eBISS European Business Intelligence Summer School

Data mining for local patterns
Toon Calders

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Outline

PART I: Frequent itemset mining

- **Definition & Applications**
- **Algorithms for Frequent Itemset Mining**
- **Extensions to other pattern types**

PART II:

- **Pattern explosion & Redundancy problem**
- **Methods to remove redundancy**
  - **Condensed representations**
  - **Statistical methods**
  - **Minimal Description Length**

Motivation

- **Originally stated in the context of *Market – Basket Analysis***
  - **Data consists of transactions**

  - **Find unexpected associations between sets of products.**
    - **Store layout**
    - **Promotions**
    - **...**

Definitions

- **Transaction database:**

TID	a	b	c
1	1	1	0
2	1	1	1
3	0	0	1

or, more compact

TID	Items
1	a, b
2	a, b, c
3	c

Itemset I = set of items

supp(I) = # transactions containing I

EXAMPLE:

supp(abc) = 1

supp(ab) = 2

Association Rules and Confidence

- **Association rule: X=>Y**
  - **X, Y non-empty itemsets**
  - **Meaning: *"Occurrence of X implies Y"***

E.g.   A,B => C
  - ***"People who buy A and B, tend to buy C as well"***

- **Confidence: "*Strength*" of the implication X=>Y**

    conf(X=>Y) = support(XY) / support(X)

- **Support of a rule X=>Y = support(XY)**

Association Rule Mining Problem

- **Given:**
  - **transaction database D**
  - **$0 \leq$ minsup $\leq |D|$**
  - **$0 \leq$ minconf ≤ 1**

- **Find all rules X=>Y such that**
  - **support(X=>Y) $\geq$ minsup**
  - **conf(X=>Y) $\geq$ minconf**

Association Rule Mining Problem

- **Minsup = 3**
- **Minconf = 65%**

Rule	Support	Confidence
A => B	4	67%
B => A	4	~~57%~~
A => C	3	~~50%~~
C => A	3	~~50%~~
B => C	5	71%
C => B	5	83%
A => BC	~~2~~	~~40%~~
AB => C	~~2~~	~~50%~~
AC => B	~~2~~	66%
B => AC	~~2~~	~~29%~~
BC => A	~~2~~	~~40%~~
C => AB	~~2~~	~~33%~~

TID	Items
1	A, B
2	B, C
3	B, C
4	A, B
5	A
6	B, C
7	A, C
8	A, B, C
9	A, B, C

Association Rule Mining Problem

- **Typical approach:**
  - **First find all itemsets I s.t. support(I) $\geq$ minsup**
  - **Then: for all subsets X of I:**
    - **Test if confidence(X=>(I / X)) $\geq$ minconf**

Frequent Itemset Mining Problem:

Given:

- **Database D**
- **$0 \leq$ minsup $\leq$ |D|**

Find: all itemsets I such that

- **support(I) $\geq$ minsup**

Association Rule Mining

- **Minsup = 3**
  Minconf = 65%

TID	Items
1	A, B
2	B, C
3	B, C
4	A, B
5	A
6	B, C
7	A, C
8	A, B, C
9	A, B, C

→

Frequent Itemsets

set	supp
A	6
B	7
C	6
AB	4
AC	3
BC	5
ABC	~~2~~

Association Rule Mining

- **Minsup = 3**
  Minconf = 65%

TID	Items
1	A, B
2	B, C
3	B, C
4	A, B
5	A
6	B, C
7	A, C
8	A, B, C
9	A, B, C

→

Frequent Itemsets

set	supp
A	6
B	7
C	6
AB	4
AC	3
BC	5
ABC	~~2~~

Rules

Rule	Conf
A => B	67%
B => A	~~57%~~
A => C	~~50%~~
C => A	~~50%~~
B => C	71%
C => B	83%

Other Measures of Rule Quality

- **Confidence often criticized:**
  - **Beer => Snack   (300)   75%**
  - **Beer => Diapers (200)   50%**

- **However:**
  - **Overall population:**
    - **86% buys snack**
    - **42% buys diapers**

Beer has a negative effect on snacks, and a positive effect on diapers !

TID	Items
1-100	Beer, Snack
101-200	Beer, Diapers, Snack
201-300	Beer, Diapers
301-400	Beer, Snack
401-500	Diapers, Snack
501-600	Snack
601-700	Snack

Other Measures of Rule Quality

- **Alternative measure:**
  - **Lift(X=>Y) = conf(X=>Y) / (support(Y) / |D|)**
    I.e., by which factor does P(Y) change if X is present?

  - **Beer => Snack   0.87**
  - **Beer => Diapers 1.72**

- **There exist many other measures as well:**
  - **Statistically based**
  - **Information theory based**

TID	Items
1-100	Beer, Snack
101-200	Beer, Diapers, Snack
201-300	Beer, Diapers
301-400	Beer, Snack
401-500	Diapers, Snack
501-600	Snack
601-700	Snack

Statistical-Based Measure: X²-test

- **X²-test for dependency between X and Y:**

Example: Beer => Snack

observed

	¬X	X	
¬Y	0	100	100
Y	300	300	600
	300	400	700

Expected (indep.)

	¬X	X	
¬Y	42.9	57.1	100
Y	257.1	342.9	600
	300	400	700

TID	Items
1-100	Beer, Snack
101-200	Beer, Diapers, Snack
201-300	Beer, Diapers
301-400	Beer, Snack
401-500	Diapers, Snack
501-600	Snack
601-700	Snack

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}.$$

Statistical-Based Measure: X²-test

observed

	¬X	X	
¬Y	0	100	100
Y	300	300	600
	300	400	700

Expected
(indep.)

	¬X	X	
¬Y	42.9	57.1	100
Y	257.1	342.9	600
	300	400	700

$X^2 = 87.5$

- **P-value = probability of having a X^2 value at least as big as what we observe, *by chance***

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  - **Condensed representations**
  - **Statistical methods**
  - **Minimal Description Length**

Why Itemset/Association Rule Mining?

- **Explorative data analysis**
  - **Find associations beyond simple correlation**
  - **Compute huge amounts of statistics at the same time**
  - **Changes in patterns can be significant**

Machine log

World cup website visits

Why Itemset/Association Rule Mining?

- **Input to other data mining algorithms**

- **Finding name variations**
  - **Transaction = set of names co-occurring with at least 3 other names**

Why Itemset/Association Rule Mining?

- **Data Summarization**
  - **What are the frequent patterns in my data?**
  - **Abstract away from infrequent patterns**

Illustration: eBISS registration

- **Data: at eBISS registration**
  - **Highly interested in …**

- **1 student → 1 transaction**
  - **Items = topics the student is highly interested in**

- **Result: toy dataset with 14 items & 36 transactions**
  - **Example: 0 0 0 1 1 0 1 1 0 0 1 1 1 1**
    - **→ { Ontologies, Semantic web, IR, DM, Graph mining, Cloud computing, Dist. Comp., Map Reduce }**

Illustration: eBISS registration

```
0 0 0 1 1 0 1 1 0 0 1 1 1 1       1 1 0 0 0 0 0 0 0 1 0 0 0 1
1 1 0 1 0 0 0 1 1 0 0 1 1 1       1 1 0 0 0 0 1 0 0 1 0 0 0 1
1 1 1 1 1 0 1 1 1 1 1 0 0 0       0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 0 0 0 1 1 1 0 0 0 0 1 1       0 0 1 1 0 0 1 0 1 0 0 0 1 1
1 1 0 0 0 0 0 1 0 0 0 1 0 0       1 1 1 0 0 0 1 1 1 0 1 1 1 1
0 0 0 0 0 0 1 1 0 1 1 0 0 1       0 1 0 0 1 0 0 0 0 0 0 0 0 0
1 1 0 0 1 0 0 1 0 0 0 0 0 0       0 0 0 0 0 0 0 1 1 0 1 0 1 0
1 1 0 0 0 0 1 1 0 0 0 0 0 0       1 1 1 1 1 0 1 1 1 0 0 0 0 0
0 1 0 0 0 0 0 0 0 1 1 1 1 0       0 0 0 0 0 0 1 0 0 0 0 0 1 1
0 0 0 0 0 0 0 1 0 0 0 1 1 0       1 1 0 0 0 0 0 1 1 0 0 1 0 0
1 1 0 1 1 0 0 1 0 0 0 0 0 0       0 1 1 1 0 0 0 1 1 0 0 0 0 0
0 0 0 1 1 1 1 1 0 0 0 0 0 0       1 1 0 0 0 1 1 0 0 0 0 0 1 1
1 1 0 0 0 0 0 1 1 0 0 0 0 0       1 1 1 0 0 0 1 1 1 1 1 0 0 0
0 0 0 0 0 1 0 1 1 0 0 0 0 0       0 0 0 0 0 0 1 1 1 1 1 0 1 0
1 1 0 0 0 0 0 1 1 1 1 1 1 1       0 0 0 0 0 0 1 1 1 1 1 0 1 0
1 1 0 1 1 0 0 0 0 0 0 0 0 0       0 0 0 1 1 0 0 1 0 1 1 0 1 0
1 1 0 0 0 0 0 0 0 1 0 0 0 1       1 1 1 0 0 0 0 0 1 1 0 1 0 0
1 1 0 0 0 0 0 0 0 1 0 0 0 1       1 1 0 0 0 0 1 1 1 1 1 0 0 0
1 1 0 0 0 0 0 0 0 1 0 0 0 1
```

Frequent Sets (support 14 or more)

25 DW

24 DM

22 DB

16 IR

16 Visual analytics

15 Graph databases

14 Distributed computing

14 Map Reduce

22 DB DW

15 DW DM

14 DB DM

14 Visual analytics DM

14 DB DW DM

Illustration: eBISS registration

Lift	Conf	Supp	Rule
1.48	1.0	22	DB => DW
1.41	0.92	11	Graph mining => DM
2.31	1.0	7	GIS => Visual analytics
2.88	0.7	7	Ontologies => Semantic web
2.88	0.78	7	Semantic web => Ontologies
3.17	0.86	6	Semantic web, DM => Ontologies
3.08	0.75	6	Ontologies, DM => Semantic web
2.64	1.0	5	DB, Dist. Comp. => Map Reduce

Illustration: eBISS registration

Lift	Conf	Supp	Rule
1.54	0.5	8	IR => Graph mining
1.41	0.46	11	DM => Graph mining
1.85	0.6	9	Graph databases => Graph mining
2.06	0.67	8	IR, DM => Graph mining
3.08	1.0	8	DM, Graph databases => Graph mining

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Monotonicity Principle

If X $\subseteq$ Y, then support(X) $\geq$ support(Y)

Apriori

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

Candidates

A 0   B 0   C 0   D 0

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

A 0   B 1   C 1   D 0

{}

Apriori

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

A 0   B 2   C 2   D 0

{}

Apriori

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

A 1   B 2   C 3   D 1

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

A 2   B 3   C 4   D 2

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

A 2   B 4   C 4   D 3

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

Candidates

AB   AC   AD   BC   BD   CD

A 2   B 4   C 4   D 3

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

AB 1   AC 2   AD 2   BC 3   BD 2   CD 2

A 2   B 4   C 4   D 3

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

Candidates

ACD   BCD

AB 1   AC 2   AD 2   BC 3   BD 2   CD 2

A 2   B 4   C 4   D 3

{}

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

minsup=2

FI

ACD 2

AC 2   AD 2   BC 3   BD 2   CD 2

A 2   B 4   C 4   D 3

{}

Apriori Algoritme

Apriori
Input: **minsup,** *D*
Output: **Set of all frequent itemsets F**

k := 1
C_1 **:= { {A} | A is an item }**
Repeat until (C_k **= {}) {**
   Count support of all itemsets in C_k in 1 scan over D
     F_k **:= { I $\in$ C_k : I is frequent};**
   Generate new candidates
     C_{k+1} **:= { I : |I| = k+1 and all J $\subset$ I with |J|=k are in F_k};**
   k:=k+1
}
Return $\cup_{i=1...k-1}$ F_i

Apriori: Summary

- **Candidate generation is optimal:**
  - **If only information we can get from the database is whether or not an itemset I is frequent**
  - **Number of database scans is minimal (parallel queries to the database)**

- **What if:**
  - **we can load database into memory and transform the database?**
  - **we know the frequencies?**

Depth-First Algorithms

- **Depth-First algorithms:**
    - **+ allow for more efficient counting**
        - **are based on divide-and-conquer**
    - **- do not fully exploit monotonicity principle**

- **Counting all itemsets with item a?**

    → First reduce the database; remove all transactions without a

- **Counting all itemsets without a?**

    → Remove a from the database

Divide-and-Conquer

All itemsets with d

All itemsets $\longrightarrow$ All itemsets with c, without d

All itemsets with b, without c or d

All itemsets with a, without b, or c, or d

Divide-and-Conquer

All itemsets

All itemsets with d
→ All itemsets with c → …
→ …
→ All itemsets with b, without c → …
→ All itemsets with a, without b or c

All itemsets with c, without d → … → …

All itemsets with b, without c or d → …

All itemsets with a, without b, or c, or d

Divide-and-Conquer

Only transactions with d

All itemsets with c → …
→ …

All itemsets with b, without c → …

All itemsets with d → All itemsets with a, without b or c

All itemsets

All itemsets with c, without d → …
→ …

All itemsets with b, without c or d → …

All itemsets with a, without b, or c, or d

Item d no longer needed

Divide-and-Conquer

Find all frequent itemsets with d

TID	a	b	c
3	1	0	1
4	1	1	1
5	0	1	0

a, b, c, ac

Find all frequent itemsets

TID	a	b	c	d
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

a, b, c, d
ad, bd, cd, acd
ac, bc

Find all frequent itemsets without d

TID	a	b	c
1	0	1	1
2	0	1	1
3	1	0	1
4	1	1	1
5	0	1	0

ac, bc

Skip FPGrowth

Divide-and-Conquer

MineFrequent(DB)

1. **F := { a | a is frequent in DB }**
2. **If |F|<2 return F**   ←   **base case**
3. **Remove infrequent items from DB**
4. **For every frequent item a, *except the last*:**
   a. **DB[a] = { (tid,T\{a}) | (tid, T) $\in$ DB, a in T }**
   b. **F[a] := MineFrequent(DB[a])**   ←   **recursion**
   c. **F := F $\cup$ { I $\cup$ {a} | I $\in$ F[a] }**
   d. **Remove a from DB**
5. **return F**

> Often skipped; a. then becomes:
> DB[a] = { (tid,T$\cap$O) | (tid, T) $\in$ DB, a in T },
> where O is the set of items not yet processed

Depth-First Algorithms

minsup = 2

TID	a	b	c	d
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

F
a, b, c, d
ad, bd, cd, acd
ac, bc

Depth-First Algorithms

- **Main difference between different algorithms:**
  - **Way to represent the database**
    - **Trie; tid-lists; …**

- **Database representation should allow for:**
  - **Selecting transaction containing a specific item**
  - **Building the conditional databases**

- **Most depth-first algorithms rely on an in-memory data structure**
  - **Random access important**

FP-growth Algorithm

- **Use a compressed representation of the database using an FP-tree**

- **Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets**

FP-Tree

Transaction
Database

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Header table

Item	Pointer
A	
B	
C	
D	
E	

null

A:7

B:3

B:5

C:1

D:1

C:3

C:3

D:1

E:1

D:1

E:1

D:1

D:1

E:1

E:1

Depth-First Algorithms

- **Building the conditional database:**
  - **DB[a] = { (tid,T∩O) | (tid, T) ∈ DB, a in T };**
    O is the set of items not yet processed

TID	a	b	c	d
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

TID	a	b	c
3	1	0	1
4	1	1	1
5	0	1	0

TID	a	b
1	0	1
2	0	1
3	1	0
4	1	1

TID	a
1	0
2	0
4	1
5	0

(Order d→c→b→a)

FP-Tree Operations: Example

DB;
Order e,d,c,b,a

{}

A:7

B:3

B:5

C:1   D:1

C:3

D:1

C:3

D:1

D:1   E:1

E:1

D:1

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

DB[d]

How to create the FPTree of DB[d] from the FPTree of DB?

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

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FP-Tree Operations

DB;
Order e,d,c,b,a

DB[d]

Item table:

Item	Pointer
A	
B	
C	
D	
E	

Tree node labels: {}, A:7, B:3, B:5, C:1, D:1, C:3, D:1, C:3, D:1, D:1, E:1, E:1, D:1

DB[d] tree: {}, A:1, B:1, C:1

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FP-Tree Operations

DB;
Order e,d,c,b,a

DB[d]

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FP-Tree Operations

DB;
Order e,d,c,b,a

DB[d]

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FP-Tree Operations

DB;
Order e,d,c,b,a

DB[d]

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FP-Tree Operations

DB;
Order e,d,c,b,a

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

DB[d]

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FP-Tree Operations

DB;
Order e,d,c,b,a

DB[d]

Item	Pointer
A	
B	
C	
D	
E	

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

TID	Items
2	{B,C}
3	{A,C}
4	{A}
6	{A,B,C}
9	{A,B}

FPGrowth – Complete Example

- **Step 1: create Initial FPTree**

minsup = 2

TID	a	b	c	d
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

FPGrowth – Complete Example

FPGrowth Summary

- **Depth-first algorithm**
    - **Divide-and-conquer strategy**
    - **+ More efficient counting**
        - **Reduce database in every step**
    - **- Not fully exploiting monotonicity**
- **FPTree data structure**
    - **+ Allows for quickly projecting the database**
    - **- Kept in-memory**
- **Overall: if database fits in memory, depth-first algorithms rule**

Frequent Itemset Mining: Summary

- **Useful for exploration, feature selection, association discovery**

- **Many efficient algorithms exist**
  - **Monotonicity principle central property in all algorithms**
  - **General-to-specific exploration of the search space**
  - **Breadth-first algorithm: Apriori**
  - **Depth-first algorithm: FPGrowth**

Outline

PART I: Frequent itemset mining

- **Definition & Applications**
- **Algorithms for Frequent Itemset Mining**
- **Extensions to other pattern types**

PART II:

- **Pattern explosion & Redundancy problem**
- **Methods to remove redundancy**
  - **Condensed representations**
  - **Statistical methods**
  - **Minimal Description Length**

Other Types of Patterns

- **Sequences**
- **Graphs**
- **Dynamic graphs**

Some Molecules from the NCI HIV Database

Common Fragment

Other Types of Patterns

- **Sequences**
  - **Mining sequences of alarms**

- **Graphs**
  - **Finding common structures**
    - **Socially relevant**

- **Dynamic graphs**
  - **How do social graphs grow?**
  - **Patterns explaining growth over time.**

Other Types of Patterns

- **Sequences**
- **Graphs**
- **Dynamic graphs**

- **Breadth-first algorithms usually no longer work for more complex pattern types:**
  - N^K **sequences of size K with N items**
  - $N^K 2^{K*(K-1)}$ **directed graphs with N labels and K nodes**
- **Cannot hold this many patterns in memory**
  - **Monotonicity check requires random access**
- **Therefore: most algorithms are depth-first**

Other Types of Patterns

- **Sequences**
- **Graphs**
- **Dynamic graphs**

Generate(P)

> **If supp(P) ≥ minsup :**
>
> > **Write P to output**
> >
> > **Successors = extend(P)**
>
> **For c in Successors:**
>
> > **Generate(C)**

Sequence Mining

- **Input data: database of sequences**
  - **Sequence of alarms in an event log**
  - **Order in which students followed courses**
  - **Text = sequence of words**

- **Two settings:**
  - **One large string**
  - **Database of strings**

- **Algorithms are very similar as for frequent itemset mining**

Sequence Mining: Example

sequences
DB, DM, IR, DB II
DB, DM, IR, DB, DB II
DB, DB II, DM, IR
DB, DB, DB, DM

DB→DM→IR→DM: 0   DB→DM→IR→DBII: 2

DB→DM→IR→DB: 0

DB→DM→IR→IR: 0

DB→DM→DB: 0   DB→DM→DM: 0   DB→DM→IR: 3   DB→DM→DBII: 2

DB→DB: 2   DB→DM: 4   DB→IR:3   DB→DBII: 3

DB:4   DM:4   IR:3   DB II:3

Sequence Mining: Example

sequences
DB, DM, IR, DB II
DB, DM, IR, DB, DB II
DB, DB II, DM, IR
DB, DB, DB, DM

DB→IR→DB: 1   DB→IR→DM: 0   DB→IR→IR: 0   DB→IR→DBII: 2

DB→DB: 2   DB→DM: 4   DB→IR:3   DB→DBII: 3

DB:4   DM:4   IR:3   DB II:3

Sequence Mining: Example

sequences
DB, DM, IR, DB II
DB, DM, IR, DB, DB II
DB, DB II, DM, IR
DB, DB, DB, DM

DB→DBII→DB: 0   DB→0DBII→DM: DB→DBII→IR: 0   DB→DBII→DBII: 2

DB→DB: 2   DB→DM: 4   DB→IR:3   DB→DBII: 3

DB:4   DM:4   IR:3   DB II:3

Sequence Mining: Example

sequences
DB, DM, IR, DB II
DB, DM, IR, DB, DB II
DB, DB II, DM, IR
DB, DB, DB, DM

… … … …

DB:4   DM:4   IR:3   DB II:3

Other Types of Patterns

- **Sequences**
- **Graphs**
- **Dynamic graphs**

- **Common problems:**
  - **How to generate all candidates without duplicates**
  - **How to count efficiently**
  - **Notion of "support" is not always straightforward**
    - **Must be anti-monotone and efficient to compute**

Generate Graphs w.o. Duplicates

Generate Candidates w.o. Duplicates

- **Canonical representation**

  (1,2), (2,3), (3,4), (2,4) abab

  (1,2), (2,3), (2,4), (3,4) abab

Generate Candidates w.o. Duplicates

- **Canonical representation**

    (1,2), (2,3), (3,4), (2,4) abab

    (1,2), (2,3), (2,4), (3,4) abab

    (1,2), (2,3), (3,4), (2,4) abba

    (1,2), (2,3), (2,4), (3,4) abba

    ...

- **Canonical representation = lexicographically <u>first</u>**

    0 1 0 0
    0 0 1 1   0100001100010000 = 34308
    0 0 0 1
    0 0 0 0

Generate Candidates w.o. Duplicates

- **Generating successors:**
  - **Look at all direct successors of the pattern:**

Candidate successors

…

  - **For all successors, look at all the predecessors that could have generated it**

Candidate "real father"

Generate Candidates w.o. Duplicates

- **Generating successors:**
  - **For all successors, look at all the predecessors that could have generated it**

  - **Find the canonical representation: pick the first one**

656; abab   56; abab   656; bbaa

Generate Candidates w.o. Duplicates

- **Generating successors:**
  - **Find the canonical representation; pick the first one**

656; abab   56; abab   656; bbaa

- **Only that pattern is allowed to generate the successor**
  - **Avoid the generation of duplicates while exploring of the search space depth-first**

Other Types of Patterns

- **Sequences**

- **Graphs**

- **Dynamic graphs**

- **Common problems:**
  - **How to generate all candidates without duplicates**
  - **How to count efficiently**
  - **Notion of "support" is not always straightforward**
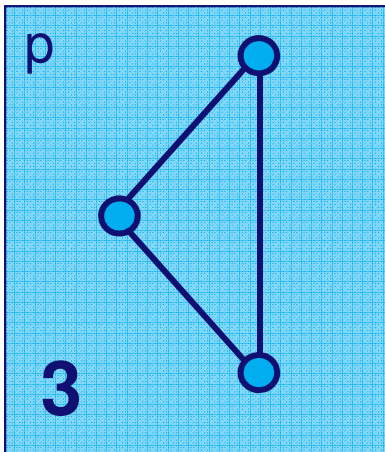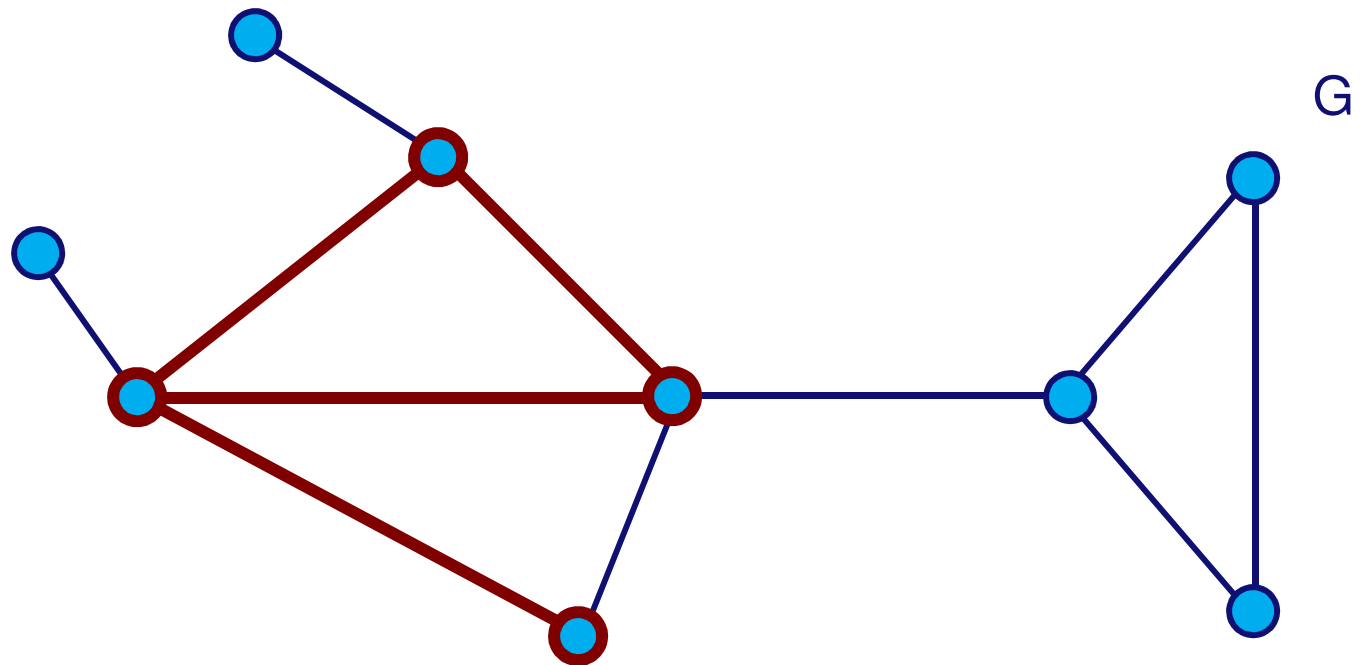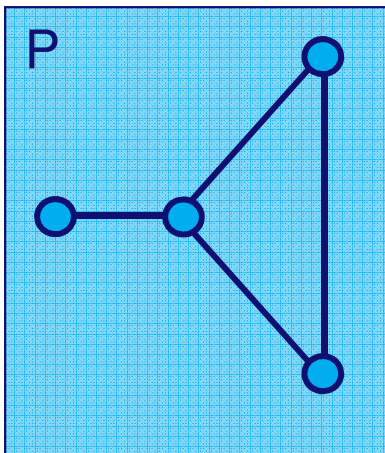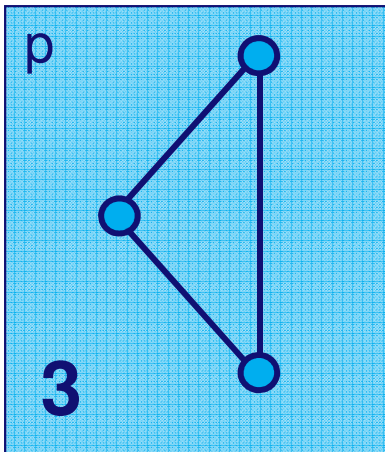   - **Must be anti-monotone and efficient to compute**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

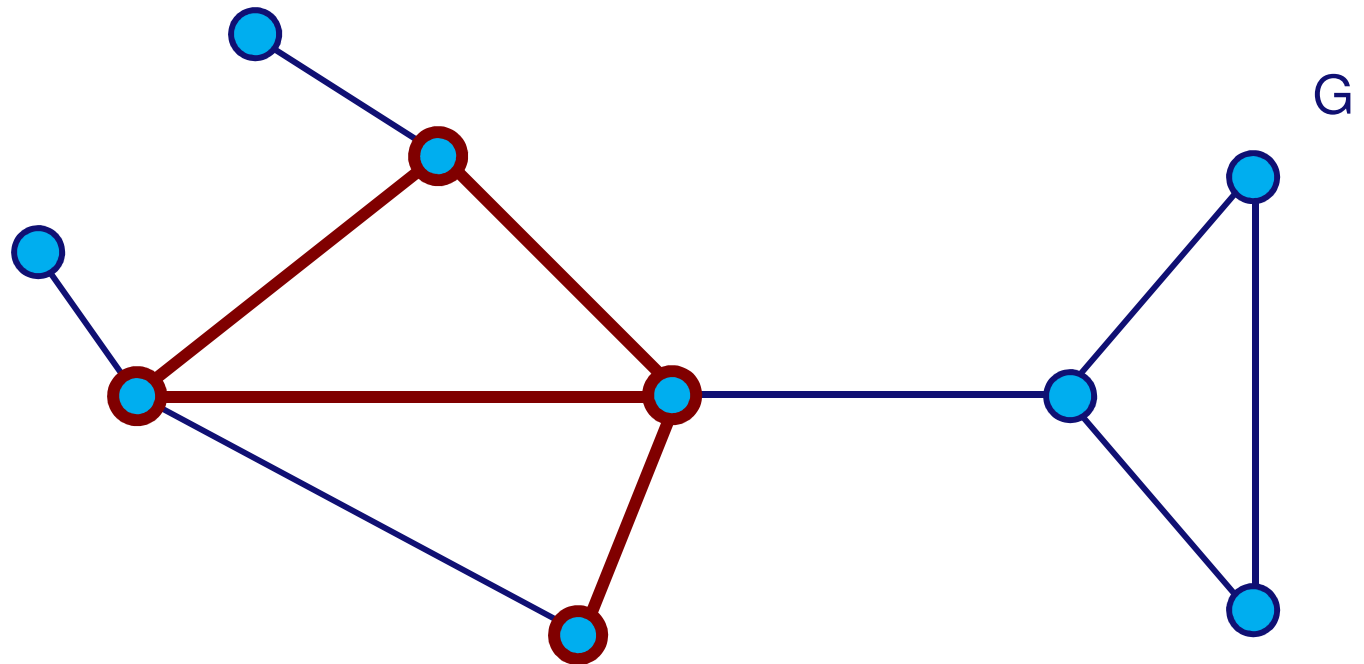# Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

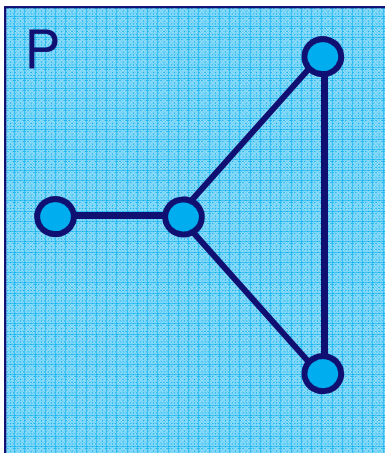# Problems with Frequency
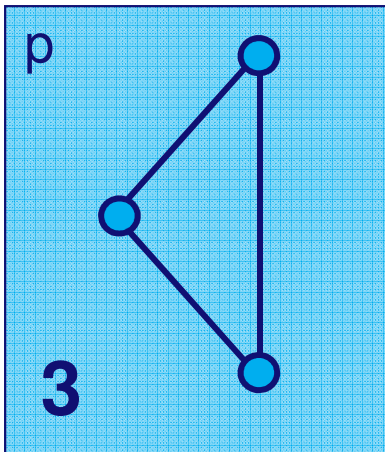
- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency
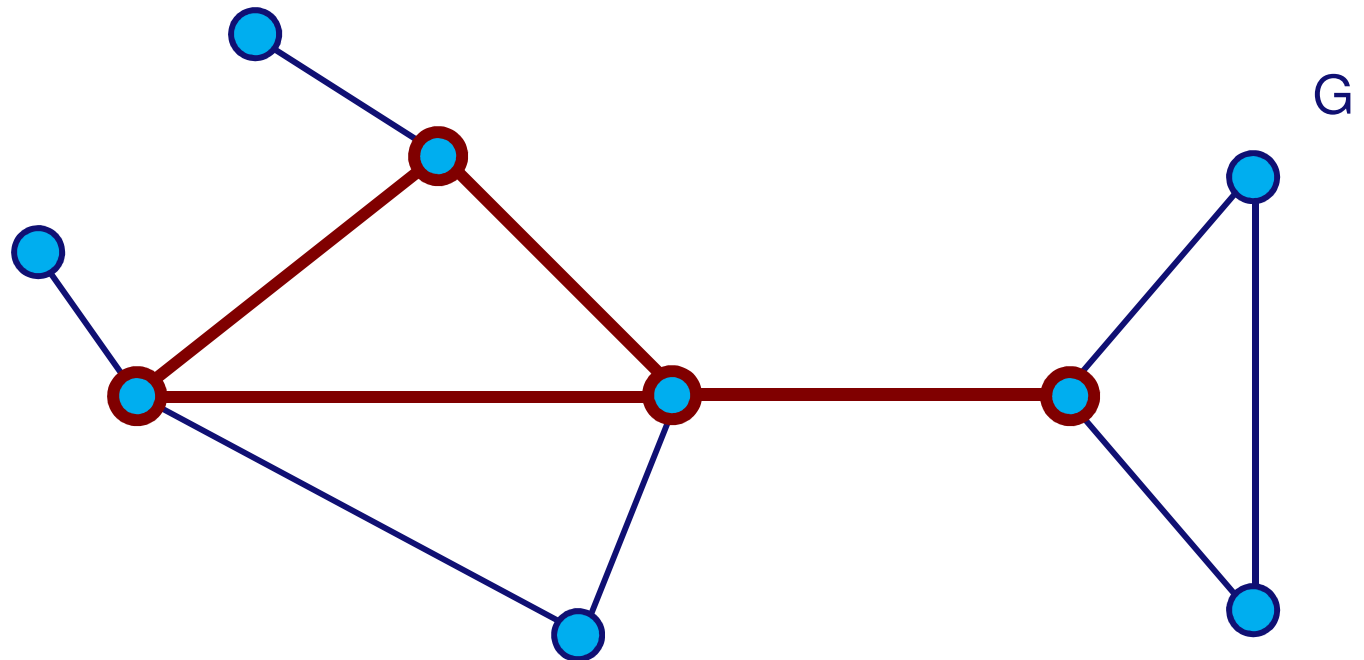
- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency
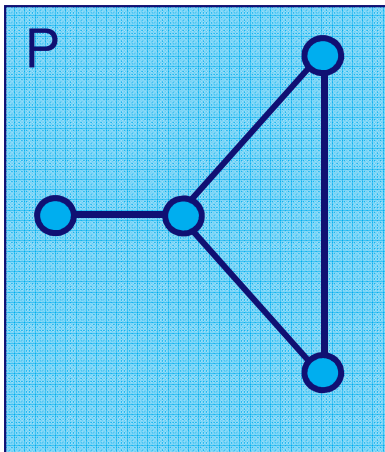
- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency
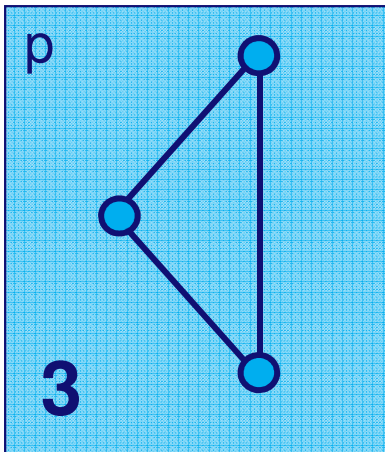
- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency
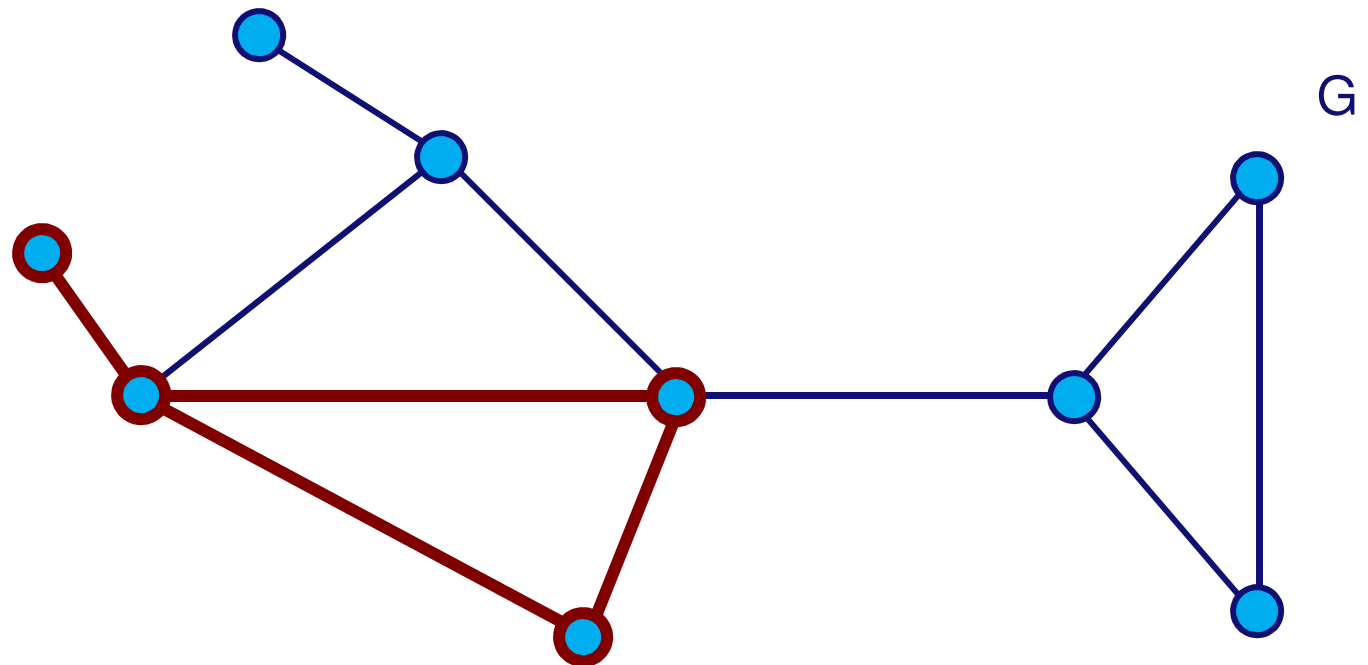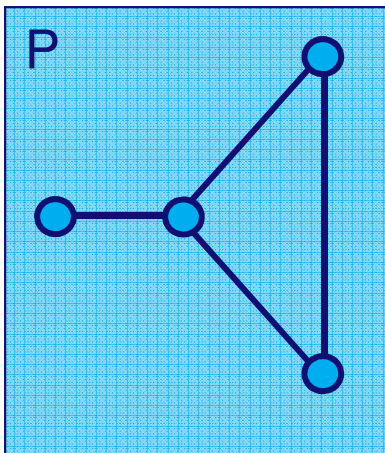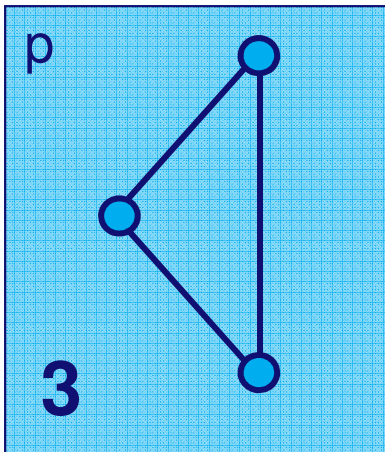
- **Counting instances does not work !**
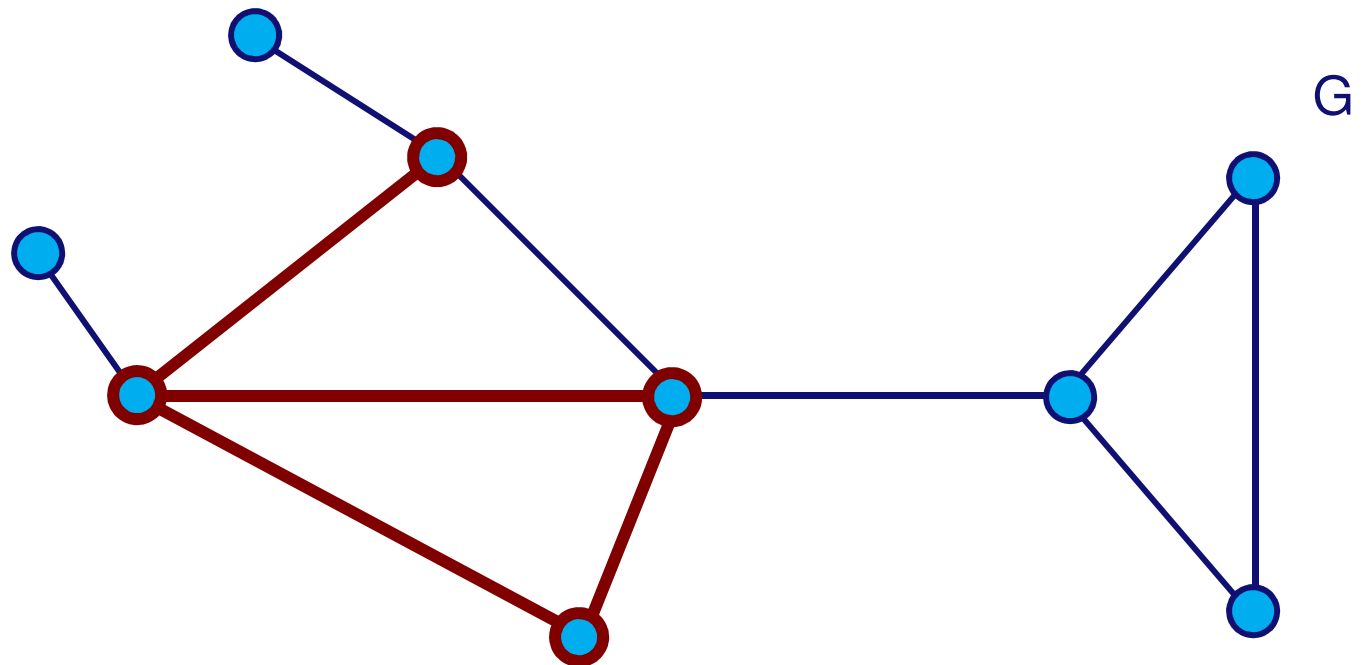
Problems with Frequency

- **Counting instances does not work !**

Problems with Frequency

- **Counting instances does not work !**
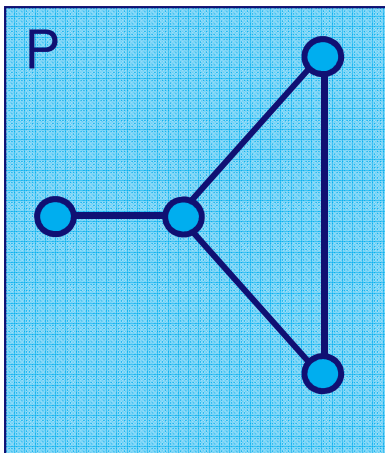 - **Counter-intuitive**

p

3

Less frequent than

P

10

?!

- **Algorithms rely critically on *anti-monotonicity* for pruning the search space**

Overlap Graph

- **Most algorithms for single graph mining base themselves on the *overlap-graph*:**

Example:

overlap graph of   p   in   G   :

Overlap Graph

- **Most algorithms for single graph mining base themselves on the *overlap-graph*:**

Example:

overlap graph of   p   in   G   :

Overlap Graph

- **Most algorithms for single graph mining base themselves on the *overlap-graph*:**

Example:
   overlap graph of   in   :

Overlap Graph

- **Most algorithms for single graph mining base themselves on the *overlap-graph*:**

Example:

overlap graph of   p   in   G   :

G_p

Overlap Graph

- **Most algorithms for single graph mining base themselves on the *overlap-graph*:**

Example:
overlap graph of in :

Overlap Graph

- **Summarizes all instances, and *describes* how they *overlap***
  - **vertex ←→ instance**
  - **edge ←→ overlap**
- **Notion extends straightforwardly to instances in labeled/directed graphs**
- **Yet, overlap graph is *always* an *unlabeled, undirected graph***

Maximum Independent Set

- **Anti-monotone measure on overlap graph:**
  - **size of the *Maximum Independent Set* of the overlap graph**

Data graph   **pattern**   **overlap graph**

Summary: Extension to Other Pattern Types

- **Many extensions of frequent itemset mining exist**
  - **Sequences, partial orders, trees, graphs**

- **Most algorithms are depth-first**
  - **Too many patterns of same size for breadth-first**

- **Extensions become much more challenging**
  - **Pattern generation without duplicates**
  - **Define a good support measure**
  - **Counting support efficiently**

Outline

PART I: Frequent itemset mining

- Definition & Applications
- Algorithms for Frequent Itemset Mining
- Extensions to other pattern types

PART II:

- **Pattern explosion & Redundancy problem**
- **Methods to remove redundancy**
  - Condensed representations
  - Statistical methods
  - Minimal Description Length

Illustration: Tags dataset

Pictures from: Xirong Li, Cees G.M. Snoek, and Marcel Worring, *Learning Social Tag Relevance by Neighbor Voting*, IEEE Transactions on Multimedia, volume 11, issue 7, page 1310-1322, 2009

- **Flickr tags dataset**
  - **Question: What are popular tags?**

Illustration: Tags dataset (top-most frequent)

796 street	477 telephone
713 bridge	453 canon
661 night	433 kitchen
552 city	431 airplane
532 people	430 ship
527 water	423 new
521 the	409 blue
517 bus	404 of
495 dog	395 harbour
489 boat	387 cityscape
487 sky	381 flying

Illustration: Tags Dataset (Longest)

304 flight aeroplane travel aircraft plane

319 flight aeroplane aircraft plane

308 aeroplane travel aircraft plane

305 flight travel aircraft plane

304 flight aeroplane travel plane

304 flight aeroplane travel aircraft

639 airport aircraft plane

630 and black white

511 war protest demonstration

489 aeroplane aircraft plane

Redundancy Problem

- **Frequent itemset / Association rule mining**
  = find all itemsets / ARs satisfying thresholds

- **Only some itemsets / association rules are interesting**
  - **Many are redundant**

    smoker → lung cancer

    smoker, bald → lung cancer

    pregnant → woman

    pregnant, smoker → woman, lung cancer

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Compact Representation of Frequent Itemsets

- **Some itemsets are redundant because they have identical support as their supersets**

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1																									0	0	0
5	1	1	1																									0	0	0
6	0	0	0																									0	0	0
7	0	0	0																									0	0	0
8	0	0	0																									0	0	0
9	0	0	0																									0	0	0
10	0	0	0																									0	0	0
11	0	0	0																									1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

304 flight aeroplane travel aircraft plane

304 flight aeroplane travel plane

304 flight aeroplane travel aircraft

- **Number of frequent itemsets** $= 3 \times 2^{10} - 2 = 3070$

- **Need a compact representation**

Closed Itemsets

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

- **The *support-set* of an itemset I is:**

  sset(I) := { TID | (TID,J)$\in$ D, I $\subseteq$ J }

- **Itemset I and J are said to be *equivalent* if:**

  sset(I) = sset(J)

Example:

sset(A) = { 3, 4 }

sset(AC) = { 3, 4 }

sset(BC) = { 1, 2, 4 }

Closed Itemsets

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

- **Let [I] denote the *equivalence class* of itemset I**
  - **For all J $\in$ [I] : support(J) = support(I)**
    - **support(J) = | sset(J) | = | sset(I) |**
  - **[I] has a unique maximal element max([I])**
    - **If X $\in$ [I], Y $\in$ [I], then also X$\cup$Y $\in$ [I]**
  - **The *closure cl(I)* of an itemset I is defined as max([I])**
  - **A set I is *closed* if I = cl(I)**

Example:

[ACD] = { A, AC, AD, ACD, CD }; hence ACD is closed

TID	A	B	C	D
1	0	1	1	0
2	0	1	1	0
3	1	0	1	1
4	1	1	1	1
5	0	1	0	1

Closed Itemsets

- **All sets in the same equivalence class have the same support**
  - **Occur in the same transactions**
- **Maximal element in an equivalence class is unique**
  - **If two itemsets occur in the same transactions, then so does their union**

- **Frequent Closed Itemset representation:**

  $\{ I \mid I \in F \text{ and } I \text{ is closed} \}$

Benefit of Condensed Representations

(d) Mushroom

Disadvantages of the "Combinatorial Method"

- **Still too many rules/itemsets remain**
  - **Rules where head and tail are independent remain**

  conf(smoking => lung cancer) = 20%

  conf(smoking & blue eyes => lung cancer) = 20%

  - **Highly frequent items form together frequent itemsets**
    - → **not very surprising**

- **Need a way to quantify what is "surprising"**
  - **Depends on what we expect**

Closed Sets – Tags Dataset

304 flight aeroplane travel aircraft plane

319 flight aeroplane aircraft plane

308 aeroplane travel aircraft plane

305 flight travel aircraft plane

~~304 flight aeroplane travel plane~~

~~304 flight aeroplane travel aircraft~~

639 airport aircraft plane

630 and black white

511 war protest demonstration

489 aeroplane aircraft plane

458 ussmidway sandiego aircraftcarrier

449 aviation aircraft plane

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Interestingness Depends on Expectation

Figure 2: Examples of two 0–1 datasets, $\mathcal{D}_1$ and $\mathcal{D}_2$. In both cases we are interested in the correlation between columns (attributes) X and Y. The significance of the correlation result might depend on the overall context of the dataset

Picture from: A. Gionis, H. Mannila, T. Mielikäinen, P. Tsaparas: Assessing data mining results via swap randomization. TKDD 1(3): (2007)

The Modeling Method

Background knowledge →

List of patterns & statistics

← Surprising patterns

Update →

Statistical model
Expresses what you expect given the known patterns

Surprising given model?

yes

no

pattern

Database Of patterns

ECOLE POLYTECHNIQUE

Example: Statistical Model

Pregnant => female
Smoking: 20%   Cancer: 10%
Pregnant: 1%   Female: 40%

Statistical model
Update
Expresses what you expect given the known patterns

Surprising patterns

Surprising given model?

yes

pattern

no

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

ECOLE POLYTECHNIQUE

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%

Surprising patterns

Update

$P(s,c,p,f) = P(s).P(c).P(p,f)$

yes

pattern

Surprising given model?

no

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%

Update

$P(s,c,p,f) = P(s).P(c).P(p,f)$

Surprising patterns

2%

yes

Surprising given model?

pattern

S&C: 8%

no

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%

Update

$P(s,c,p,f) = P(s).P(c|s).P(p,f)$

Surprising patterns

yes

pattern

Surprising given model?

no

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

ECOLE POLYTECHNIQUE

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%

Surprising patterns

Update

$P(s,c,p,f) = P(s).P(c|s).P(p,f)$

0.2%

yes

Surprising given model?

no

pattern

S&P: 0.1%

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

ECOLE POLYTECHNIQUE

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%
P => S : 10%

Update

$$P(s,c,p,f) = P(s|p).P(c|s).P(p,f)$$

Surprising patterns

Surprising given model?

yes

no

pattern

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%
P => S : 10%

Surprising patterns

Update

$P(s,c,p,f) =$
$P(s|p).P(c|s).P(p,f)$

19.5…%

yes

Surprising given model?

no

pattern

S&F:15%

S	C	P	F	Supp
X	X			8%
X			X	0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%
P => S : 10%
F => S : 15%

Update

$P(s,c,p,f) = P(s|p).P(c|s).P(p,f)$

Surprising patterns

yes

pattern

Surprising given model?

no

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
X	X		X	8%

Example: Statistical Model

Example: Statistical Model

P => F
S: 20%   C: 10%
P: 1%   F: 40%
S => C : 40%
P => S : 10%
F => S : 15%

Update

$P(s,c,p,f) = P(s|p).P(c|s).P(p,f)$

Surprising patterns

8.125%

yes

Surprising given model?

no

pattern

C&F:8%

S	C	P	F	Supp
X	X			8%
X		X		0.1%
X			X	15%
X		X	X	0.1%
	X		X	8%

MTV – Statistics Based Filter

geo geotagged lat lon
airplane plane flying aircraft
boat ship
city nyc new york
two people
and white black
night exposure long
b w
protest demonstration
airplane flying aviation
san francisco
diamondclassphotographer flickrdiamond

Tell me what I need to know: Succinctly summarizing data with itemsets. *Michael Mampaey, Nikolaj Tatti, and Jilles Vreeken.* In Proceedings of the 17th ACM SIGKDD Conference on Knowledge Discovery and Data Mining (KDD), 2011.

Outline

PART I: Frequent itemset mining
- Definition & Applications
- Algorithms for Frequent Itemset Mining
- Extensions to other pattern types

PART II:
- **Pattern explosion & Redundancy problem**
- **Methods to remove redundancy**
  - **Condensed representations**
  - **Statistical methods**
  - **Minimal Description Length**

Minimal Description Length

- **A good model helps us to compress the data and is compact**
  - **Let L(M) be the description length of the model,**
  - **Let L(D|M) be the size of the data when compressed by the model**

- **Find set of patterns (model M) that minimizes:**

  L(M) + L(D|M)

- **Explicit trade-off; making a model more specific:**
  - **Increases L(M),**
  - **Decreases L(D|M)**

Skip app

Minimal Description Length: Example

- **Determining the intrinsic cardinality of a time series**

- **More segments will make the model more accurate**
  - **What is the optimal number of segments?**

Minimal Description Length: Example

- **What is the optimal number of segments?**
- **Increasing the number of segments**
  - **Increase model complexity**
    = # bits to describe the model = L(M)
  - **Decrease residuals**
    = less bits for encoding the error = L(D|M)

- **Optimal point is determined by minimizing**
  L(M) + L(D|M)

- **L(M) + L(D|M) = amount of structure that can be exploited *usefully***

Application: Deep Sleep Prediction

- **Based on ECG data predict if patient in deep sleep**
  - **Less intrusive than EEG**

A First Result

- **Use L(M) + L(D|M) to characterize regularity of the sequence**
  - **Window slides over the ECG; continuously compute L(M)+L(D|M) for the best model**

Stage	Mean
Wake	0.5119
REM	0.4596
N1	0.4700
N2	0.3256
N3	0.2053

PHILIPS

Minimal Description Length

- **We can use patterns to code a database**

D

TID	Items
1	A
2	C
3	C
4	A,B
5	A,B,C
6	A,B,C

L(M)

pattern	code
A	00
B	01
C	10
AB	11

L(D|M)

TID	Items
1	00
2	10
3	10
4	11
5	1110
6	1110

- **Find set of patterns that minimizes L(M)+L(D|M)**
  - **Heuristic approach**

Minimal Description Length

- **Rank itemsets according to how well they can be used to compress the dataset**
  - **Property of a set of patterns**

- **The "Krimp" algorithm was the first to use this paradigm in itemset mining**
  - **Assumes a seed set of patterns**
  - **A subset of these patterns is selected to form the "code book"**
  - **The best codebook is the one that gives the best compression**

Vreeken, Jilles, Matthijs Van Leeuwen, and Arno Siebes. "Krimp: mining itemsets that compress." *Data Mining and Knowledge Discovery* 23.1 (2011): 169-214

Tags Dataset - MDL

- jet landing gear airliner jetliner jetliners planes aeroplanes engines aircrafts airliners les avions tail motors cockpit fuselage flaps rudder aeroplano vliegtuig avi

- boat ship

- geo geotagged lat lon

- http library gov congress loc identifier hdl pnp purl elements

- airplane flying

- two people

- bridges bridgepix bridgepixing bridging

- photograph d set slr close nikonstunninggallery camera heigan martin mh

- jets aircraft airplanes aeroplane avion wings nose flugzeug

- nyc new york

- b w

- white black

- protest demonstration

- exposure long

- emergency fire truck vehicle

Summary: Redundancy problem

- **Output of frequent set mining not useful in itself**
  - **Lots of redundant patterns**

- **Methods to remove redundancy**
  - **Element of "surprise"**
  - **Statistical: model expectation**
  - **MDL: how much structure can be exploited efficiently**

- **Mainly aimed towards summarization**
  - **Although also applications in change detection**

Summary

- **Frequent itemset mining**
  - **Simple definition, high complexity**
  - **Breadth-first and Depth-first algorithms**
  - **Many extensions to other pattern types**

- **Pattern explosion problem**
  - **Too many, redundant patterns are generated**
  - **Condensed representations → subset of all patterns**
    - **"Combinatorial" approach insufficient**

- **Recently new techniques emerged**
  - **statistically and MDL based**
  - **Model expectation / benefit of a set of patterns**

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FPGrowth:

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Jilles Vreeken, Matthijs van Leeuwen, Arno Siebes: Krimp: mining itemsets that compress. Data Min. Knowl. Discov. 23(1): 169-214 (2011)

Thank You for Your Attention!