A New Weight-Restricted DEA Model Based on PROMETHEE II

2nd International MCDA workshop on PROMETHEE: Research and case studies

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- DEA & MCDA
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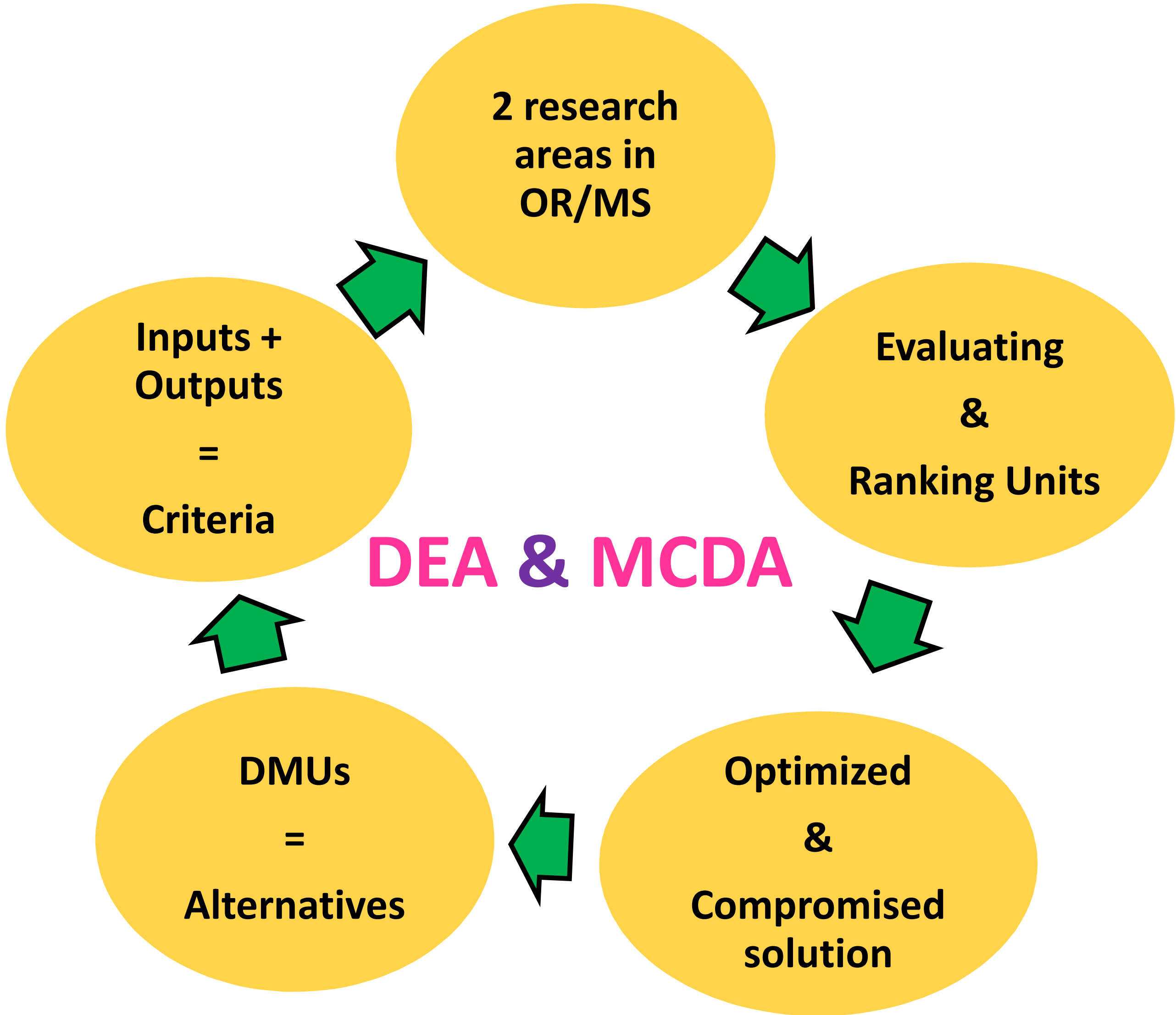
- MCDA: PROMETHEE II
- Synergies
- Objective
- Methodology
- Numerical Examples
- The main advantages of this work & further ideas


2 research areas in OR/MS
Evaluating & Ranking Units
Optimized & Compromised solution
DMUs = Alternatives
Inputs + Outputs = Criteria
DEA & MCDA

DEA

- Non-parametric and non-statistical method Combining several measures of inputs and outputs into a single measure of efficiency

- ***Generating automated weights by model***

- CCR, BCC, Additive, FDH, Super efficiency, …

MCDA

- A decision making tool in the presence of conflicting criteria and absence of optimal solution: ***Sorting***, ***Ranking*** and ***Choosing*** alts

- ***Assigning pre-determined weights to Criteria***

-MAUT, AHP, Outranking (ELECTRE, *PROMETHEE*), Interactive

Ranking and Selecting between bank branches, health care centers (Flokou, A. et al., 2010), educational institutions (Salerno, C., 2006), localization of a factory (Vaninsky, A., 2008), proper ways for a project, …

- **Shanghai ranking** (Academic Ranking of World Universities, Shanghai Jiao Tong University, 2007), (Jean-Charles Billaut, Denis Bouyssou, Philippe Vincke, 2009)
- **FIFA world ranking**
- **Country's ranking in Globalization**
- **Largest producing countries of agricultural commodities, …**

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A DEA example:

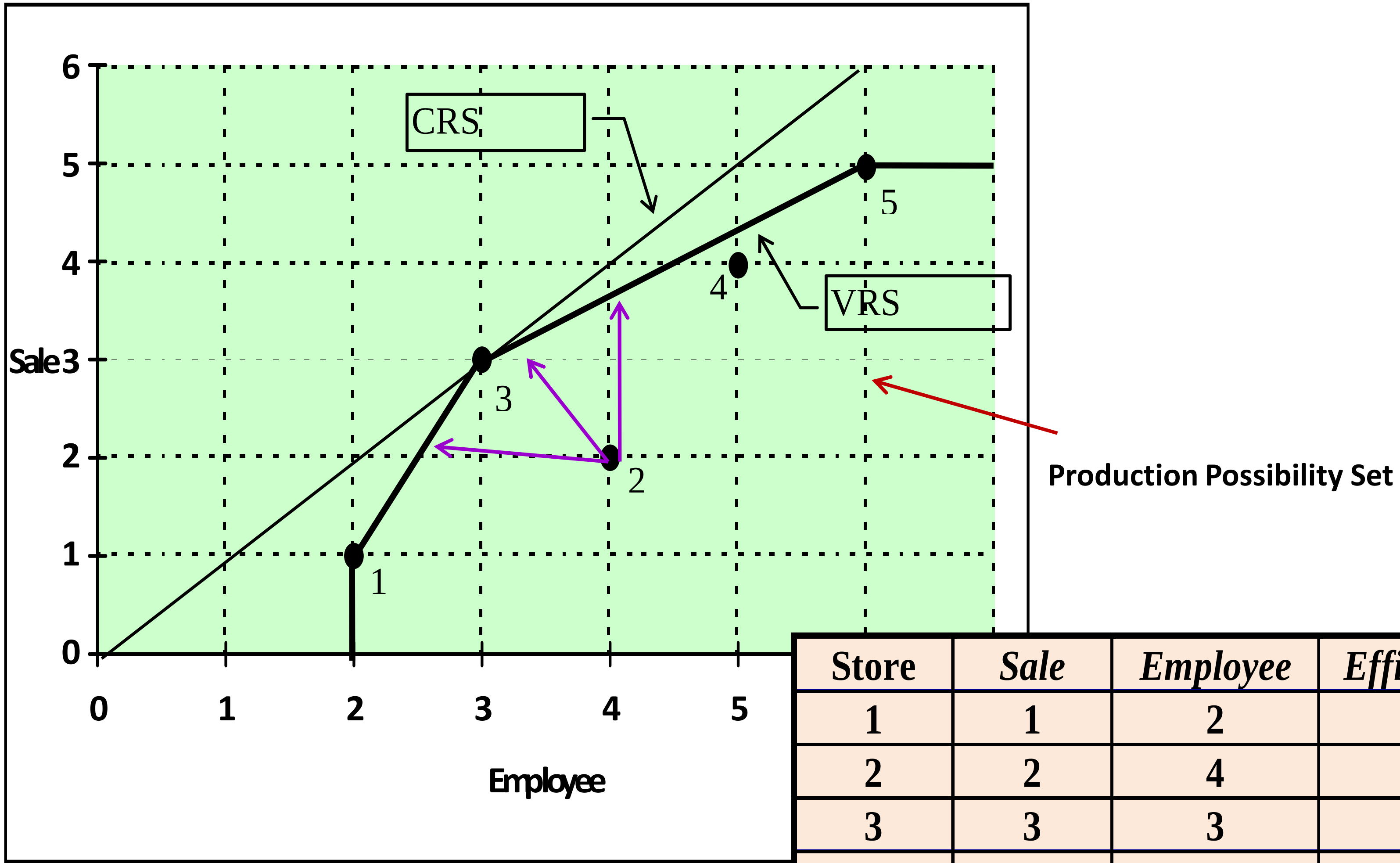

Figure 1- Efficient frontier

Store	*Sale*	*Employee*	*Efficiency*
1	1	2	0.5
2	2	4	0.5
3	3	3	1
4	4	5	0.8
5	5	6	0.83

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BCC Input-Oriented

Envelopment model	Multiplier model

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io}, i = 1,2,\dots,m;$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, r = 1,2,\dots,s;$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, j = 1,2,\dots,n.$$

$$\max \quad z = \sum_{r=1}^{s} \mu_r y_{ro} + u_o$$

s.t.

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} + u_o e \leq 0$$

$$\sum_{i=1}^{m} \nu_i x_{ij} = 1$$

$$\mu_r, \nu_i \geq \varepsilon > 0, u_o \text{ free in sign}$$

BCC Output-Oriented

Envelopment model	Multiplier model

$$\max \phi + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io}, i = 1,2,\dots,m;$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \phi y_{ro}, r = 1,2,\dots,s;$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, j = 1,2,\dots,n.$$

$$\min q = \sum_{i=1}^{m} \nu_i x_{io} - \nu_o$$

s.t.

$$\sum_{i=1}^{m} \nu_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \nu_o e \geq 0$$

$$\sum_{r=1}^{s} \mu_r y_{ro} = 1$$

$$\mu_r, \nu_i \geq \varepsilon > 0, \nu_o \text{ free in sign}$$

Table 1- Different BCC models (Cooper et al. , 2004)

Some difficulties in DEA

- No common set of weights

- No strict bounding for weights (probability of having non-realistic answers):

- Some inputs or outputs can be characterized by low or high weight values;
- Contradiction with a priori information offered by the Decision Maker (DM).

- DMUs can not be ranked with such a weights, which may vary from unit to unit

Weight Restricted DEA models

- **Thompson et al. (1986):** assessing the efficiency of physics laboratories (AR),
- **Dyson and Thanassoulis (1988):** eliminating use of zero weights (RA),
- **Wong and Beasley (1990):** introducing virtual weights DEA models,
- **Roll and Golany (1993):** using generated weights of DEA model,
- **Takamura and Tone (2003):** using the judgments of people,
- **Ueda (2000,2007):** suggesting a canonical correlation analysis,
- **Dimitrov and Sutton (2012):** proposing a symmetric weight assignment technique.

Using MCDA in DEA to determine bounds

❖ *DEA and AHP:*

- **Shang et Sueyoshi (1995):** using subjective ***AHP*** results in ***DEA*** to rank and select between flexible manufacturing systems: the pareto solutions of ***DEA*** and the subjectivity of ***AHP***

- **Sinuany-Stern et al. (2000):** suggesting two stage ***AHP/DEA*** ranking model: removing the pitfalls of Shang et Sueyoshi but does not incorporate the DM preferences

- **Takamura and Tone (2003):** integrating ***AR*** and ***AHP***: 1. providing criteria weights for each DM by ***AHP***, 2. employing ***AR*** to limit them: more than one DM

- **Liu (2003):** Combining ***DEA*** and ***AHP*** to integrate two objective and subjective weight restrictions method

- **Han-Lin Li and Li-Ching Ma (2008):** Developing an iterative method of ranking DMUs by integrating ***DEA***, ***AHP*** and ***Gower plot***

Some unwillingness of AHP

- Lack of undeniable foundations on the utility preferences of the DM (Saati, 1986, Barzilai et al., 1987, Dyer, 1990, Winkler, 1990);

- No special graphical tool;

- Subjectivity: constructing a pair wise comparison matrix based on DM's preferences. From the view point of a DM: easier to use some models with less subjectivity to evaluate different alternatives (Sinuany-Stern et al., 2000).

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❖ *DEA and MACBETH:*

- **Junior (2008):** Employing ***MACBETH*** as a ***MCDA*** tool to produce the bounds of the weights and adding these restrictions to a virtual weight ***DEA*** model to evaluate the alternatives/DMUs.

  MACBETH: a MCDA approach to help an individual or a group, quantifying the relative attractiveness of options by qualitative judgements about differences in value (Bana e Costa et al., 1993)

❖ Causing a contradicted result with MACBETH ranking. To avoid this weakness: adding some extra constraints to the virtual weight restrictions

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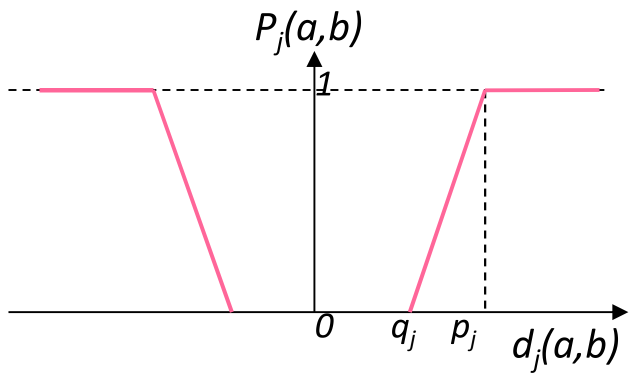
PROMETHEE II

- **J. P. Brans (1982):** based on ***pair wise comparisons***: allowing a DM to ***rank completely*** a finite set of n actions that are evaluated over a set of *k* criteria:
  - For each criterion f_j , j=1,2,…,k:
    - Preference function P_j
    - Weight w_j



  - ***Preference degree*** of *a* over *b*:

$$\boxed{\pi(a,b)=\sum_{j=1}^{k} w_j\ P_j(a,b)}$$

- ***Net flow score***

$$\phi(a) = \sum_{j=1}^{k} w_j \cdot \phi_j(a)$$

with

$$\phi_j(a) = \frac{1}{n-1} \sum_{b \in A} \left[P_j(a,b) - P_j(b,a) \right]$$

- ***Unicriterion net flow score***

Weight Stability Intervals (Mareschal, B. (1988))

- ***what is the impact of changing a given weight value in a computed ranking?***

Determination of exact weight values is often a cognitive complex task for the DM.

↓

Purpose of WSI:

Preserve the preference ranking of a subset of alternatives: ***automated*** generation of intervals limits (confirming the robustness of *PROMETHEE II* outputs, typically the first alternative).

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Synergies between *DEA* & *PROMETHEE*

a) Common problems encountered in *DEA* and *PROMETHEE:*

- Rank reversal

b) *DEA* applied to *PROMETHEE:*

- A quantitative comparison between the weighted sum and PROMETHEE II using DEA (Bagherikahvarin M., De Smet Y., 75th MCDA Conference, Tarragona, Spain, 2012)

- Defining new possible weight values in PROMETHEE VI: a procedure based on Data Envelopment Analysis (Bagherikahvarin M., De Smet Y., 1st International MCDA workshop on PROMETHEE, Brussels, Belgium, 2014)

c) *PROMETHEE* applied to *DEA:*

- Complete ranking in DEA by PROMETHEE II

- Weighted DEA model based on PROMETHEE II

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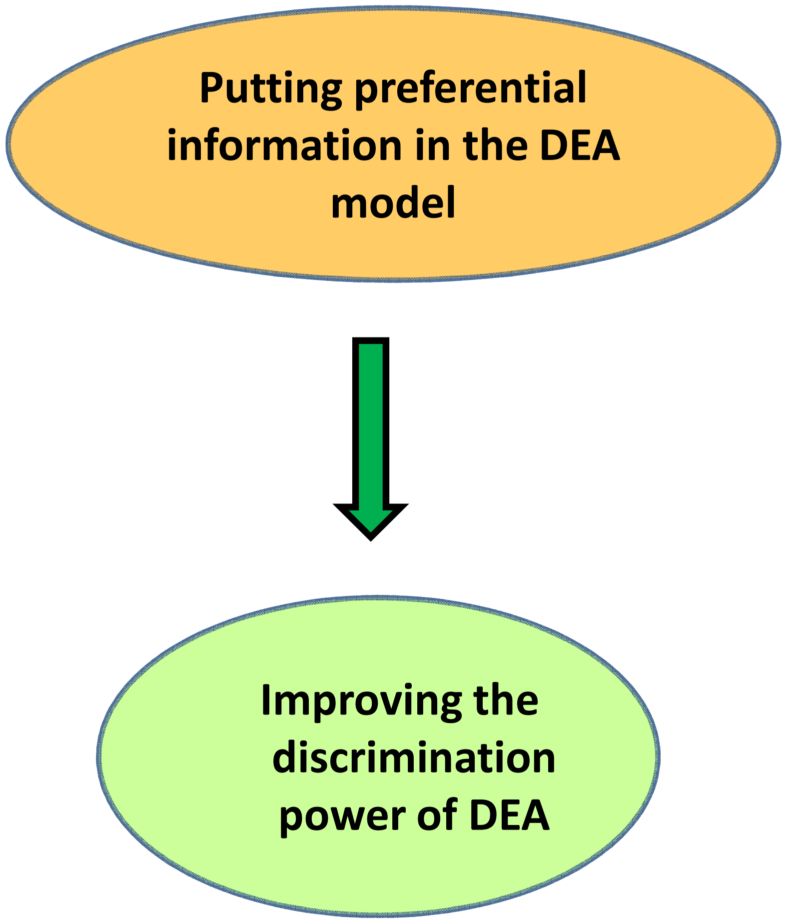
Weighted DEA model based on PROMETHEE II



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❖ The steps of algorithm:

1. The algorithm inputs: an evaluation table, preference functions and parameters (indifference, preference thresholds and weights);

2. PROMETHEE II: Net flow scores, Unicriterion net flow scores and WSI;

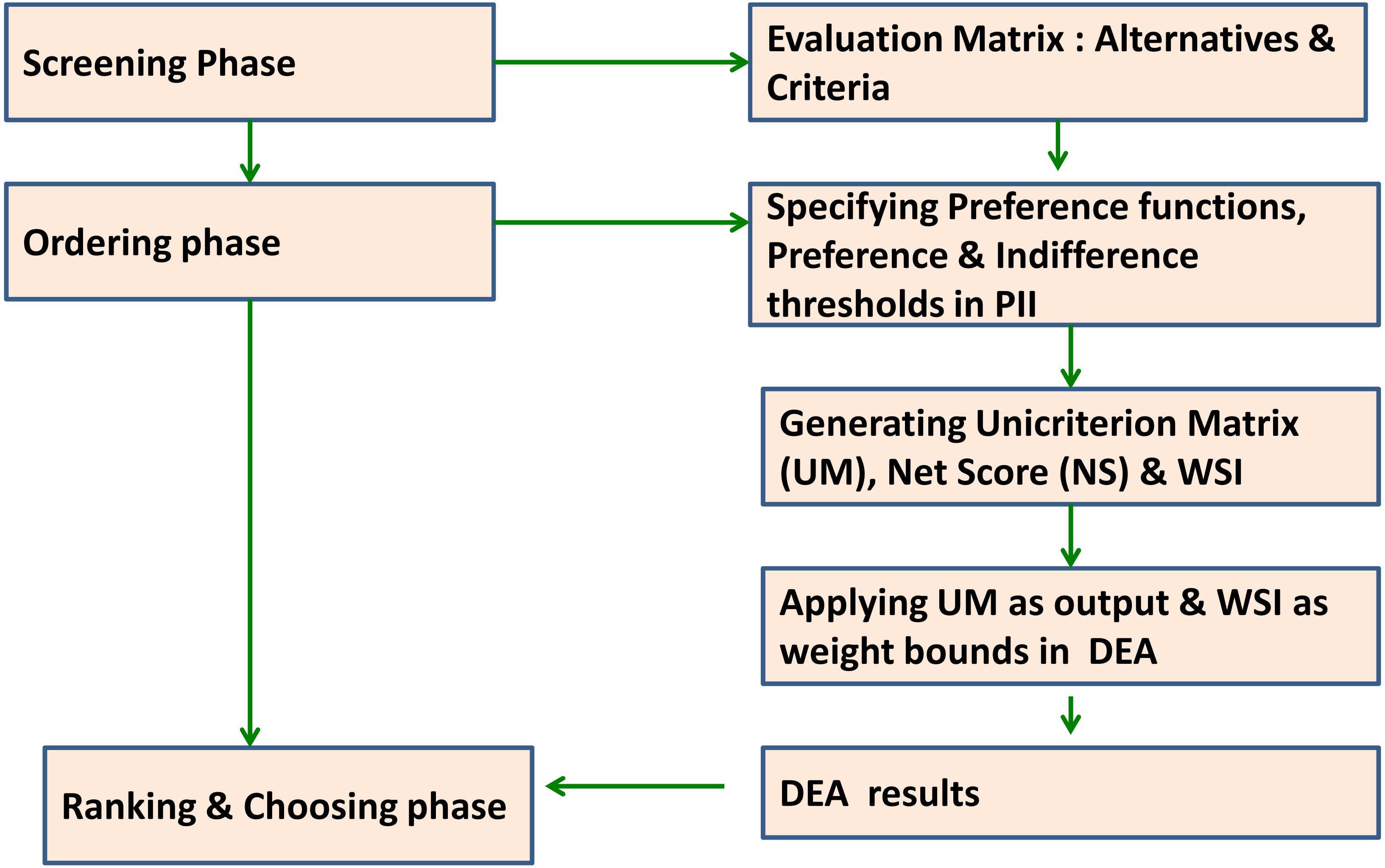
3. **Maximize** bet flow score and **Restrict** DEA weights by PROMETHEE II WSI in the first level;

4. Induce a DEA ranking;

5. Use a super efficiency model to present a complete ranking.

(MACBETH (Junior, H. V., 2008) and ELECTRE (Madlener R. et al. 2006) has been proposed such a method

The decision making framework



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PROMETHEE II Weighted CCR model *(PIIWCCR)*

$$E_o = \mathrm{Max}[\phi(a_o) = \sum_{j=1}^{k} w_j \varphi_j(a_o)]$$

such that

$$\sum_{j=1}^{k} w_j \varphi_j(a_i) \leq 1; i = 1,\ldots,n,$$

$$w_j^- \leq w_j \leq w_j^+;$$

$$w_j \geq 0$$

w_j^- and w_j^+ are WSI in PROMETHEE II

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Irrigation management (to choose a water pricing policy) (Yilmaz and Yurdusef, 2011):

- Comparing 36 alternatives according to 7 criteria:

  C_1 (crops profitability), C_2 (used water efficiency), C_3 (social impact including employment), C_4 (initial cost), C_5 (maintenance cost), C_6 (irrigation water volume used), C_7 (pollution effect)

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Min/Max	Max	Max	Max	Min	Min	Min	Min
Type	Linear	Linear	Linear	Linear	Linear	Linear	Linear
Thresholds	q=0.1,p=1	p=0.5	q=0.5,p=1	q=0,p=0.29	q=0.1,p=0.26	q=0,p=0.26	q=0,p=0.46
Weights	0.3	0.25	0.09	0.1	0.1	0.1	0.06

Table 2- Irrigation management (Yilmaz and Yurdusef, 2011)

WSI in level 1

Criteria	Min weight	Value	Max weight
C_1	0.036	0.3	1
C_2	0	0.25	0.407
C_3	0	0.09	0.529
C_4	0	0.1	0.502
C_5	0	0.1	1
C_6	0	0.1	0.383
C_7	0	0.06	1

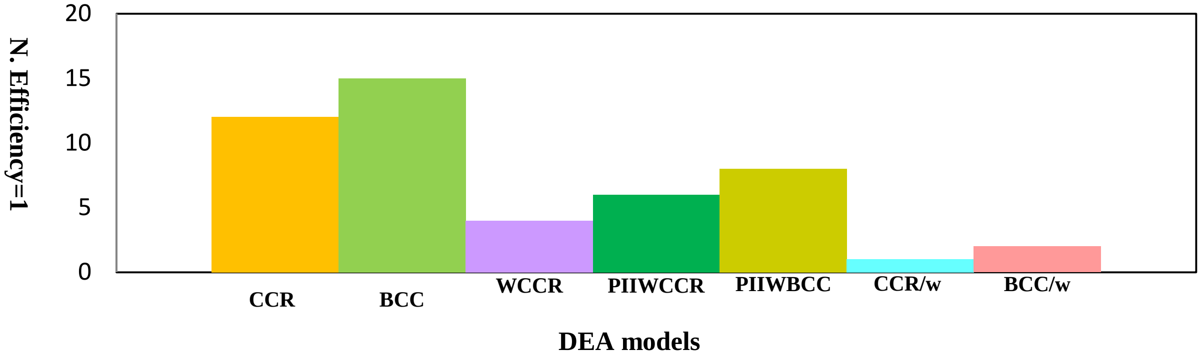
Table 3- Irrigation management, **WSI**

Rank	EL.3	PR.II	SE-WCCR	PIIWCCR	PIIWBCC	RCCR/w, CCR/w	BCC/w
1	26	26	26	26	26	26	26
2	28	34	34	34	4	28	30
3	2	30	4	4	28	2	28
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.
34	9	3	23	11	23	21	21
35	11	7	7	19	11	23	11
36	23	11	19	23	9	11	23

Table 4- Irrigation management (Yilmaz and Yurdusef, 2011)

	EL. 3	PR.II	CCR	BCC	CCR/w	BCC/w
PIIWCCR	0.807	0.914	0.877	0.898	0.824	0.877
PIIWBCC	0.800	0.826	0.871	0.854	0.769	0.803

Table 5- Spearman correlation at the 0.01 level



The number of efficient DMUs was reduced:

CCR (12), PIIWCCR (6)
BCC (15), PIIWBCC (8)

• Medium sized companies in Brussels

Comparing 75 companies according to 6 criteria

(Revenue (Turnover), cash-flow and employees: absolute and relative growth)

“Gazelles” ranking in March 2014: assigning a rank to each criterion in each company (during 4 years): obtaining final score by adding the rank of each company in each classification of criteria (the growth value of each criterion during 4 years)

PROMETHEE II, BCC, GAZELLES, New weighted DEA model:

The WSI in level 1

1. ***H & M logistic*** always the best (DMUs 1 (H&M Log.), 14 (Lubrizol), 37 (BBC Corp.) always between the best)

2. Decreasing the number of efficient units: BCC (7), PIIWBCC (3)

3. Approximating the result of DEA and PROMETHEE II by maximizing the net flow score of PROMETHEE II in a DEA problem:

$$\Downarrow$$

More correlation

- r=1

	SE-PIIWCCR	PR.II	SE-CCR	BCC	Gazelles
SE-PIIWCCR	1	**0.884**	0.857	0.860	0.807
PR.II		1	**0.622**	0.623	0.991
SE-CCR			1	0.989	0.641
BCC				1	0.645
Gazelles					1

Table 6- Spearman correlation at the 0.01 level (r=1)

- r=3

	PIIWCCR	PIIWBCC	PR.II	SE-CCR	BCC
PIIWCCR	1	0.906	**0.962**	0.643	0.650
PIIWBCC		1	**0.961**	0.695	0.702
PR.II			1	**0.622**	**0.623**
CCR				1	0.998
BCC					1

Table 7- Spearman correlation at the 0.01 level (r=3)

Moore Stephens

- ***Moore Stephans:*** **3d place in PROMETHEE II** ranking: fixing the stability level of problem in its rank, 3

less equal efficient DMUs and more correlation between rankings: PIIWCCR and PIIWBCC (1)

•Wellbeing in Wallonia

Comparing the level of wellbeing in 132 municipalities of Wallonia according 13 criteria (Charlier, J. et al., 2014)

Centers ***Tintigny*** and ***Ottignies-LLN*** always the best

less equal efficient DMUs: BCC (85), PIIWBCC (25)

	SE-PIIWCCR	PIIWBCC	SE-WCCR	PR.II	SE-CCR	BCC
SE-PIIWCCR	1	0.844	0.394	0.881	0.376	0.486
PIIWBCC		1	0.444	0.810	0.51	0.529
SE-WCCR			1	0.182	0.915	0.446
PR.II				1	0.136	0.281
SE-CCR					1	0.500
BCC						1

Table 8- Spearman correlation at the 0.01 level (r=1)

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The main advantages of our model

- Discrimination power of ***DEA*** is increased by using ***PROMETHEE II WSI*** in a ***DEA*** model;

- The DM does not have to fix bounds to ***DEA*** weights which is found a difficult task;

- ***PROMETHEE II*** lets generate different ***WSI*** in ***different levels***: higher level, less efficient equal units, more correlation;

- As expected ***approximation*** of ***PROMETHEE II*** and ***DEA*** is possible through our model (more correlation).

Further deepening ideas:

❖ Applying the stability intervals in proportional form in DEA;

❖Using partial or subset stability intervals of PROMETHEE in DEA;

❖Proposing this model in D-sight software;

❖…

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Thanks for your attention