# Big Graph Processing Systems: Reachability Indexes

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Graphs

#### Plain reachability

Plain graph



#### **Path-constraint reachability**



## Agenda

- 1. Reachability on **plain** graphs
  - a. A panoramic view of reachability indexes
  - b. Milestones
- 2. Reachability on edge-labeled graphs
  - a. Techniques
    - Alternation-based path constraints
    - Concatenation-based path constraints
  - b. Challenges

## Section I: Plain Reachability



Is there a path from vertex 14 to vertex 20?

#### Plain reachability query

- Q(s, t) on a directed graph G
  - Checking the existence of a path from s to t in G
- Boolean query
  - Either *true* or *false*
- Fundamental graph operator [Sah17]
  - Inferring the relationships among objects

E.g., querying protein-protein interaction in biology networks [Yil]

E.g., querying related works in citation networks [Yil]

[Sah17]S. Sahu et al. The ubiquity of large graphs and surprising challenges of graph processing. VLDB J. 29(2-3): 595-618 (2020)[Yil10]H. Yildirim et al. GRAIL: Scalable Reachability Index for Large Graphs. Proc. VLDB Endow. 3(1): 276-284 (2010)

## Reachability query processing

- Query evaluation
  - Online traversal: BFS, DFS, and BiBFS
  - Problem: graphs are large
- An index for reachability queries
  - Reachability index



#### Example: Vertex 10 reaches vertex 20 as the

#### Naive index: transitive closure

target





# source

	10	11	12	13	14	15	16	17	18	19	20
10		Т	т	Т	Т	Т	Т	Т	Т	Т	Т
11			т	Т	т	т	Т	Т	Т	Т	т
12											
13			т			Т	Т	Т	Т	т	т
14			т			Т	Т	Т	Т	Т	т
15			т			Т	Т	Т	Т	Т	т
16			т			т	Т	Т	Т	Т	т
17			т			т	Т	Т	Т	Т	т
18			т			т	Т	Т	Т	Т	т
19											т
20											















[Tar72] R. Tarjan. Depth-First Search and Linear Graph Algorithms. SIAM J. Comput. 1(2): 146-160 (1972)





#### From Tree to DAG Q1: How about multiple trees? r: virtual root $\rightarrow$ : virtual edge Assigning a virtual root $\rightarrow$ : tree edge DAG 16)

2: How about non-tree edges? Inheriting the intervals
Inheriting the intervals
9 19 7 18 20

V	Interval
10	[0, 6]
11	[0, 5]
12	[3, 4]
13	[2, 2]
14	[0, 1]
15	[0, 0]
16	[7, 10]
17	[3, 3]
18	[7, 8]
19	[9, 9]
20	[7, 7]

From Tree to DAG		V	Interval	
	Q2: How about non-tree edges?	10	[0, 6]	Merging intervals
	Inheriting the intervals	11	[0, 5]	
11		12	[3, 4], [2,2], [0,0]	[2, 4], [0, 0]
r		13	[2, 2], [0, 0]	
$\begin{pmatrix} 6\\ 10 \end{pmatrix}$	10 Tree Cover	14	[0, 1]	
		15	[0, 0]	
5	17 $8$ $7$ $(18)$ $(20)$	16	[7, 10], [3, 3], [0,0], [3, 4], [2,2]	[7, 10], [2, 4], [0, 0]
		17	[3, 3], [0,0]	
(14)		18	[7, 8], [0, 0]	
$\smile$		19	[9, 9], [7, 7]	
		20	[7, 7]	

[Agr89] R. Agrawal et al. Efficient management of transitive relationships in large data and knowledge bases. SIGMOD Conference 1989: 253-262

## Complexity

- Index size: O(n<sup>2</sup>)
- Indexing time: O(nm)
- Query time: O(log n)
- Bottleneck:
  - A larger number of intervals caused by **non-tree edges**
- Q: how to reduce the number of intervals?

#### Reducing the number of intervals

- Bounding the number of intervals
  - GRAIL [Yil10]: exactly k intervals by computing k spanning trees
  - Ferrari [Seu13]: at most k intervals by merging non-adjacent intervals
- Incomplete indexes
  - **False positives** for query processing using indexes
- Resort to online search
  - Guided DFS by querying the incomplete indexes

#### Other techniques based on tree cover

- Dual-labeling [Wan06]
  - Compressing transitive closure for non-tree edges
- GRIPP [Tri07]
  - Recursive querying intervals of rooted spanning trees



#### Rethinking of transitive closure





source

	S	U	Т
S		1	1
U			1
Т			

We can derive the existence of p(s, t)using p(s, u) and p(u, t)

#### Rethinking of transitive closure

target





With the deriving, we only need to record p(s, u), p(v, u), p(u, w), and p(u, t).

#### 2-Hop labeling

- Assigning  $L(v) = (L_{in}(v), L_{out}(v))$  for each v in G,
  - $\forall$  u ∈ L<sub>in</sub>(v), ∃ a path from u to v
  - $\forall$  w ∈ L<sub>out</sub>(v), ∃ a path from v to w
- Vertex s reaches t in G, if and only if
  - $\circ \quad \text{Case 1: } \exists \ t \in L_{\text{out}}(s) \text{, or}$
  - $\circ \quad \text{Case 2: } \exists \ s \in L_{in}(t), \text{ or }$
  - $\circ \quad \textbf{Case 3: } L_{in}(t) \cap L_{out}(s) \neq \varnothing$
- Index size:  $\sum_{v \in V} |L_{in}(v)| + |L_{out}(v)|$



v	L <sub>in</sub> (v)	L <sub>out</sub> (v)
10	Ø	11, 12, 15
11	Ø	15
12	10, 11, 15, 16	Ø
13	11	15
14	11	15
15	Ø	15
16	17, 15	12, 15, 18
17	15	18
18	15	15
19	15	Ø
20	15, 17, 18	Ø

[Coh02] E. Cohen et al. Reachability and distance queries via 2-hop labels. SODA 2002: 937-946

2-hop labeling



Q(10, 20) = true, $L_{out}(10) \cap L_{in}(20) = 15$
Q(15, 18) = true, 15 $\in$ L <sub>in</sub> (18)
O(10, 12) = follow

Q(16, 13)	= false
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[Coh02] E. Cohen et al. Reachability and distance queries via 2-hop labels. SODA 2002: 937-946

2-hop labeling

v	L <sub>in</sub> (v)	L <sub>out</sub> (v)
10	Ø	11, 12, <mark>15</mark>
11	Ø	15
12	10, 11, 15, 16	Ø
13	11	15
14	11	15
15	Ø	15
16	17, 15	12, 15, 18
17	15	18
18	15	15
19	15	Ø
20	<mark>15</mark> , 17, 18	Ø

#### Minimum 2-hop labeling

- The number of Case 3 should be maximized
- **Minimum** 2-hop: the one with the minimum index size
- NP-hard problem [1]
- Approximated algorithm [1]
  - Bounded by a logarithmic factor
  - Complexity
    - Indexing time: O(n<sup>4</sup>)
    - Index size: O(nm<sup>1/2</sup>)
    - Query time: O(m<sup>1/2</sup>)

Impractical for real-world large graphs

### Advanced 2-hop indexing heuristics

- TFL [Che13]
  - Recursive topological folding over DAG
- DL [Jin13]
  - Vertex order for non-redundant hop vertices
- PLL [Aki13]
  - Greedy indexing according to vertex degree
- TOL [Zhu14]
  - General total order for indexing

[Che13] J. Cheng et al. TF-Label: a topological-folding labeling scheme for reachability querying in a large graph. SIGMOD Conference 2013: 193-204
 [Jin13] R. Jin et al. Simple, Fast, and Scalable Reachability Oracle. Proc. VLDB Endow. 6(14): 1978-1989 (2013)
 [Aki13] E. Akiba et al. Fast exact shortest-path distance queries on large networks by pruned landmark labeling. SIGMOD Conference 2013: 349-360

[Zhu14] A. Zhu et al. Reachability queries on large dynamic graphs: a total order approach. SIGMOD Conference 2014: 1323-1334



[Jin09] R. Jin et al. 3-HOP: a high-compression indexing scheme for reachability query. SIGMOD Conference 2009: 813-826 [Cai10] J. Cai et al. Path-hop: efficiently indexing large graphs for reachability queries. CIKM 2010: 119-128



#### Rethinking of transitive closure



- out(v): v and all the vertices that v can reach
- If u reaches v, then  $out(v) \subseteq out(u)$
- Example: 10 reaches 13,
  - out(10) = {10, 11, 12, **13**, 14, **15**, 17}
  - out(13) = {**13**, **15**}
- If out(v) ⊈ out(u), then u does not reach v
- Similarly, if in(u) ⊈ in(v), then u does not reaches v, where in(v) denotes v and all the vertices that can reach v

How to leverage the contrapositive conditions?

#### Membership testing

K-min-wise independent permutation

#### Reachability Querying: An Independent Permutation Labeling Approach

VLDB'14

Hao Wei, Jeffrey Xu Yu, Can Lu Chinese University of Hong Kong Hong Kong, China Ruoming Jin Kent State University Kent, OH, USA

**Bloom filter** 

#### Reachability Querying: Can It Be Even Faster?

Jiao Su<sup>†‡</sup>, Qing Zhu<sup>†</sup>, Hao Wei<sup>‡</sup>, and Jeffrey Xu Yu<sup>‡</sup>

TKDE'17

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## **Bloom filter labeling**

- Compute in(v) and out(v) for each v
  - In in(v) and out(v), recording the **hash codes** of vertices
- Query processing
  - set containment testing
- False positives require online traversal
  - Guided DFS with recursively querying the index

**Bloom Filter Labeling** 



Q(12, 18): False because  $out(18) \leq out(12)$ 

Q(11, 18): True, but false positive Guided DFS leads to False

V	in(v)	out(v)	
10	3	0,1,3,4,5,6	
11	3,4	0,1,3,4,5,6	
12	2,3,4,5	1,3,5,6	
13	3,4,5,6	1,6	
14	0,3,4	0,1	
15	0,1,2,3,4,5,6	1	
16	2	1,2,3,4,5,6	
17	2,3,4,5	1,3	
18	2,4	1,4,6	
19	2,5	5,6	
20	2,4,5	6	

#### Other reachability techniques

- Path-tree labeling [Jin08]: *path partition + two-dimension labeling over a planar graph*
- SCRAB [Jin12]: reachability backbone + reachability through backbone vertices
- HL [Jin13]: recursive reachability backbones
- Feline [Vel14]: *dominance drawing (no false negatives) + online search*
- Preach [Mer14]: contraction hierarchies + bidirectional online search
- O'Reach [Han21]: *partial hop labeling + topological order + existing indexes*

<sup>[</sup>Jin08] R. Jin et al. Efficiently answering reachability queries on very large directed graphs. SIGMOD Conference 2008: 595-608

<sup>[</sup>Jin12] R. Jin et al. SCARAB: scaling reachability computation on large graphs. SIGMOD Conference 2012: 169-180

<sup>[</sup>Jin13] R. Jin et al. Simple, Fast, and Scalable Reachability Oracle. Proc. VLDB Endow. 6(14): 1978-1989 (2013)

<sup>[</sup>Vel14] R. Veloso et al. Reachability Queries in Very Large Graphs: A Fast Refined Online Search Approach. EDBT 2014: 511-522

<sup>[</sup>Mer14] F. Merz et al. PReaCH: A Fast Lightweight Reachability Index Using Pruning and Contraction Hierarchies. ESA 2014: 701-712

<sup>[</sup>Han21] K.Hanauer et al. O'Reach: Even Faster Reachability in Large Graphs. SEA 2021: 13:1-13:24

### Readings

- 2 minutes
  - T. Özsu. Graph Processing: A Panoramic View and Some open Problems. Keynote at VLDB'19. (The section on reachability queries)
- 10 minutes
  - J. Su et al. *Reachability Querying: Can It Be even Faster?* In TKDE'17. (The related work section)
- Half a day
  - J. Xu yu et al. *Graph Reachability Queries: A Survey.* Managing and Mining Graph Data 2010.
- One day
  - A. Bonifati et al. Querying Graphs. Morgan & Claypool Publishers 2018. (Chapter 6.5: Reachability Indexing)
- Unlimited time
  - 9 SIGMOD/TODS + 4 VLDB + 4 ICDE/TKDE + 1 SODA + 1 EDBT, etc.

## Section II: Path-Constraint Reachability

#### Reachability queries with path-constraints

The overlapping: a single label under the Kleene operator.

#### RPQs

- Regular path queries (RPQs):
  - Having a regular expression as a constraint [Ang17]
- Reachability:
  - Checking the existence of a path that can satisfy a path constraint
- The Kleene operator: either \* or +
- Two types (so far)
  - LCR: alternation-based reachability
  - RLC: concatenation-based reachability





#### LCR (label-constrained reachability) queries

- LC (label constraint)
  - $\circ \quad (I_1 \cup ... \cup I_k)^{+}, \text{ where } \cup \text{ is disjunction}$
- LCR query (s, t, LC)
  - Checking whether s reaches t
  - Checking whether the path only contains edges with labels in the LC
- Boolean query
  - Returning either True or False

- Supported languages
  - SPARQL
  - PGQL
  - openCypher





## LCR query evaluation

- Online traversal
  - DFS, BFS, or BiBFS, visiting only edges with labels in the LC
  - Unfeasible for large graphs
- An index for LCR queries
  - LCR indexes
- Index-based evaluation for Q(s, t, LC)
  - **Path-label set** from s to t is mandatory
- Redundancy of path-label sets?



Two path-label sets from Alice to Kim

- 1. {friendOf}
- 2. {friendOf, follows}

Do we need to record both of them?

 ${friendOf} \subset {friendOf, follows} \subseteq a given constraint$ 

### Sufficient path-label set (SPLS)

- Definition [Jin10]
  - The **minimal subsets** of all the path-label sets from u to v



- Free for merging [1], i.e., distributive
  - Computing **SPLS**(p(u, w)) by using **SPLS**(p(u, v)) and **SPLS**(p(v, w))
  - SPLS from u to w: {a, b, e}, {a, b, d}, <del>{a, b, c, e}, {a, b, c, d}</del>, {a, d, e}, and {a, d}

## GTC (Generalized transitive closure)

- GTC [1]: transitive closure with sufficient path-label set
  - For each (u, v):
    - recording whether u reaches v, and
    - SPLS(u, v)
- Problems:
  - Too much time to compute
  - Too much space to store
- How to efficiently compute and effectively compress GTC?



#### GTC compression using spanning tree

- Path characterization [1]:
  - $\circ$  Case 1: (u, x) or (y, v) is a tree edge
  - Case 2: neither (u, x) nor (y, v) is a tree edge
    - Partial GTC

Query processing: 

- Case 2: partial GTC
- Case 1: spanning tree + partial transitive closure







## Efficient GTC computation

- Observations:
  - redundant path-label sets do not need to be expanded
- Dijkstra-like algorithm [Zou14]
  - Simulating distance using **distinct** labels
- Example:
  - two path-label sets from 1 to 5 {a} and {a,c}
  - {a,c} can be pruned



а



#### Landmark index

- Landmark vertices
  - High degree vertices, e.g., hubs
- Landmark indexing [Val17]
  - Computing GTC for each landmark
- Query processing
  - BFS + Index lookup



## Label constrained 2-hop labeling

- The free for merging properties
  - SPLS
  - 2-hop labeling
- LC 2-hop [Pen20]
  - SPLS + PLL [Aki13]
- Example: Q(3, 6, {r, b})
  - 0 (1, {r,b}) in L<sub>out</sub>(3)
  - **(1,{r})** in L<sub>in</sub>(6)



V	L <sub>in</sub> (v)	L <sub>out</sub> (v)
1		
2	(1, {r})	(1, {b})
3	(1, {g})	(1,{r,b}), (2,{r})
4	(1,{r,g}), (1,{r,b}), (2,{g}), (3,{b})	
5	(1,{r,g}), (2,{g})	(4,{g})
6	(1,{r}), (2,{r,g}), (5,{r})	
7	(1,{r})	

[Pen20] Y. Peng et al. Answering billion-scale label-constrained reachability queries within microsecond. Proc. VLDB Endow. 13(6): 812-825 (2020) [Aki13]. Akiba et al. Fast exact shortest-path distance queries on large networks by pruned landmark labeling. SIGMOD Conference 2013: 349-360



#### Person **External Entity** Account P12 knows WorksKo, MONS P10 P11 knows knows KNONS knows P13 holds. holds credits debits

Money laundering analysis:

Do accounts 14 and 19 have the repeated outside-inside money transferring pattern?

#### PGQL

P16

PATH out\_in AS (:Person) -[:debits-]> (:EnternalEntity) -[:credits]-> (:Person) SELECT \* FROM MATH (s) -/:out\_in+/-> (t) WHERE ID(s) = 14 AND ID(t) = 19

#### RLC (recursive label-concatenated) queries

- CL (concatenated labels)
  - $\circ$   $(I_1, ..., I_k)^+$ , where labels are **concatenated**
- RLC query (s, t, CL) [Zha22]
  - Checking whether s reaches t
  - Checking whether the path matches the given CL pattern
- Boolean query
  - Returning either True or False
- Supported languages
  - SPARQL
  - PGQL

- Path semantics
  - Arbitrary paths
- RLC queries appear quite often in timeout query logs [Bon19]

#### A Reachability Index for Recursive Label-Concatenated Graph Queries

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Abstract-Reachability queries checking the existence of a path from a source node to a target node are fundamental operators for querying and processing graph data. Current approaches for index-based evaluation of reachability queries either focus on plain reachability or constraint-based reachability with only alternation of labels. In this paper, for the first time we study the problem of index-based processing for recursive labelconcatenated reachability queries, referred to as RLC queries. These queries check the existence of a path that can satisfy the constraint defined by a concatenation of at most k edge labels under the Kleene plus. Many practical graph database and network analysis applications exhibit RLC queries. However, their evaluation remains prohibitive in current graph database engines.

efficiently process RLC queries. The RLC index checks whether the source vertex can reach an intermediate vertex that can also reach the target vertex under a recursive label-concatenated constraint. We propose an indexing algorithm to build the RLC index, which guarantees the soundness and the completeness of Comprehensive experiments on real-world graphs show that the the following. RLC index can significantly reduce both the offline processing cost and the memory overhead of transitive closure, while improving query processing up to six orders of magnitude over online traversals. Finally, our open-source implementation of the RLC index significantly outperforms current mainstream graph engines for evaluating RLC queries.

Index Terms-reachability index, graph query, graph databases, RLC queries

#### I. INTRODUCTION

Graphs have been the natural choice of data representation in various domains [1], e.g., social, biochemical, fraud de- patterns of money transfers between these accounts. The RLC tection and transportation networks, and reachability queries query  $Q1((A_{14}, A_{19}, (\text{debits, credits})^+)$  evaluates to are fundamental graph operators [2]. Plain reachability queries true because of the existence of the path (A14, debits, E15, check whether there exists a path from a source vertex to a credits,  $A_{17}$ , debits,  $E_{18}$ , credits,  $A_{19}$ ). Another extarget vertex, for which various indexing techniques have been ample is  $Q2(P_{10}, P_{13}, (knows, knows, worksFor)^+)$  that proposed 3-18. To facilitate the representation of different evaluates to false because there is no path from  $P_{10}$  to  $P_{13}$ types of relationships in real-world applications, edge-labeled satisfying the constraint. graphs and property graphs, where labels can be assigned to edges, are more widely adopted nowadays than unlabeled query logs, e.g., Wikidata Query Logs [24], which is the graphs. Such advanced graph models allow users to add path largest repository of open-source graph queries (of the order constraints when defining reachability queries, which play a of 500M queries). In particular, RLC queries often time out key role in graph analytics. However, current index-based in these logs 24 thus showing the limitations of graph query approaches focus on constraint-based reachability with only al- engines to efficiently evaluate them. Moreover, Neo4j (v4.3) ternation [19]-[23]. In this paper, we consider for the first time [25] and TigerGraph (v3.3) [26], two of the mainstream graph reachability queries with a path constraint corresponding to a data processing engines, do not yet support RLC queries in



Fig. 1. A social and professional network on which RLC queries instantiated

We introduce the RLC index, the first reachability index to concatenation of edge labels under the Kleene plus, referred to as recursive label-concatenated queries (RLC queries). RLC queries call for a novel indexing technique due to inherently different path constraints compared to either plain reachability queries or alternation-based reachability, respectively. To furquery execution and avoids recording redundant index entries. ther motivate RLC queries, we present a running example in

> Running example. Figure 1 shows a property graph inspired by a real-world use case encoding an interleaved social and professional network along with information of bank accounts of persons. RLC queries can be used to detect fraud and money laundering patterns among financial transactions. For instance, the query  $Q1(A_{14}, A_{19}, (\text{debits, credits})^+)$ checks whether there is a path from account  $A_{14}$  to  $A_{19}$ such that the label sequence of the path is a concatenation of an arbitrary number (one or more) of occurrences of (debits, credits), which can lead to detect suspicious

RLC queries are also frequently occurring in real-world



https://github.com/g-rpgs/rlc-index





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### Challenges for reachability indexes with path constraints

- 1. Limited resources
  - Partial index + Guided online search
- 2. Beyond static graphs
  - Dynamic graphs
    - Append-only graphs
    - Fully dynamic graphs
  - Streaming graphs [Pac20]
- 3. Distributed graphs

- [Pac20] A. Pacaci et al. Regular Path Query Evaluation on Streaming Graphs. SIGMOD Conference 2020: 1415-1430
   [Bon19] A. Bonifati et al. Navigating the Maze of Wikidata Query Logs. WWW 2019: 127-138
- [Lib12] L. Libkin et al. Regular path queries on graphs with data. ICDT 2012: 74-85
- [Ros22] C. Rost et al. Distributed temporal graph analytics with GRADOOP. VLDB J. 31(2): 375-401 (2022)

- 4. More regular expressions [Bon19]
- 5. Upper and lower bound of hops
- 6. REM [Lib12]: topology + data
- Temporal graph query with time interval [Ros22]



# Thank you and Q&A