Plane
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2- PROMETHEE II method and Gaia Plane



→ *Ranking by Total Preoder (Global Ranking)*

Alternatives set: $A = \{ a_1, a_2, \dots, a_n \}$

Criteria set: $F = \{ f_1, f_2, \dots, f_k \}$

Criteria weight set: $W = \{ w_1, w_2, \dots, w_k \}$

	f_1	f_2	...	f_k
a_1	$f_1(a_1)$	$f_2(a_1)$...	$f_k(a_1)$
a_2	$f_1(a_2)$	$f_2(a_2)$...	$f_k(a_2)$
:	:	:	:	:
a_n	$f_1(a_n)$	$f_2(a_n)$...	$f_k(a_n)$

The aim is to find the alternative with $\max \{ f_1(x), f_2(x), \dots, f_k(x) \mid x \in A \}$

The procedure is:

→ $\forall a, b \in A: d_j(a, b) = f_j(a) - f_j(b)$

2- PROMETHEE II method and Gaia Plane

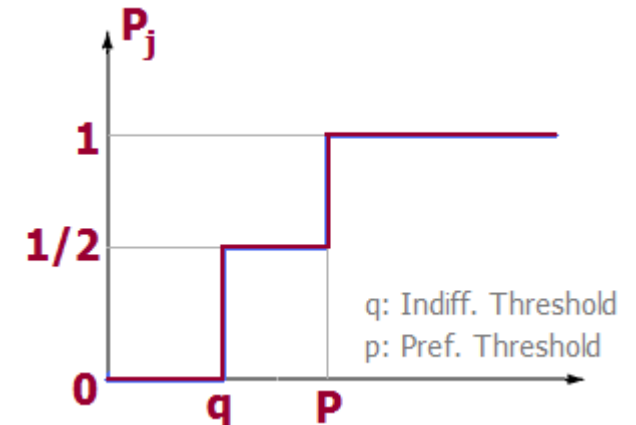
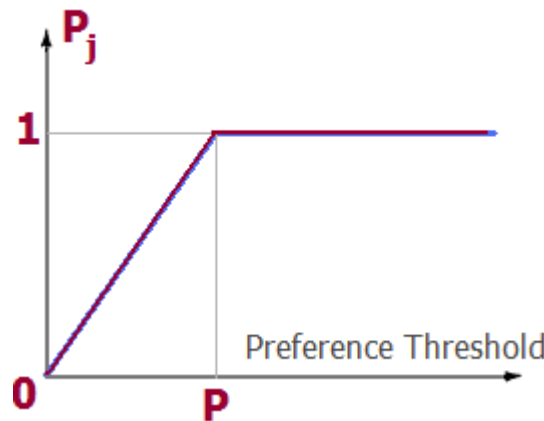


→ Define a Preference function by criterion:

Examples:

$$P_j(\mathbf{a}, \mathbf{b}) = P_j [d_j(\mathbf{a}, \mathbf{b})]$$

$$(0 \leq P_j(\mathbf{a}, \mathbf{b}) \leq 1)$$



→ Preference Index:

$$\pi(\mathbf{a}, \mathbf{b}) = \sum_{j=1}^k P_j(\mathbf{a}, \mathbf{b}) \cdot w_j$$

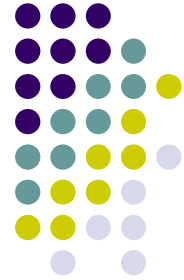
→ Outgoing flow:

$$\Phi^+(\mathbf{a}) = \frac{1}{n-1} \sum_{x \in A} \pi(\mathbf{a}, x)$$

→ Incoming flow:

$$\Phi^-(\mathbf{a}) = \frac{1}{n-1} \sum_{x \in A} \pi(x, \mathbf{a})$$

2- PROMETHEE II method and Gaia Plane

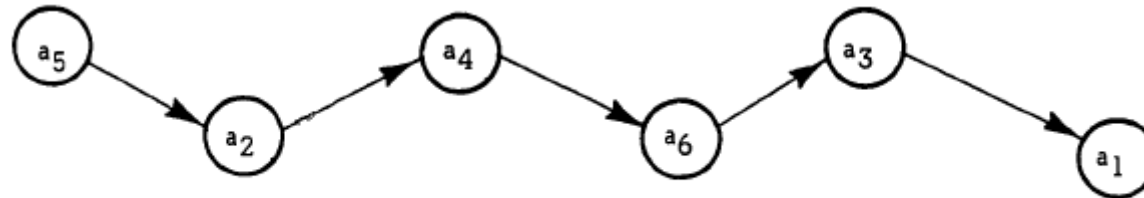


→ *The net flow:*

$$\Phi(\mathbf{a}) = \Phi^+(\mathbf{a}) - \Phi^-(\mathbf{a})$$
$$\Phi(\mathbf{b}) = \Phi^+(\mathbf{b}) - \Phi^-(\mathbf{b})$$

\mathbf{a} outranks \mathbf{b} → *iff* $\Phi(\mathbf{a}) > \Phi(\mathbf{b})$

\mathbf{a} is indifferent to \mathbf{b} → *iff* $\Phi(\mathbf{a}) = \Phi(\mathbf{b})$



Total PROMETHEE II Relation

2- PROMETHEE II method and Gaia Plane



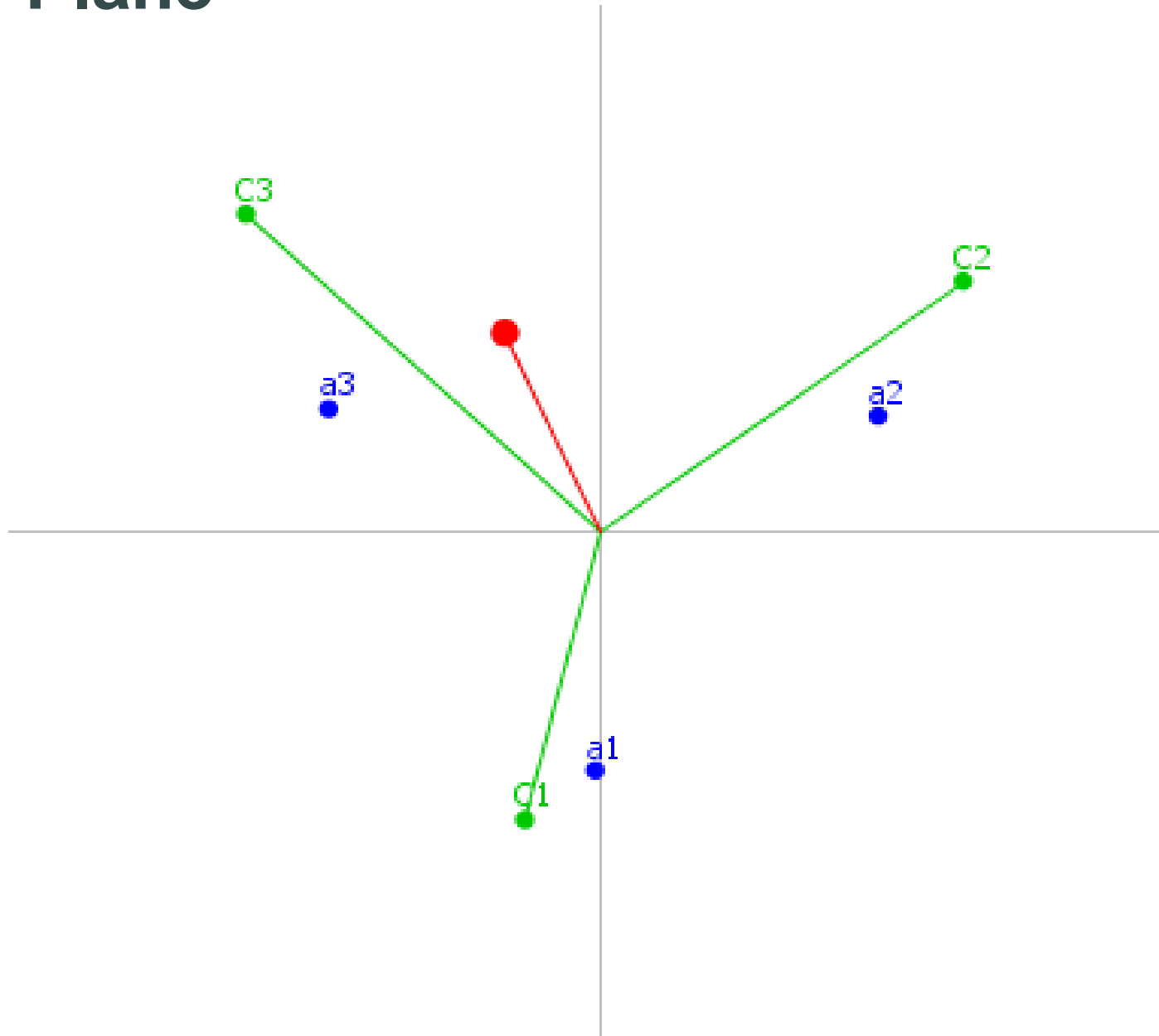
→ *GAIA Plane (D-Sight)*

*Multicriteria decision problem: -3 alternatives
-3 criteria*

	C1	C2	C3
a1	14,3	10,0	9,5
a2	11,0	15,0	10,0
a3	12,0	10,0	15,0

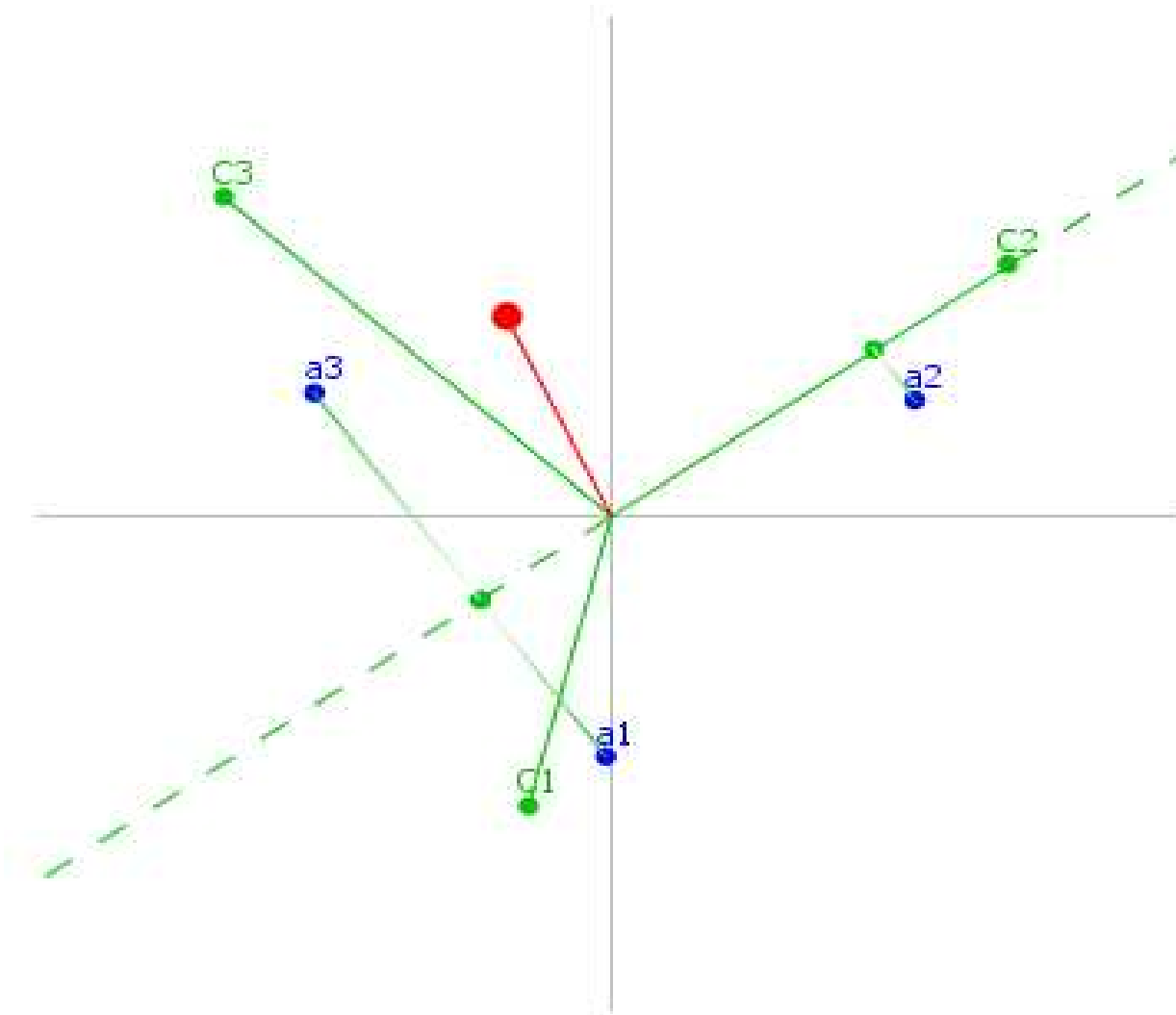
In this example, each alternative has the best score on 1 given criterion

2- PROMETHEE II method and Gaia Plane

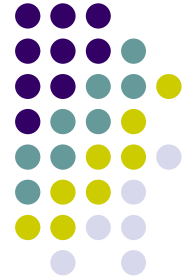


2- PROMETHEE II method and Gaia Plane

→ *Reading GAIA Plan*



We make the projection of each alternative on a given axis in order to get an idea of their importance relative to this axis



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3- Temporal PROMETHEE II and Gaia Plane



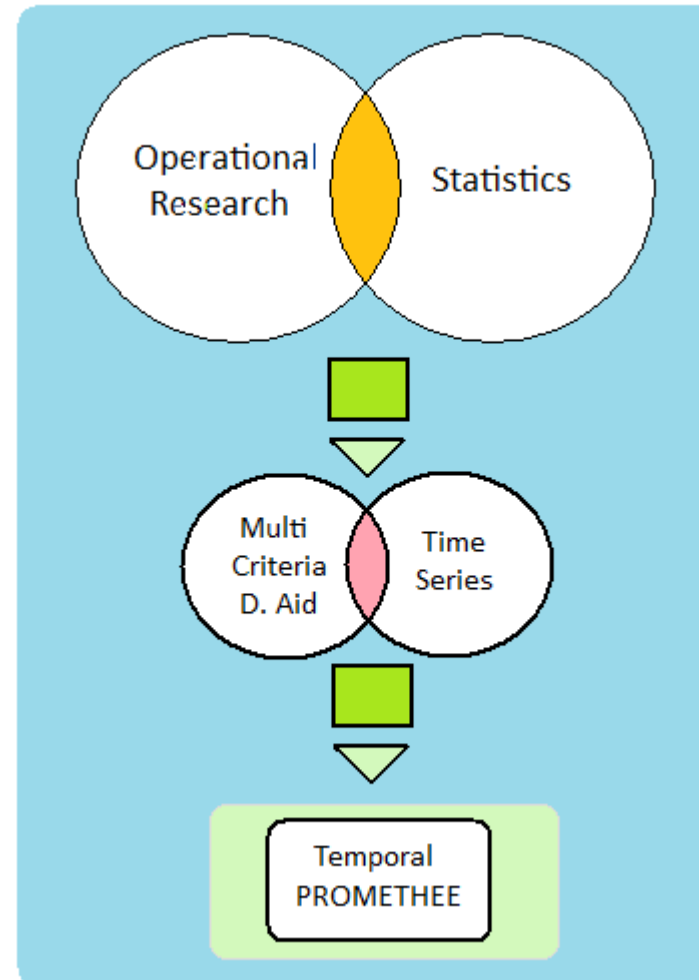
→ One year of research.

Junction of two fields:

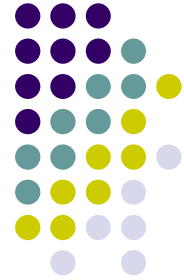
- ✓ Operational research
- ✓ Statistics

More specifically:

- ✓ Multicriteria decision aid
- ✓ Stochastic time series



3- Temporal PROMETHEE II and Gaia Plane

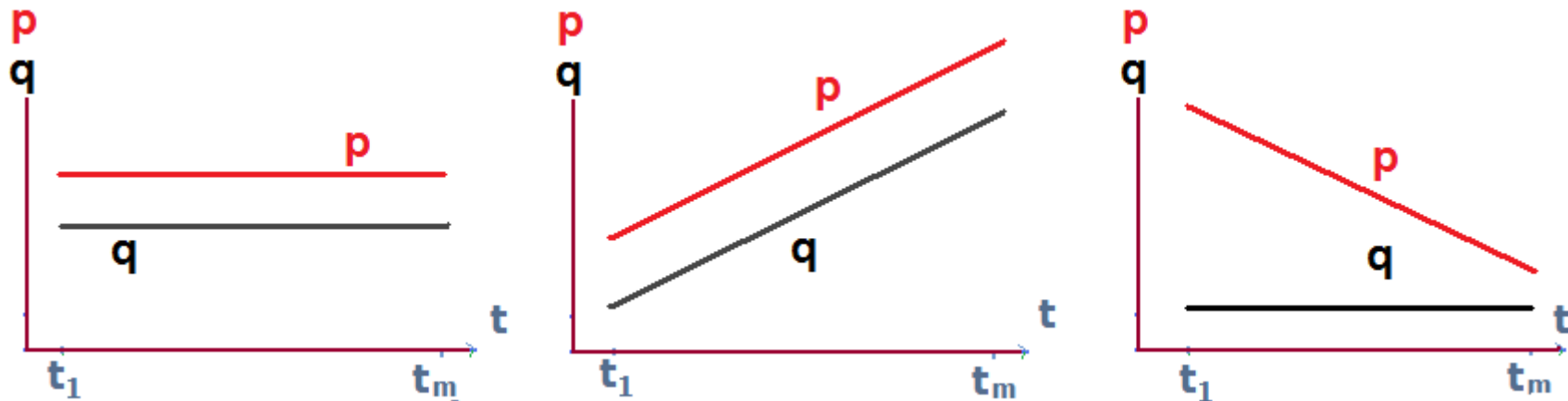


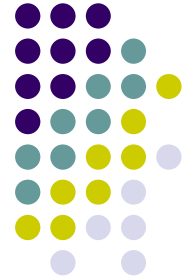
→ Procedure:

- | | |
|----------------------------|---|
| 1- Alternative set: | $A = \{ a_1, a_2, \dots, a_n \}$ |
| Criteria set: | $F = \{ f_1, f_2, \dots, f_k \}$ |
| Criteria weight set: | $W = \{ w_1, w_2, \dots, w_k \}$ |
| Instants set: | $T = \{ t_1, t_2, \dots, t_m \}$ |
| Instant weight set: | $Vt = \{ V_1, V_2, \dots, V_m \}$ |

2- Defining a preference function (Conventional PROMETHEE)

3- Defining a function of dynamic threshold per criterion





4- Computing the instantaneous net flow (Promethee II) :

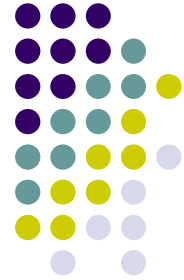
$$\Phi_{t_1}(a) = \Phi_{t_1}^+(a) - \Phi_{t_1}^-(a) \quad (\text{for each alternative } a)$$

5- Computing the global ranking over the set of instant T:

$$\Phi_{A,T}(a) = (\mathbf{V}_1 \cdot \Phi_{t_1}(a) + \mathbf{V}_2 \cdot \Phi_{t_2}(a) + \dots + \mathbf{V}_t \cdot \Phi_{t_t}(a)) / S$$

$$\text{with: } S = \mathbf{V}_1 + \mathbf{V}_2 + \dots + \mathbf{V}_m$$

6- Temporal GAIA Plane: ...



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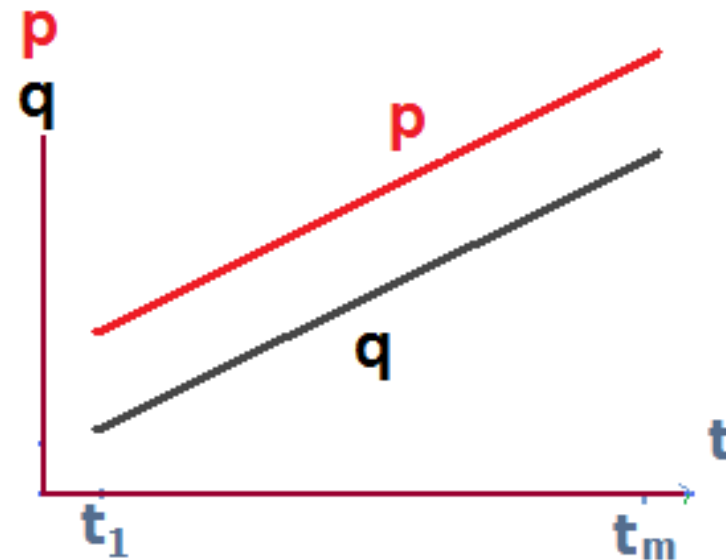
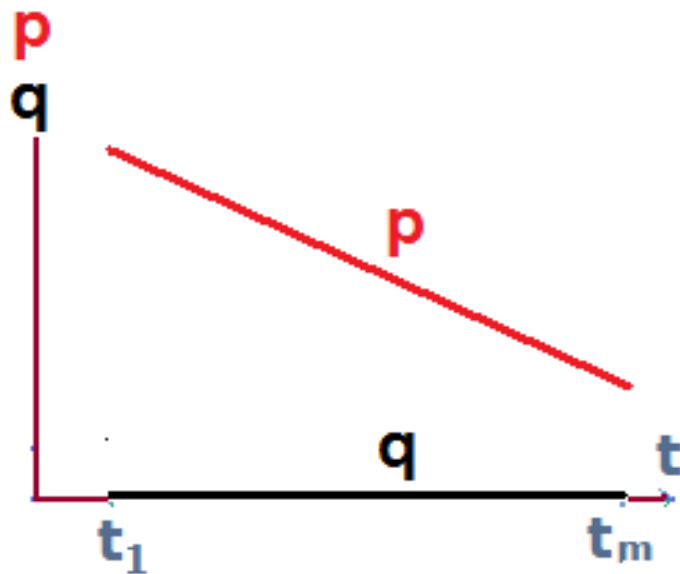
4- Dynamic Preference Threshold



→Temporal PROMETHEE: Define a Dynamic Preference function by criterion:

$$P_{j,t}(a,b) = P_{j,t} [d_{j,t} (a,b)]$$

$$0 \leq P_{j,t} (a,b) \leq 1$$



4- Dynamic Preference Threshold

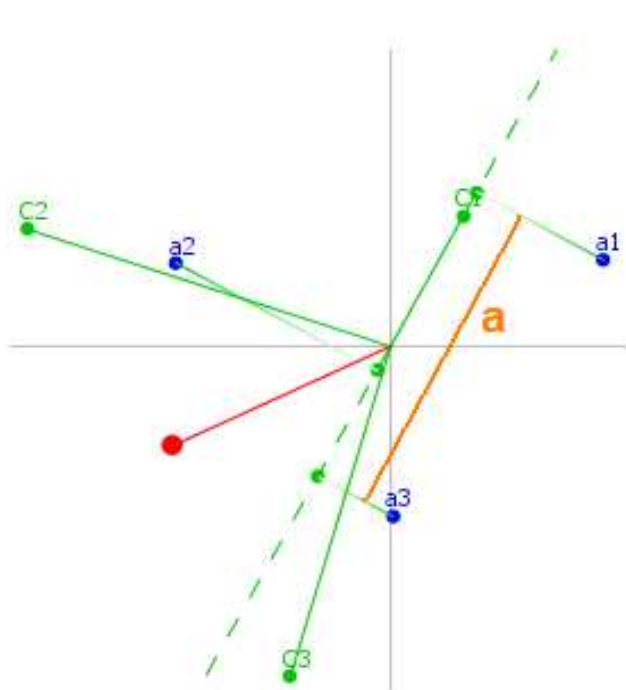


→Effect of a dynamic threshold on Gaia Plane with:

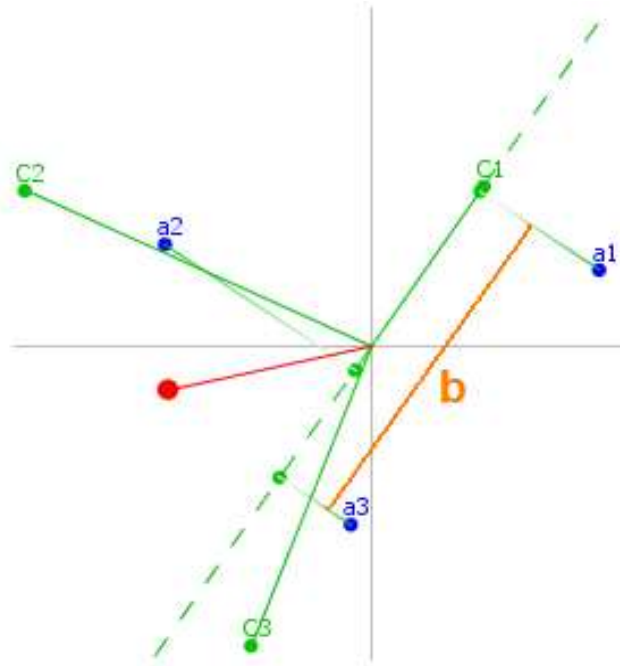
- 3 alternatives assessed on 3 criteria
- V_Shape function is chosen as preference function ($q = 0$)
- The criteria have the same weight
- No alternative evaluations over time
- Only C1 has dynamic** (decreasing) preference threshold

	C1	C2	C3
a1	14,3	10,0	9,5
a2	11,0	15,0	10,0
a3	12,0	10,0	15,0

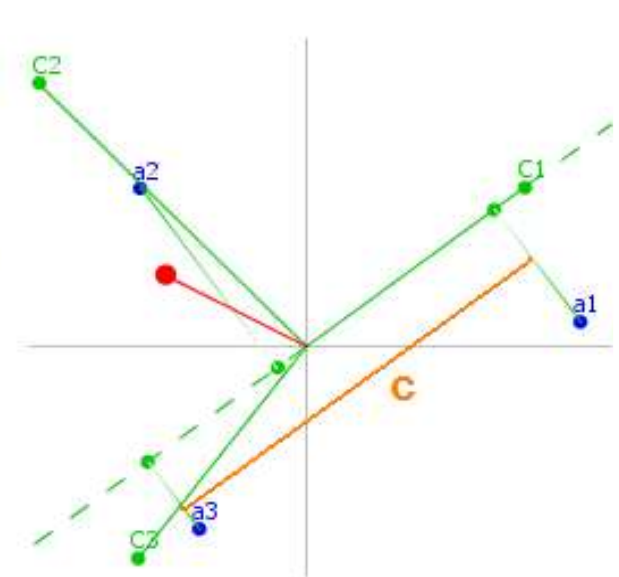
4- Dynamic Preference Threshold



Preference threshold= 35



Pref. thresh= 25



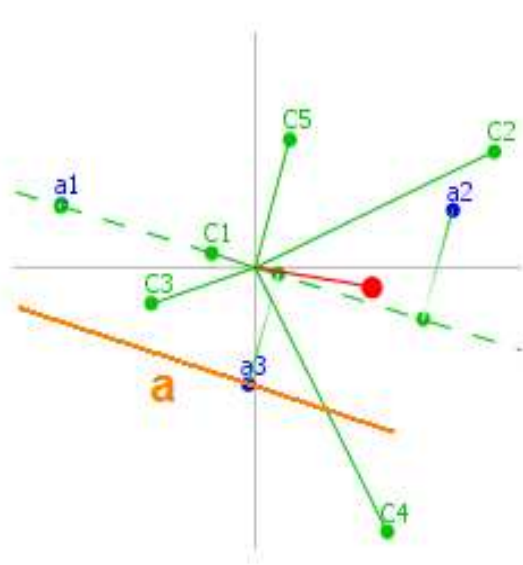
Pref. thresh= 15

→ Criterion 1 gets longer with decreasing preference threshold, because:
 $a < b < c$

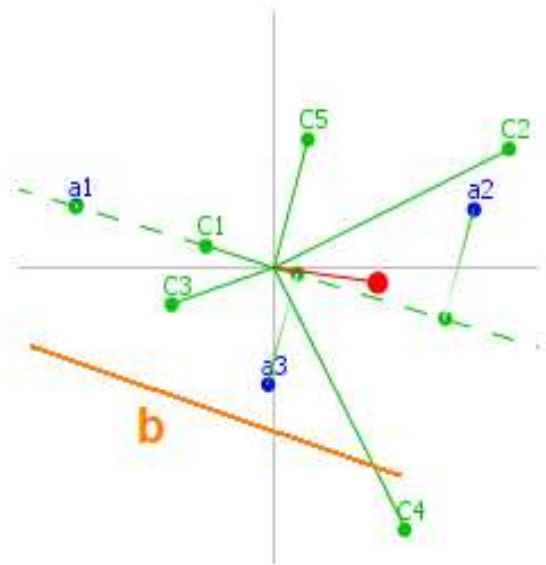
4- Dynamic Preference Threshold



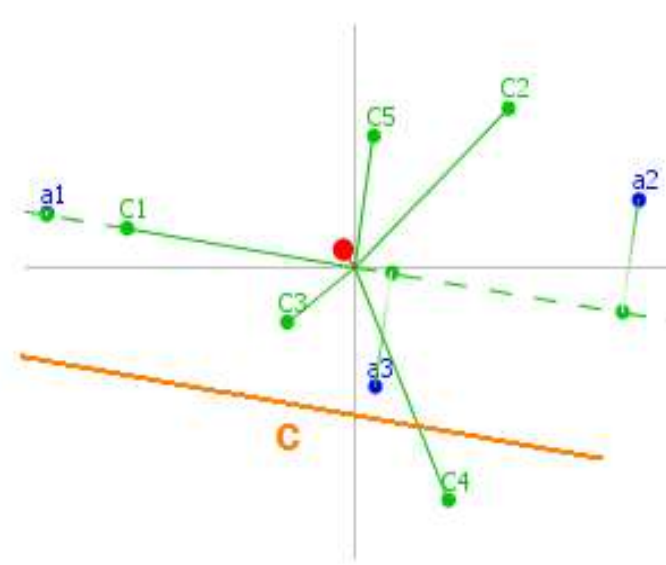
→ Here, we will repeat the same experience but with 5 criteria:



Pref. thresh = 35



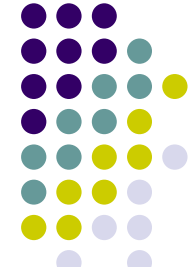
= 25



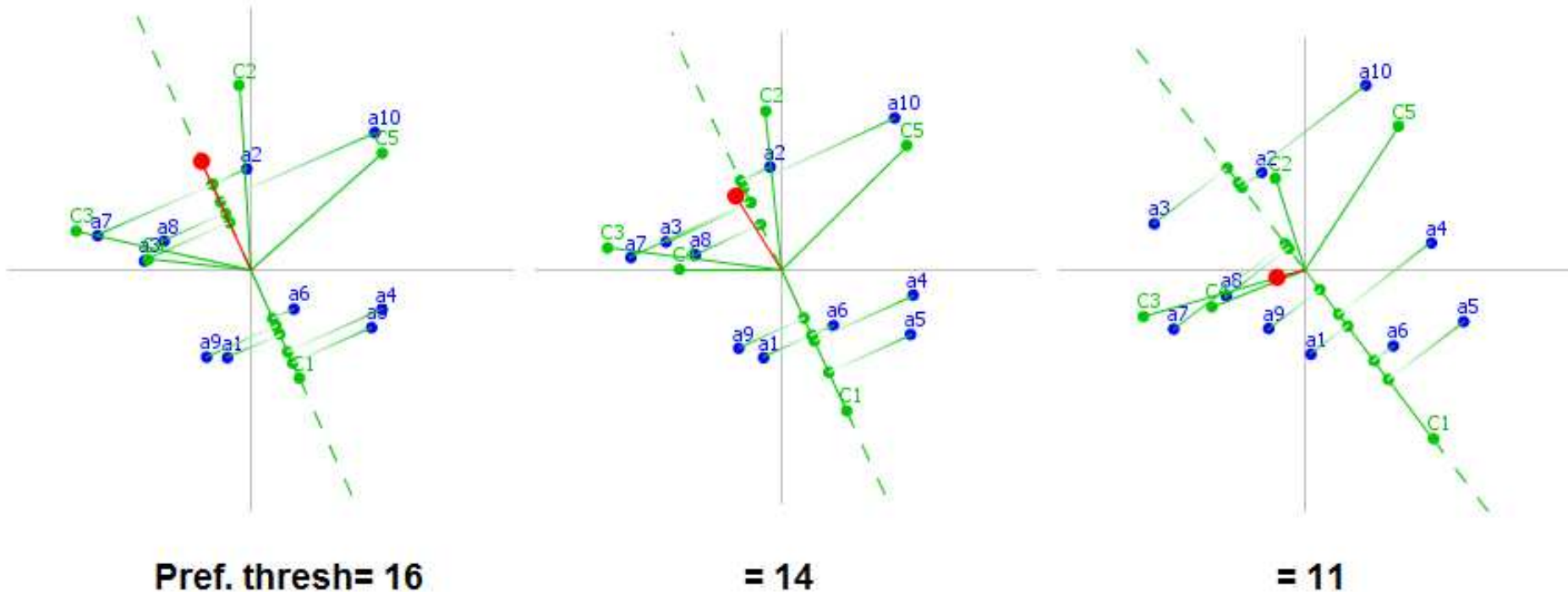
= 5

→ Criterion 1 gets longer with decreasing preference threshold for the same raison.

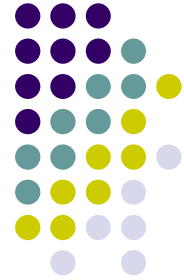
4- Dynamic Preference Threshold



→ Here, we will repeat the same experience with 10 alternatives:

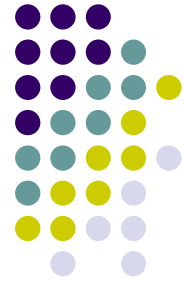


→ We can conclude that dynamic preference threshold of one given criterion has an impact on the discrimination of alternatives with respect to this criterion. More specifically, decreasing preference threshold discriminates more the alternatives.



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5- Dynamic alternatives

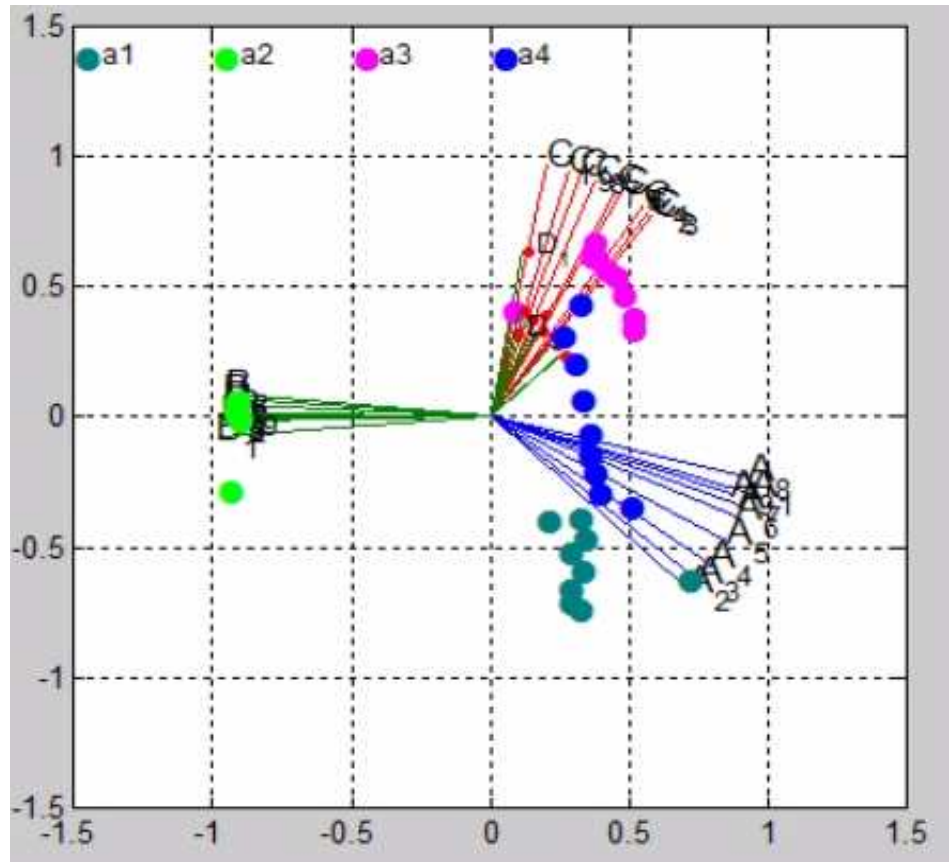


→ Effect of dynamic alternatives with:

- 4 alternatives assessed on 3 criteria
- V_Shape function is chosen as preference function
- All the criteria have the same weight
- Constant preference thresholds over time
- Only alternative a_4 evolves significantly (from the best to the worse on C3)
- During 9 moments

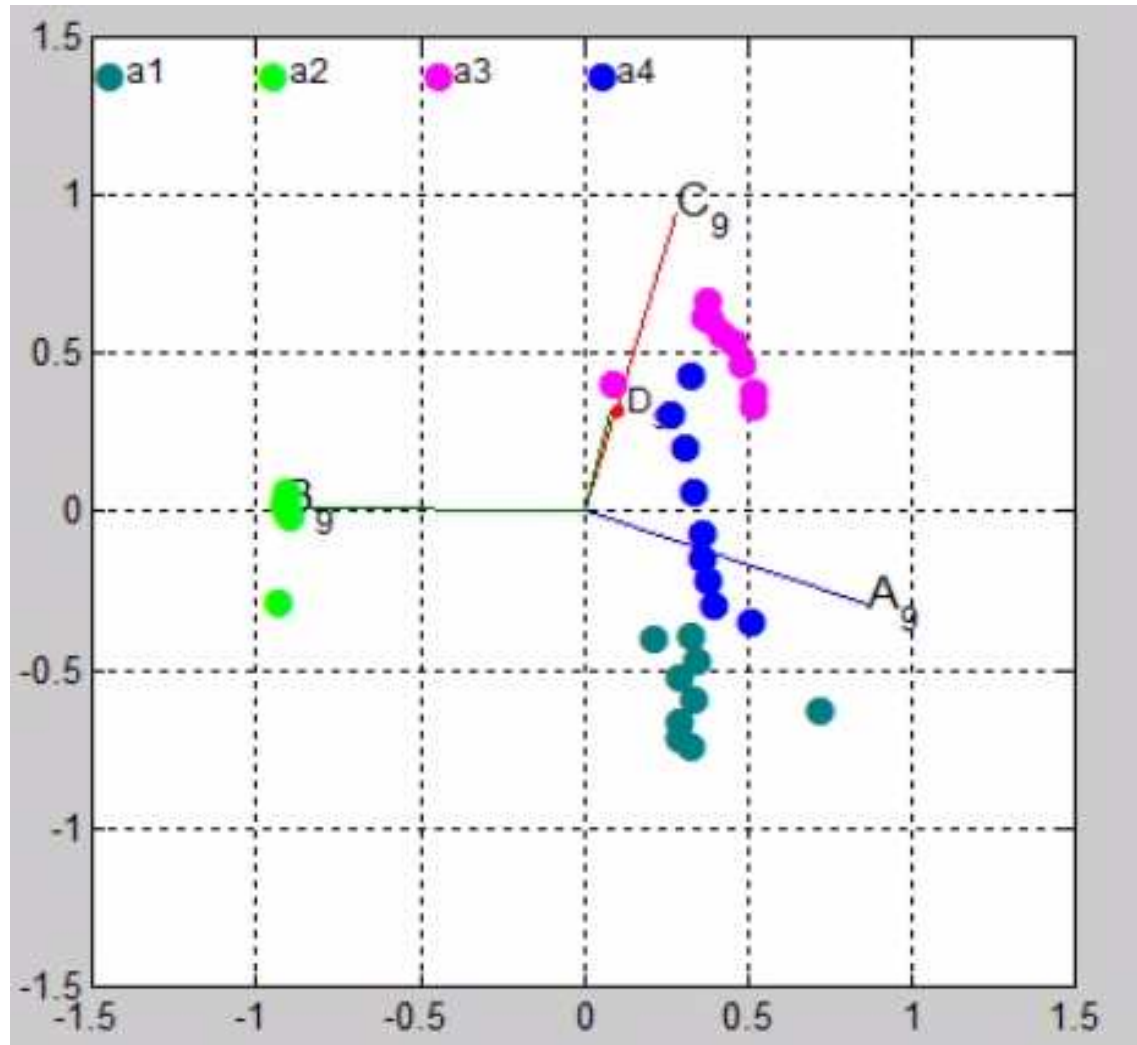
		C1	C2	C3
a1		13,0 ... 18,0	9,0 ... 12,0	9,0 ... 10,0
a2		9,0 ... 7,0	14,0 ... 16,0	9,5 ... 10,0
a3		11,0 ... 14,0	11,0 ... 9,0	16,0 ... 16,0
a4		11,0 ... 15,0	9,5 ... 6,0	16,0 ... 10,0

5- Dynamic alternatives



→ a_1 , a_2 and a_3 are almost stable in their areas while a_4 (blue one) moves away from criterion 3 (red axis) to be almost the best with respect to criterion 1 (blue axis).

5- Dynamic alternatives



Here, we took the last instant axis of each criterion

→ We can conclude that the temporal Gaia plane differentiates 2 kind of alternatives behaviours:

- Stable behaviour
- Evolving behaviour



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Assessment:

-5 players

-4 weeks

-5 criteria

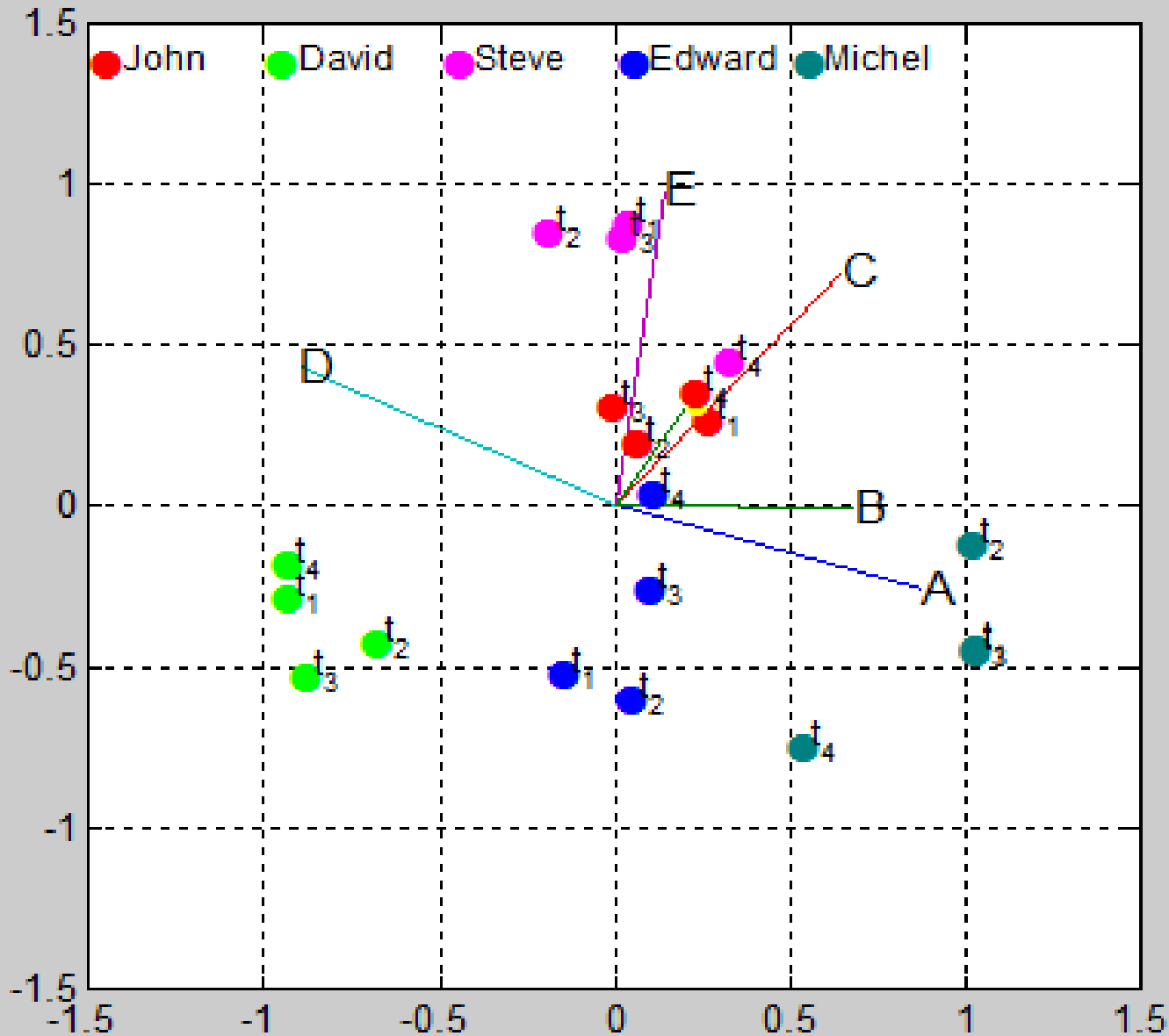


6- Illustration of Temporal Gaia Plane



	5,5 s-->4s (x10)				40 --> 50 Joules / kg				15 --> 17 W / kg (x10)				30-->50 ml.kg ⁻¹ .min ⁻¹				12 --> 15 /20			
	Speed test				Lactic capacity				Peak power				VO2max				Team work			
	A				B				C				D				E			
<u>John</u>	55	54	51	48	48	51	54	59	145	149	156	161	38	42	47	48	14	13	14	16
<u>David</u>	63	57	56	55	41	40	41	45	143	145	146	145	46	50	54	60	9	11	12	13
<u>Steve</u>	51	49	47	43	46	49	53	55	148	154	160	164	42	44	47	47	17	16	16	17
<u>Edward</u>	66	62	55	50	45	49	55	60	140	142	147	155	37	40	44	47	12	12	14	15
<u>Michel</u>	48	46	43	40	50	55	57	58	138	142	148	153	33	35	39	41	12	12	11	10

6- Illustration of Temporal Gaia Plane



Temporal Gaia Plane reflects the behaviour of each player during time



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ULB 7- Prospects

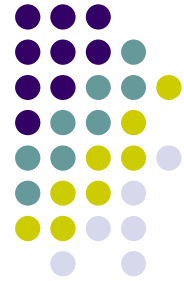
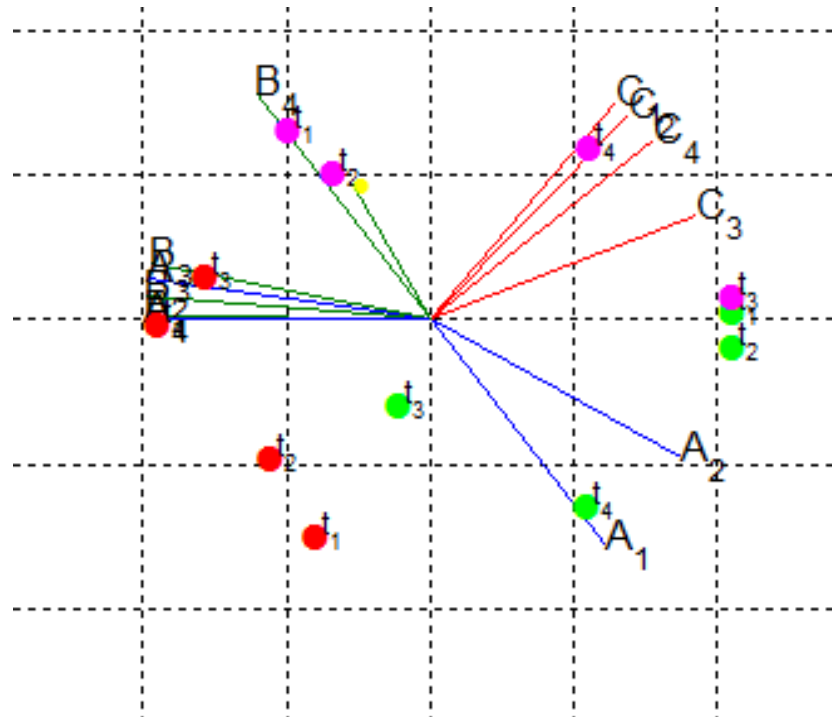
→ Gaia Plane:

If alternatives evolve abruptly:

- Gaia plane maintains ability to visualize
- We can not take the last instant axis of criteria as reference.

Example of 3 alternatives evaluated on 3 criteria during 4 moments.

In this example, **A** has changed significantly its side from instant 2 to instant 3.



ULB 7- Prospects

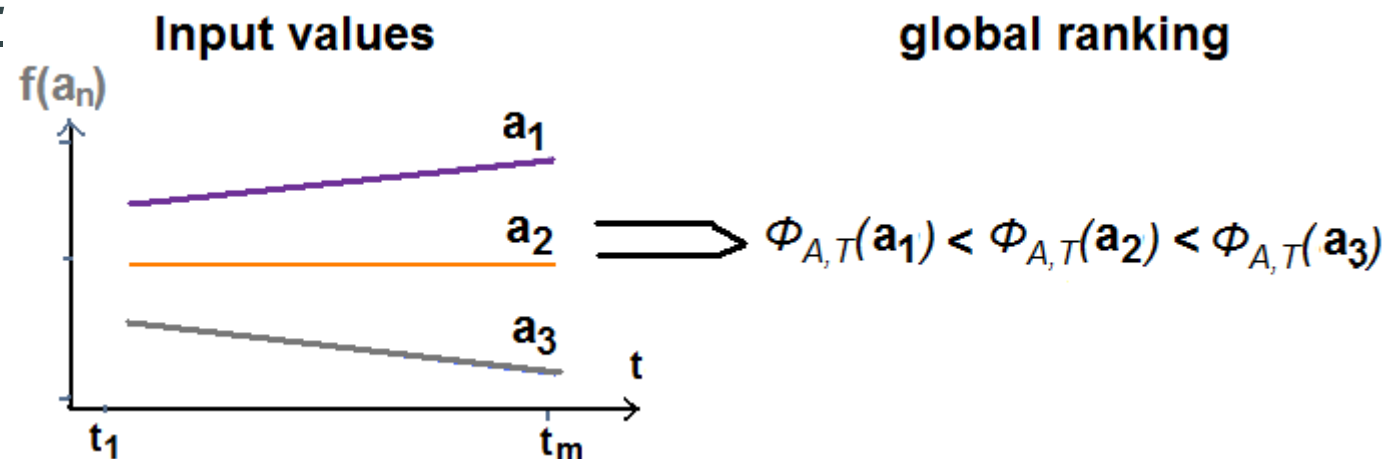


→ Demonstrated mathematical properties:

1- Dominance:

If **a** dominates **b** over all criteria, **a** must be ranked before **b** in the global ranking.

2- Monotonicity:



3- Neutrality:

The rank of **a** in the global ranking is independent on its position among the alternatives in the input.



→ The ongoing work is about how to elicitate the preferences:

- Dynamic preference thresholds
- Instants weight

A blackboard with red curtains on the sides. The text "The End" is written in white cursive on the blackboard.

The End