A summary of recent contributions in the PROMETHEE methods (from members of the SMG unit)

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International MCDA workshop on PROMETHEE: Research and case studies

Outline

- Brief reminder about PROMETHEE and GAIA
- Main topics developed since 2007 (SMG);
  - Software implementations
  - Preference elicitation
  - Sorting and clustering
  - Visual representations (application to GIS)
  - Rank reversal
- Directions for future research

PROMETHEE in a nutshell (1)

- A **finite** set of alternatives:

$$A=\{a_1,a_2,..,a_n\}$$

- A set of criteria:

$$F=\{f_1,f_2,..,f_q\}$$

- W.l.g. these criteria have to be **maximized**
- **Step 1:** computation of unicriterion preferences degrees

$$\forall a_i, a_j \in A: d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$$

$$\pi_k(a_i, a_j) = P_k[d_k(a_i, a_j)]$$

PROMETHEE in a nutshell (2)

- **Step 2**: Computation of preference degrees:

$$\forall a_i, a_j \in A : \pi(a_i, a_j) = \sum_{k=1}^{q} w_k \pi_k(a_i, a_j)$$

- **Step 3**: Computation of flow scores:

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(a_i, b)$$

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(b, a_i)$$

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$$

More formally:

$\emptyset_A^+(a_i)$, $\emptyset_A^-(a_i)$, $\emptyset_A(a_i)$

GAIA in a nutshell (1)

- We have:

$$\Phi(a_i) = \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(a_i,b) - \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(b,a_i)$$

$$= \sum_{k=1}^{q} w_k.\frac{1}{n-1}\sum_{b\in A}\pi_k(a_i,b)-\pi_k(b,a_i) = \sum_{k=1}^{q} w_k.\phi_k(a_i)$$

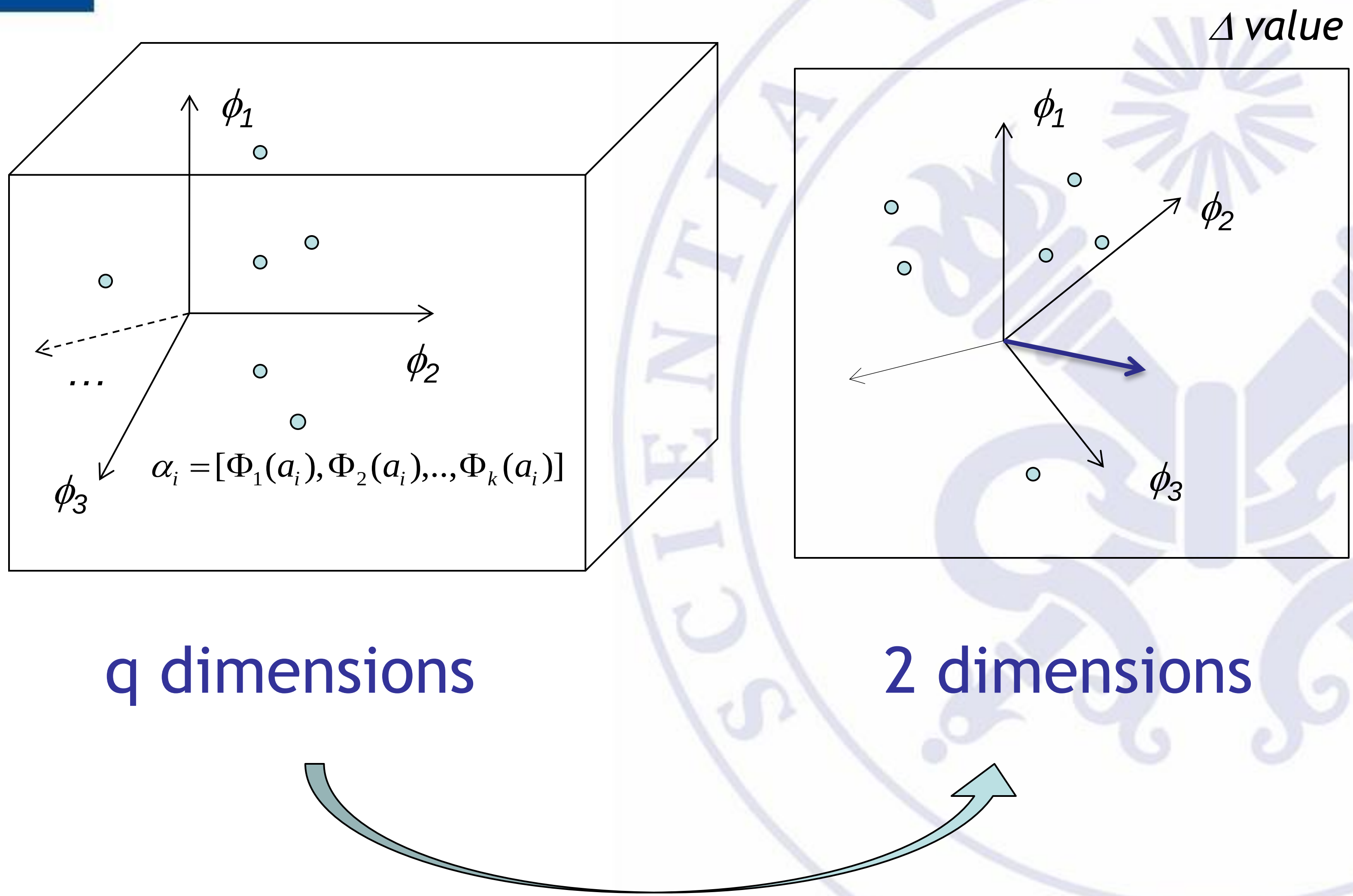
- Where

$$\Phi_k(a_i) = \sum_{b\in A}\pi_k(a_i,b)-\pi_k(b,a_i)$$

- In other words, every alternative can be represented by a vector:

$$\alpha_i = [\Phi_1(a_i),\Phi_2(a_i),..,\Phi_k(a_i)]$$

GAIA in a nutshell (2)



Principal component analysis

Software implementations

Software implementations

Hayez, Q., De Smet, Y. and Bonney, J. "***D-Sight: a new decision making software to address multi-criteria problems***" International Journal of Decision Support Systems Technologies, 4(4), 1-23 (2012)

- Long story: **PROMCALC, DECISION LAB 2000**
- Spin-off project funded by the Walloon Region (2007-2010)
- D-SIGHT desktop
- D-SIGHT web **Session C - 2nd presentation**
- Innoviris award in 2012 « ***Jeune entreprise innovante*** »
- But also Decision Deck, Smart Picker, Visual Promethee, …

D-SIGHT®

D-SIGHT Projects

Areas of Expertise

D-Sight has been developing specific expertise in certain industries over the past years. We are proud of having excellent customers, which help us developing and improving our software solutions.

Environmental Analysis

Projects Prioritization

Vendor Selection

Academia: 50 universities in 27 countries

Preference elicitation

Contributions:

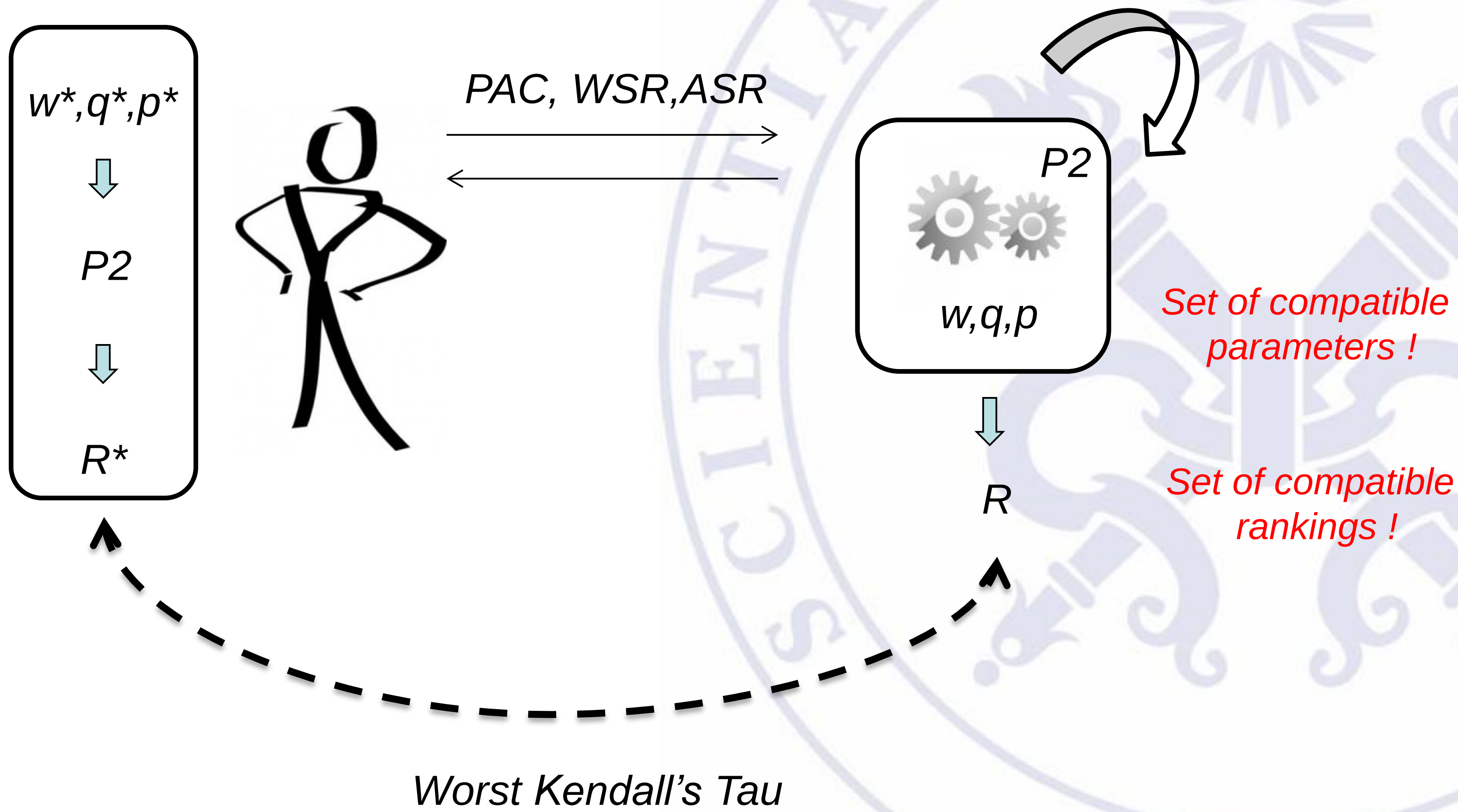
Eppe, S., De Smet, Y., Stützle, T. «**A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method** » Proceedings of ADT (2011), 56-66 (2011)

Eppe, S. et De Smet, Y. « **Studying the impact of information structure in the PROMETHEE II preference elicition process : a simulation based approach** », 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2012, Catania, Italy, July 9-13, 2012, LNAI Proceedings, 383-392, 2012

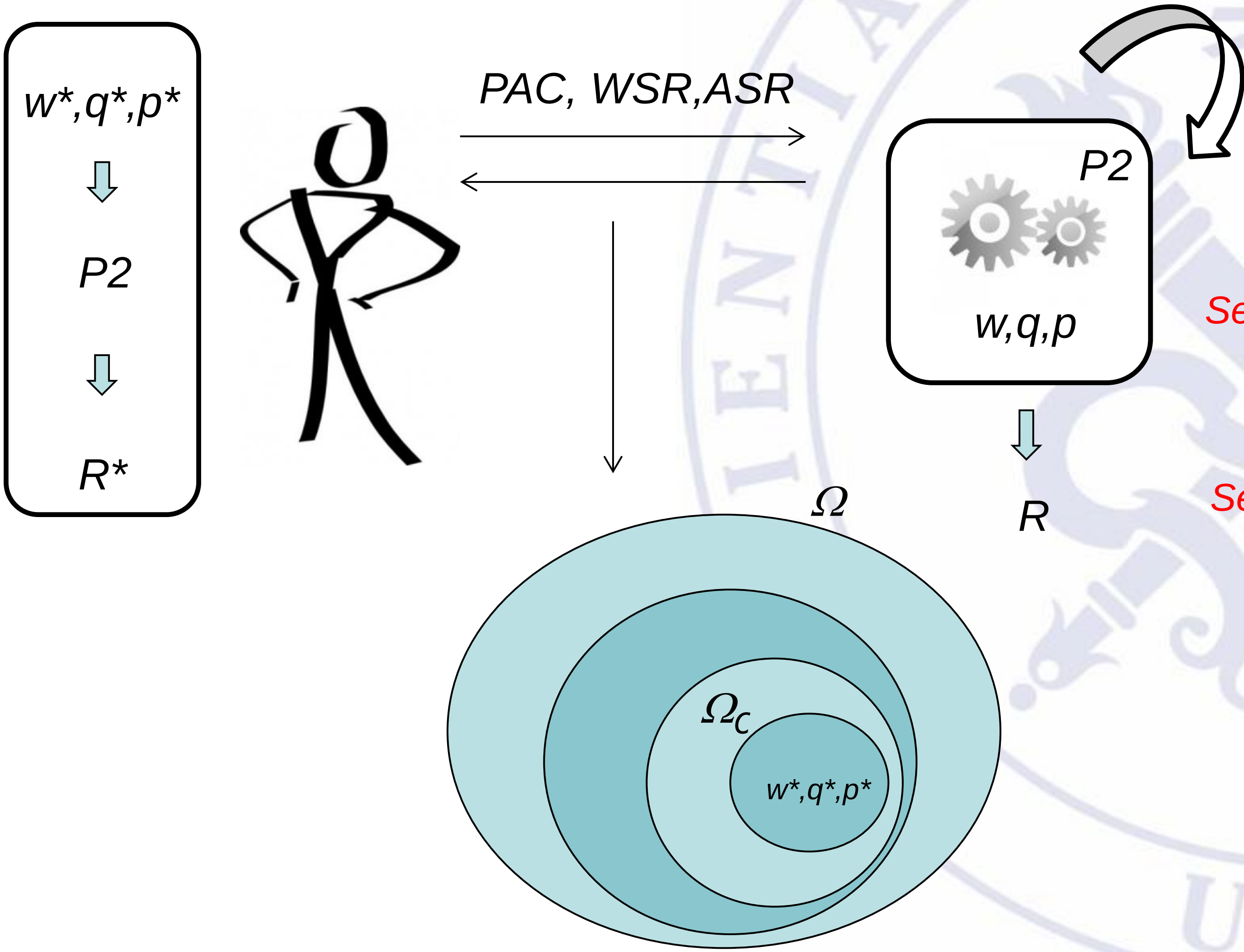
Eppe, S. and De Smet, Y. "**An adaptative questioning procedure for eleciting PROMETHEE II's weight parameters**" to appear in the International Journal of Multicriteria Decision Making (2013)

Eppe, S. and De Smet, Y. "**Approximating PROMETHEE II's net flow scores by piecewise linear value functions**" to appear in the European Journal of Operational Research (2013)

General idea (1)



General idea (2)

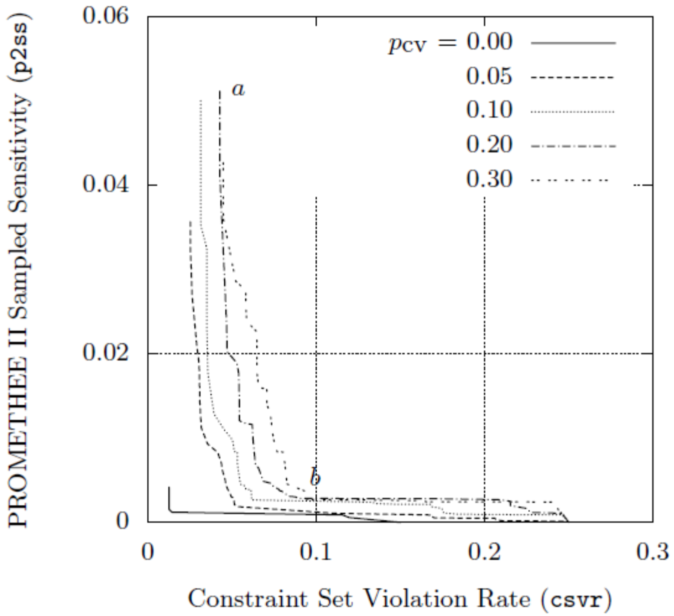


Set of compatible parameters !

Set of compatible rankings !

Eppe, S., De Smet, Y., Stützle, T. (2011) « **A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method** » Proceedings of ADT (2011), 56-66

- **Main idea**: quality and robustness
- **Distinctive feature:** the DM may communicate mistakes



(n=100,q=2)

NSGA-II

Fig. 4. This plot represents the approximated Pareto frontiers in the objective space, for 20 constraints and several values of the constraint violation rate p_{cv}, i.e., the proportion of inconsistent constraints with respect to the total number of constraints. As expected, increasing the value of p_{cv} has the effect of deteriorating the quality of the solution set both in terms of constraint violation rate and PROMETHEE II sampled sensitivity.

Eppe, S., De Smet, Y. (2012) « **Studying the impact of information structure in the PROMETHEE II preference elicitation process: A simulation based approach** » 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2012, Catania, Italy, July 9-13, 2012, LNAI Proceedings, 383-392, 2012

- **Main idea**: quantify information infrastructure …

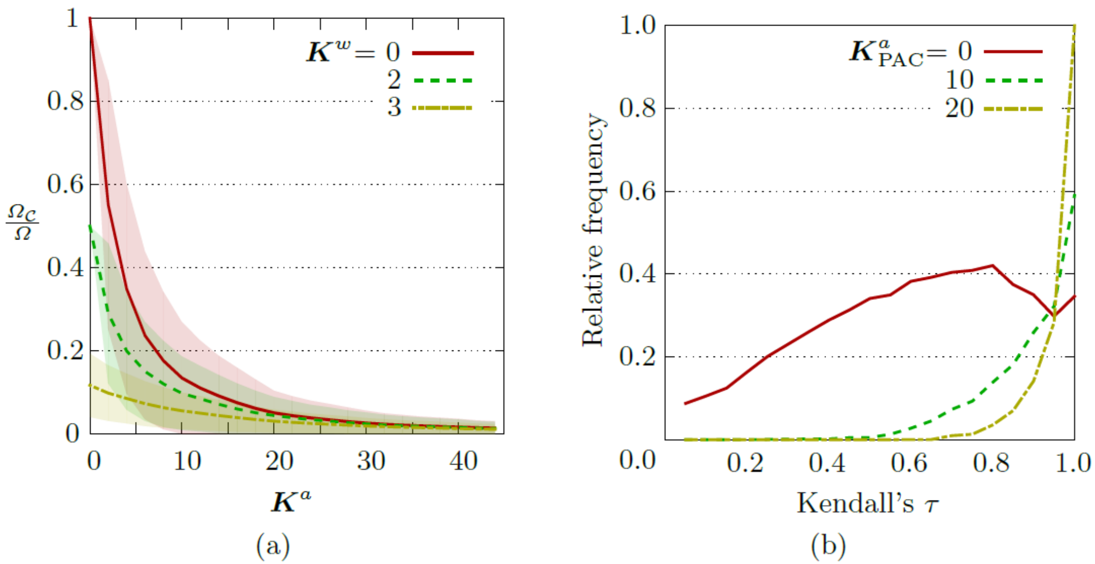


Fig. 3. **(a)** Evolution of the ratio of compatible weight domain area $\frac{\Omega_C}{\Omega}$ with respect to the whole domain of possible weights Ω, depending on the number of action constraints K^a, for different numbers of weight constraints K^w. Results are shown for 1000 randomly generated action sets ($n = 10$, $m = 3$) and PAC constraint sets. — **(b)** Distribution of all values of Kendall's τ in the compatible weights domain Ω_C, for respectively $K^a_{\mathrm{PAC}} = 0$, 10, and 20 pairwise action comparisons. No constraints on the weights relative importance are given here ($K^w = 0$).

Eppe, S., De Smet, Y. (2012) « **Studying the impact of information structure in the PROMETHEE II preference elicitation process: A simulation based approach** » 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2012, Catania, Italy, July 9-13, 2012, LNAI Proceedings, 383-392, 2012

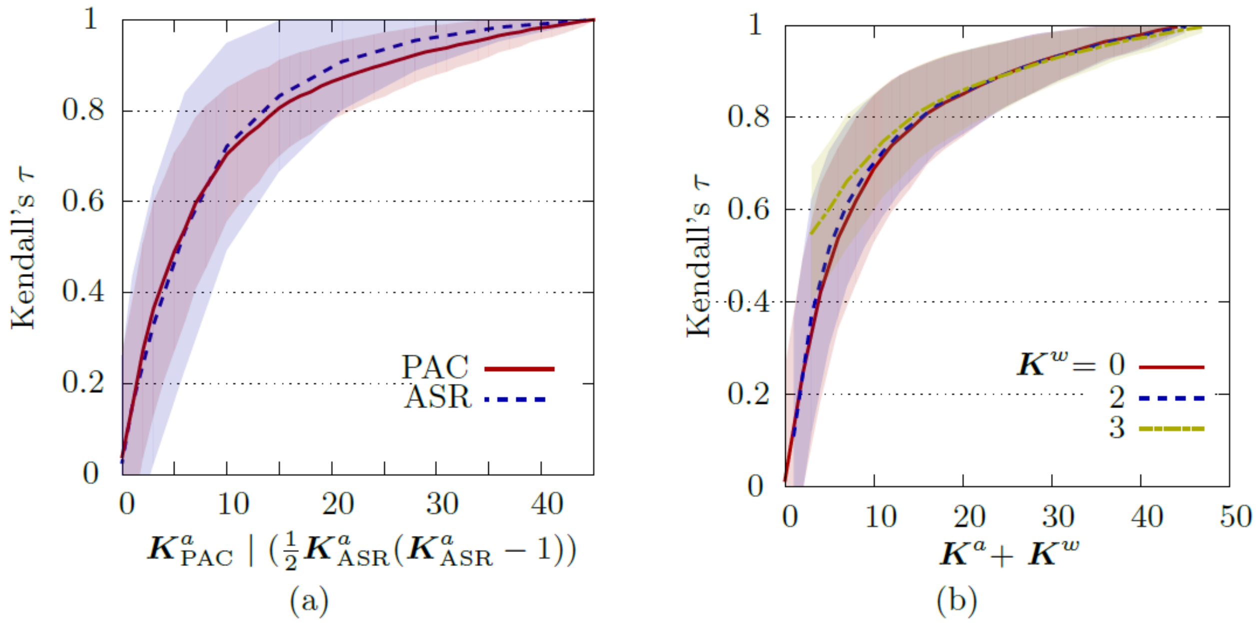


Fig. 4. (a) Evolution of worst Kendall's τ for two different information types: pairwise action comparisons (PAC) and action sub-ranking (ASR). — **(b)** Impact of "weight constraints" on the reachable quality for pairwise action comparisons (PAC). Note that the x-axis represents the sum of action *and* weight constraints, i.e. $\boldsymbol{K}^a + \boldsymbol{K}^w$.

Eppe, S., De Smet, Y. (2012) « **Studying the impact of information structure in the PROMETHEE II preference elicitation process: A simulation based approach** » 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2012, Catania, Italy, July 9-13, 2012, LNAI Proceedings, 383-392, 2012

Table 1. Number of pairwise action comparisons that have to be given by a DM to reach the desired level of quality $\underline{w}$, assuming that K^w weight constraints have already been provided. The results are shown for randomly generated 3-criteria action sets with a uniform distribution.

	$\underline{w}$						
K^w	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0	6	8	10	15	25	34	43
2	4	6	9	14	24	33	43
3	0	1	5	11	22	32	42

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

- **Main idea**: to overcome the limitations of the previous approach; pairwise comparisons have to be « well-chosen »
- **q-Eval**

Table 2 Maximum number of queries that could be generated on average with *q-Eval* for 30 requested queries for 50 randomly generated instances.

	m			
n	3	5	7	10
10	8.2	12.7	15.4	17.4
20	12.3	20.5	26.3	29.5
50	15.88	28.46	30.0	30.0
100	17.26	29.46	30.0	30.0
200	17.96	29.84	30.0	30.0

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

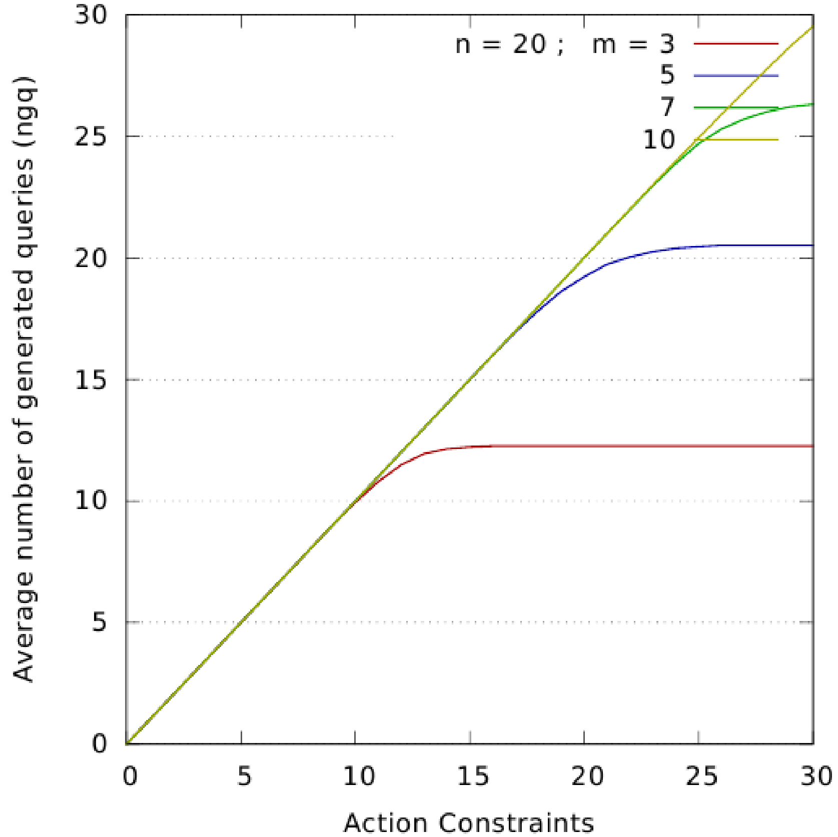


Figure 2 Evolution of the average number of constraints that can be generated with the *q-Eval* method depending on the instance size (number m of alternatives) for a given number of alternatives. Plots for a constant value of m and different number of alternatives n are very similar.

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

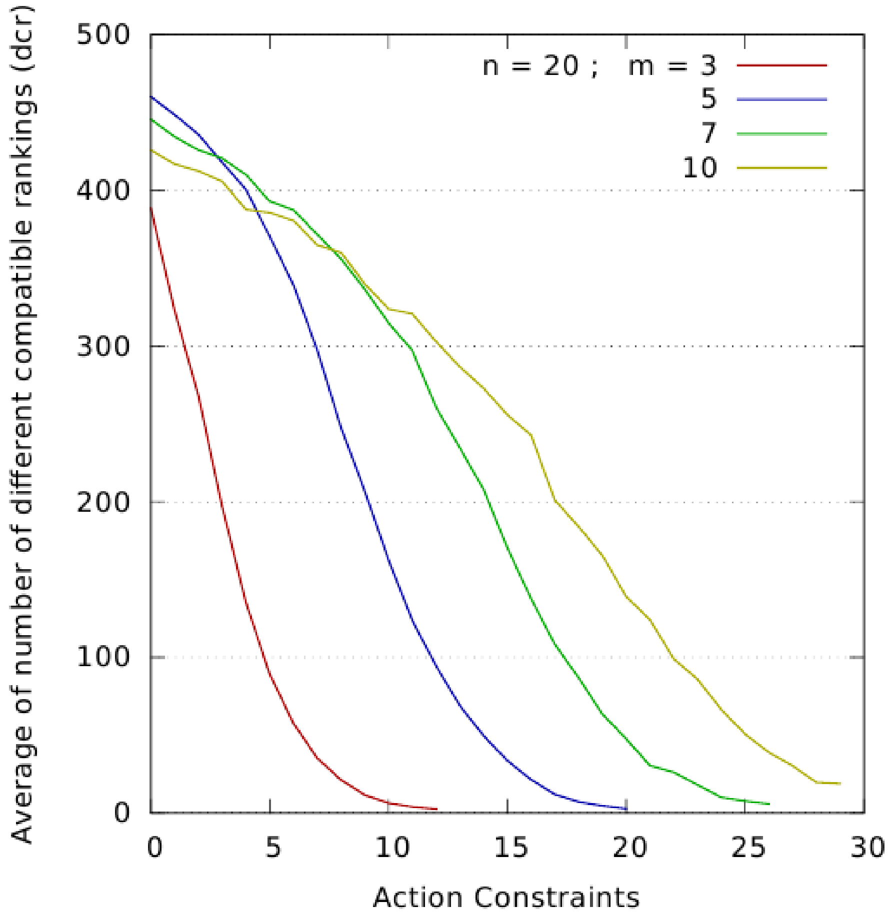


Figure 3 For $n = 20$ actions, shows the average evolution of the number of different rankings found in the sampling of the compatible parameter domain Ω, for different numbers m of criteria.

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

Table 3 This table represents the average value of the worst Kendall's correlation coefficient τ_G for the sampling of the compatible parameter domain Ω for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

	m			
n	3	5	7	10
10	0.999	0.998	0.988	0.988
20	1.000	1.000	0.999	0.994
50	1.000	1.000	0.998	0.987
100	1.000	1.000	0.997	0.983
200	1.000	1.000	0.998	0.982

Table 4 This table represents the average number of different rankings in the samples for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

	m			
n	3	5	7	10
10	1.1	1.1	2.4	3.9
20	1.0	1.5	2.2	14.1
50	4.8	18.6	204.4	358.8
100	44.6	226.7	430.8	465.9
200	187.9	432.1	477.8	490.5

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

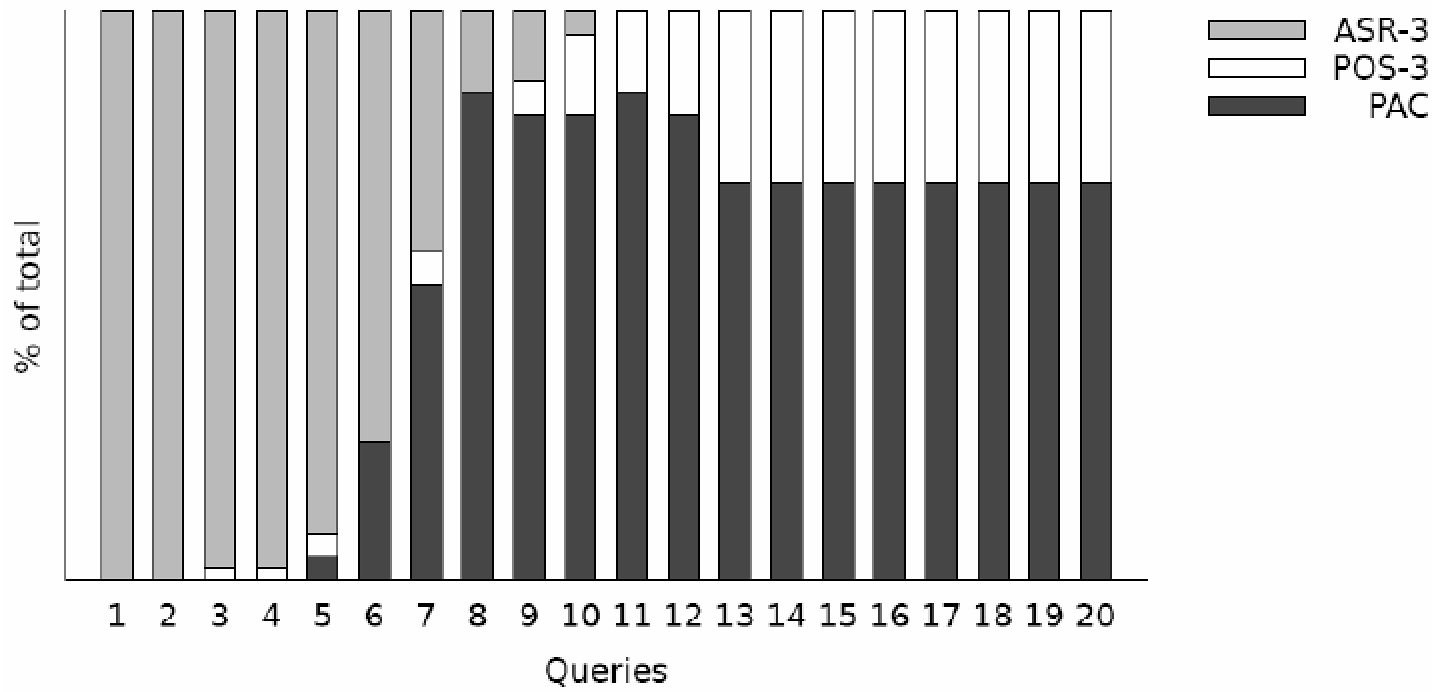
Table 12 Number of queries that has to be answered by a DM to reach the desired level of quality $\underline{\tau_G}$ for randomly generated action sets (n = 20 actions and m = 5 criteria)

		$\underline{\tau_G}$						
		0.50	*0.60*	*0.70*	*0.80*	*0.90*	*0.95*	*0.99*
Random	PAC	13	17	21	25	35	47	
Q-Eval	PAC	4	5	6	7	9	11	17
	POS-3	2	3	4	4	6	8	
	ASR-3	2	3	4	4	4	5	
	Adaptive	2	2	3	3	4	5	10

Note: The extended Q-Eval method significantly outperforms the bottom-line approach (Eppe and De Smet, 2012), presented in the first row, and the original *Q-Eval*, shown in the second row.

Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II's weight parameters** » to appear in the International of Multicriteria decision making

Figure 9 Evolution of the mean ratio of each *Q-Eval* query-type when applying the *adaptive query selection scheme*



Note: Each type of query plays a role and has its usefulness in the process at different stages of the eliciting process.

Sorting and clustering

Contributions

De Smet, Y. et Gilbart, F. « **A class definition method for country risk problem** », IS-MG 2001/13

Figueira, J., De Smet, Y. et Brans, J.P. « **MCDA methods for sorting and clustering problems : Promethee TRI and Promethee CLUSTER** » , IS-MG 2004/02

Nemery, Ph. and Lamboray, C.: "**FlowSort : A flow-based sorting method with limiting and central profiles**", TOP 16, 90-11

De Smet, Y. “**P2CLUST: an extension of PROMETHEE II for ordred clustering**” to appear in the proceedings of the 2013 IEEE International Conference on Industrial Engineering and Engineering Management, Bangkok, Thaïland, (2013)

Flowsort - main idea

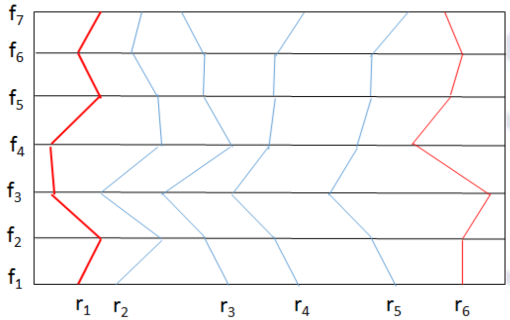
- Let us consider a set of limit (or central profiles)

$$R = \{r_1, r_2, .., r_{K+1}\}$$
$$R_i = R \cup \{a_i\}$$

- Each actions a_i is sorted according to its relative position based with respect to the profiles according to

$$\emptyset_{R_i}(a_i)$$

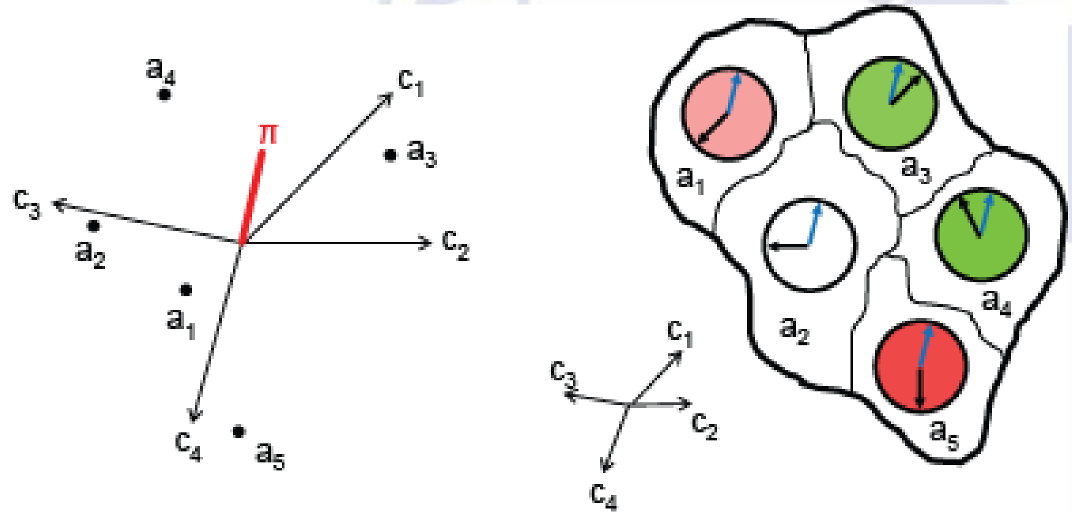
- Complete or partial sorting
- Idea behind P2CLUST Session A1 – 1st presentation



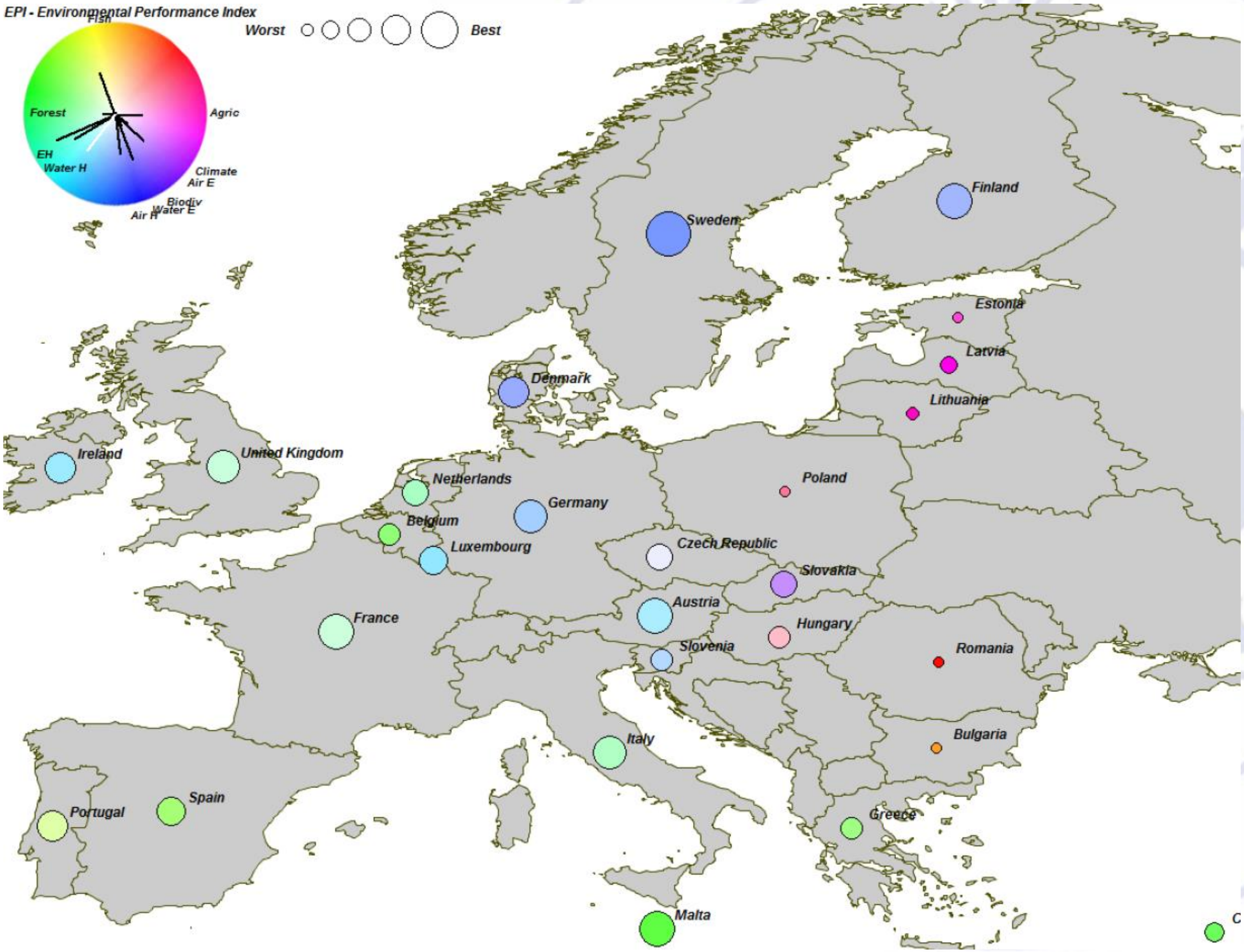
PROMETHEE and GIS
(Karim Lidouh PhD)
Session B2 - 4th presentation

Decision Clocs

Lidouh, K., De Smet, Y. et Zimányi, E. « **GAIA Map : A Tool for Visual Ranking Analysis in Spatial Multicriteria Problems** » in Proceedings of the 13th International Conference on Information Visualization 2009, Barcelona – IEEE Computer Society, 393-402, 2009



Lidouh, K., De Smet, Y. and Zimányi, E. “**An Adaptation of the GAIA Visualization Method for Cartography**”, in the proceedings of the 2011 IEEE Symposium on Computational Intelligence in Multicriteria Decision-Making, France, 29-35, 2011



Rank reversal
Session A1 – 5th presentation

Rank reversal (1)

- Rank reversal …
  - AHP: *Belton and Gear (1983), Saaty and Vargas (1984), Triantaphyllou (2001), Wang and Elhag (2006), Wijnmalen and Wedley (2009)*
  - ELECTRE: *Wang and Triantaphyllou (2005)*
  - PROMETHEE: *De Keyser and Peeters (1996)*
- **The concept of rank reversal is not fully formalized** (*add a copy of an alternative, deletion of a non discriminating criterion, deletion of an alternative, …)*
- *A direct consequence of Arrow's theorem*
- Positive results:
  - Dominance
  - Non discriminating criterion

Contributions

Mareschal, B., De Smet, Y. and Nemery, P. “**Rank Reversal in the PROMETHEE II Method : Some New Results**”, proceedings of de IEEE 2008 International Conference on Industrial Engineering and Engineering Management, Singapore, 959-963 (2008)

Roland, J., De Smet, Y. and Verly, C. “**Rank reversal as a source of uncertainty and manipulation in the PROMETHEE II ranking : a first investigation**” to appear in the proceedings of the IPMU 2012 conference (LNAI)

Verly, C. and De Smet, Y. “**Some considerations about rank reversal occurrences in the PROMETHEE methods**” accepted for publication in the International Journal of Multicriteria Decision Making.

More general result (1)

Notations: $A_x = A \backslash \{x\}$, $\Phi_x(a)$

No RR $\Leftrightarrow \big(\Phi(a) - \Phi(b)\big)\big(\Phi_x(a) - \Phi_x(b)\big) > 0$

if $\Phi(a) - \Phi(b) > \dfrac{[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$

No RR (for any action removed) if

$$\Phi(a) - \Phi(b) > \frac{\max_x [(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$$

ULB More general result (2)

➔ *RR can only occur if*

$$\Phi(a) - \Phi(b) < \underbrace{\frac{\max_x[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}}_{\substack{\textit{refined threshold} \\ \textit{(depends on the sample and (a,b))}}} \leq \underbrace{\frac{2}{n-1}}_{\substack{\textit{rough} \\ \textit{threshold} \\ \textit{(constant)}}}$$

Generalization: when k actions are removed

No RR if $\Phi(a) - \Phi(b) > \frac{2k}{n-1}$

Mareschal, B., De Smet, Y. and Nemery, P. « *Rank Reversal in the PROMETHEE II Method : Some New Results* », proceedings of de IEEE 2008 International Conference on Industrial Engineering and Engineering Management, Singapore, (2008): 959-963

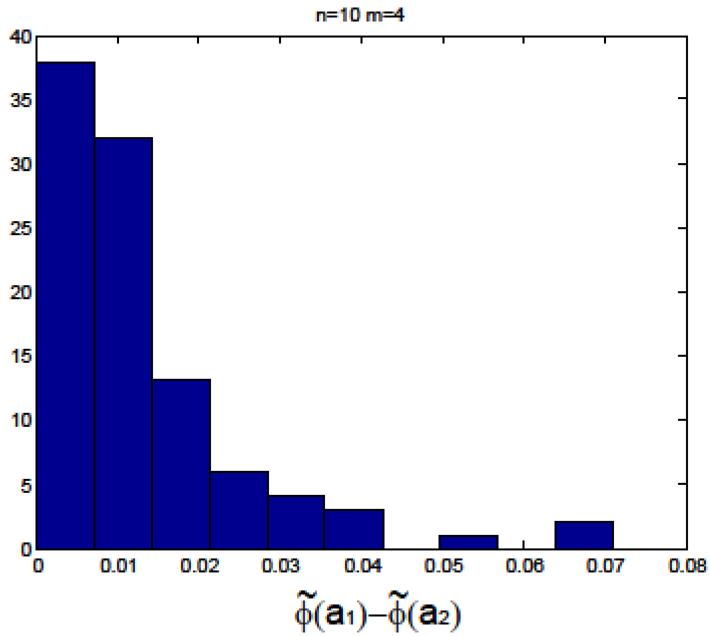
More general result (3)

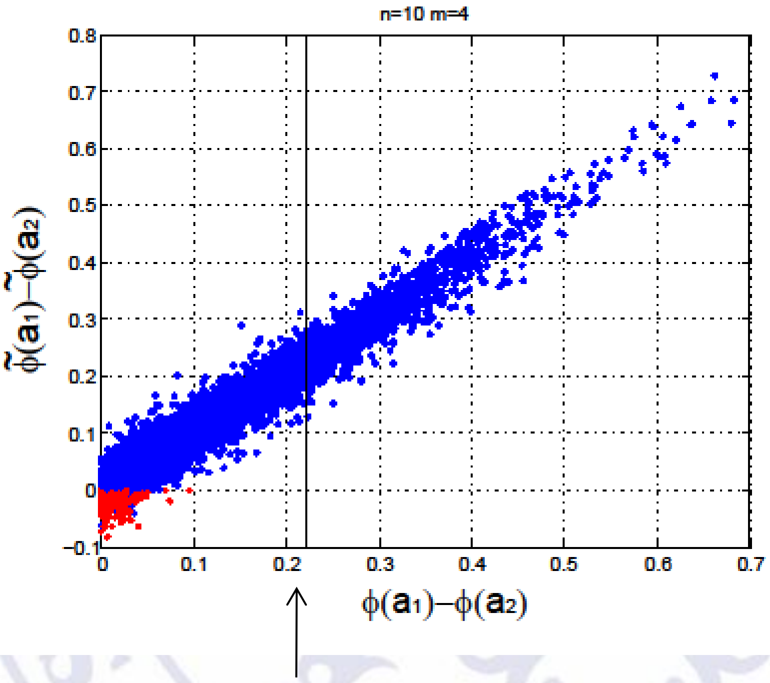
Statistical results relative to the «rough threshold» (for q = 2, DA=Unif)

n	nb RR	$b = \frac{2}{n-1}$	nb $\Delta\Phi \leq b$	nb RR $\mid \Delta\Phi \leq b$
5	2,20 %	0,50	47,4 %	4,6 %
10	0,98 %	0,22	33,5 %	2,9 %
15	0,66 %	0,14	24,7 %	2,6 %
20	0,45 %	0,10	19,9 %	2,2 %
50	0,18 %	0,04	9 %	1,9 %

Conclusion: *The number of RR occurences is really small.*

More general result (4)





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Related works for PROMETHEE I

- No rank reversal will happen between a_i and a_j if

$$|\phi^{+}(a_i)-\phi^{+}(a_j)|\geq \frac{1}{n-1}$$

$$|\phi^{-}(a_i)-\phi^{-}(a_j)|\geq \frac{1}{n-1}$$

Rank reversal = risk of manipulation

- **Joint work** with **Julien Roland** and **Céline Verly** (to appear in the proceedings of the IPMU 2012 conference)
- **Aim**: to quantify the likelihood of manipulation in a simplified version of the PROMETHEE II ranking:
  - Usual preference function and equal weights
  - Copeland scores
- **More formaly:**
  - A given decision maker has a perfect information on the evaluation table;
  - He may propose new alternatives in order to make alternative a_i the first one;
  - **Question**: how many alternatives are necessary ?

Linear mathematical program

$$\max \sum_{a \in A \cup C} y(a_s, a)$$

$$\begin{aligned}
\text{subject to: } (P_j(a_i, a_j) - 1)\overline{g_k} &< g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C,\ \forall k \in K \\
P_k(a_i, a_j)\overline{g_k} &\geqslant g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C,\ \forall k \in K \\
\pi(a_i, a_j) &= \frac{1}{q} \sum_{k \in K} P_k(a_i, a_j), \forall a_i, a_j \in A \cup C \\
\phi(a) &= \frac{1}{n + m - 1} \sum_{x \in A \cup C} \pi(a, x) - \pi(x, a), \forall a \in A \cup C \\
g_k(a) &\leqslant \overline{g_k}, \forall a \in C \\
g_k(a) &\geqslant 0, \forall a \in C \\
2(y(a_s, a) - 1) &\leqslant \phi(a_s) - \phi(a), \forall a \in A \cup C \\
2y(a_s, a) &> \phi(a_s) - \phi(a), \forall a \in A \cup C
\end{aligned}$$

Results for 10 alternatives and 3 criteria

Table 1. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives.

$j\backslash m$	1	2	3	4	5	6	7	8	9
2	7	3	0	0	0	0	0	0	0
3	37	13	7	0	0	0	0	0	0
4	57	33	17	3	0	0	0	0	0
5	83	63	40	13	0	0	0	0	0
6	90	83	60	23	0	0	0	0	0
7	90	90	77	43	10	0	0	0	0
8	100	100	87	70	37	7	0	0	0
9	100	100	97	83	63	33	3	0	0
10	100	100	97	93	83	63	33	3	0

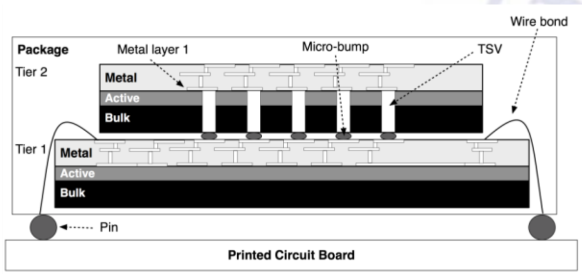
Comparison with the bound

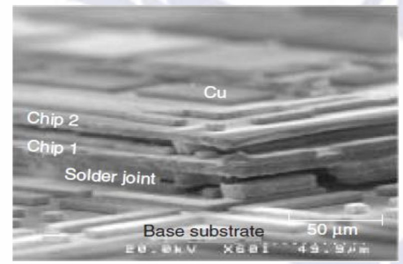
Table 3. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives while the Mareschal's bound is not reached.

$j \backslash m$	1	2
2	7	3
3	30	13
4	27	33
5	17	63
6	7	83
7	0	90
8	7	73
9	3	40
10	0	20

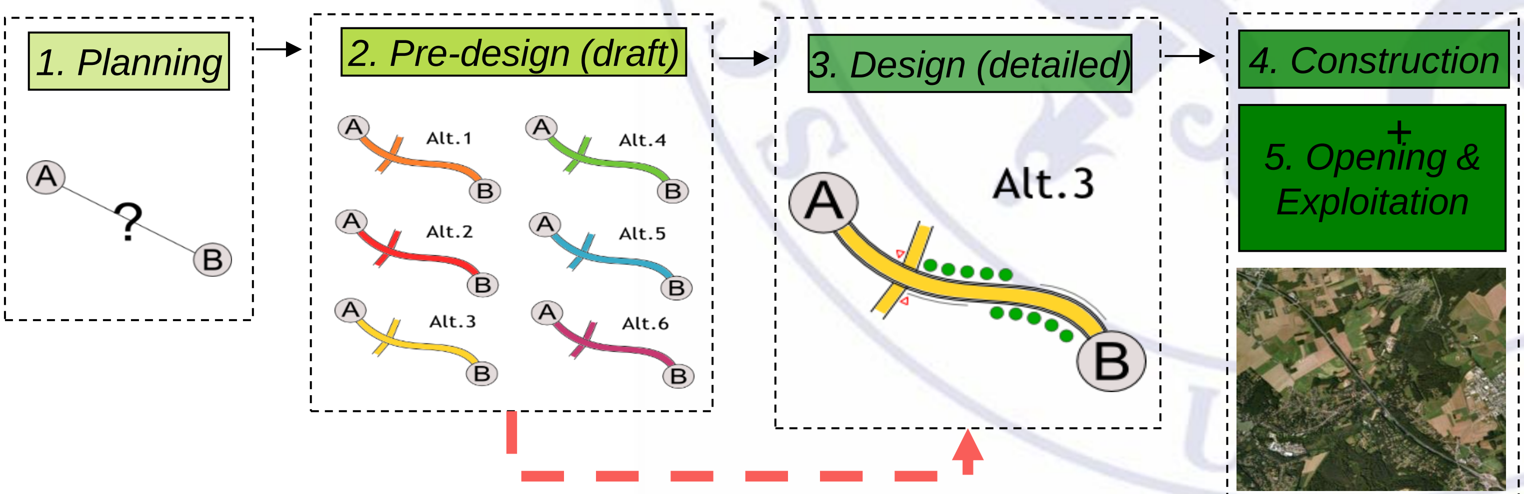
Current applications

- 3D Integrated circuits (PhD A.V. Doan) **Session B1 - 4th presentation**





- Sustainable security in road design (PhD R. Sarrazin) **Session A2 - 2nd presentation**



Future/current researches

- Rank reversal: exact conditions;
- Management of missing values;
- Synergies with Data Envelopement Analysis; (PhD Bagherikavarin) **Session A2 – 2nd presentation**
- Extension of PROMETHEE to temporal evaluations; (PhD I. Benamar).

Thank you for your attention ;-)