1. The essence of XML Schema Definitions (XSDs)

2. Basic features of XML Schema Definitions

3. Advanced features of XML Schema Definitions

4. Deterministic Regular Expressions
OUR STORY SO FAR ...

- An **XML Language** is a set of XML documents that belong to the same “application domain”
- A **schema** is a formal definition of the syntax of an XML language
- A document is either **valid** w.r.t. a schema, or not
- A **schema language** is a notation by which schemas can be defined.

The idea of validation:

![Diagram showing the process of validating an instance document against a schema]

- The **schema processor** validates the **instance document** against the **schema**.
- If the document is **valid**, it is returned as a **normalized instance document**.
- If the document is **invalid**, an **error message** is generated.
1. Cannot constrain character data
2. Specification of attribute values is too limited
3. Element and attribute declarations are context insensitive
4. Character data cannot be combined with the regular expression content model
5. The content models lack an “interleaving” operator
6. The support for modularity, reuse, and evolution is too primitive
7. The normalization features lack content defaults and proper whitespace control
8. Structured embedded self-documentation is not possible
9. The ID/IDREF mechanism is too simple
10. It does not itself use an XML syntax
11. No support for namespaces
XML Schema is W3C’s proposal for replacing DTDs

Design Principles:
- More expressive than DTD
- Use XML notation
- Self-describing
- Simplicity
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XML Schema is W3C’s proposal for replacing DTDs

**Design Principles:**
- More expressive than DTD
- Use XML notation
- Self-describing → not really
- Simplicity → not really

**Technical Requirements:**
- Namespace support
- User-defined datatypes
- Inheritance (OO-like)
- Evolution
- Embedded documentation
- ...

...
Part I: The Essence
• XML Schema is a large and complicated standard

• Its syntax in XML is really verbose …

• … so it’s easy to get lost in the beginning
LET’S START SIMPLE

- XML Schema is a large and complicated standard
- Its syntax in XML is really verbose ...
- ... so it’s easy to get lost in the beginning

Let us illustrate the essential ideas without using the actual XSD syntax
Let us focus on elements
THE ESSENCE OF XSDS

- ads under **UsedCars** must contain both **model** and **year**
- ads under **NewCars** must contain only **model**
**THE ESSENCE OF XSDS**

- **ads under** `UsedCars` **must contain both** `model` **and** `year`
- **ads under** `NewCars` **must contain only** `model`
- This XML language cannot be specified by a DTD
We need a way to distinguish between UsedCar ads and NewCar ads

- ads under **UsedCars** must contain both **model** and **year**
- ads under **NewCars** must contain only **model**
- This XML language cannot be specified by a DTD
XML Schema solves this by introducing types

- **Simple types** describe the legal values of text and attribute nodes (integer, string, date, ...)

- **Complex types** describe the content model for element nodes: each complex type is a regular expression over pairs of the form \((element\ name, type)\).

- An XML Schema is a collection of type definitions.
Example:

- **dealerType** → (UsedCars, usedType), (NewCars, newType)
- **usedType** → (ad, adType1)*
- **newType** → (ad, adType2)*
- **adType1** → (model, string), (year, date)
- **adType2** → (model, string)
THE ESSENCE OF XSDS (2)

Example:

- `dealerType` → `(UsedCars, usedType), (NewCars, newType)`
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- `adType2` → (model, string)
Example:

\[
\begin{align*}
dealerType & \rightarrow (UsedCars, usedType), (NewCars, newType) \\
usedType & \rightarrow (ad, adType1)^* \\
newType & \rightarrow (ad, adType2)^* \\
adType1 & \rightarrow (model, string), (year, date) \\
adType2 & \rightarrow (model, string)
\end{align*}
\]
Example:

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THE ELEMENT DECLARATION CONSISTENT CONSTRAINT (EDC)

Definition

XML Schema requires that within the same type definition the same element name must occur with the same type. This is called the element declaration consistent constraint (EDC).

Example
THE ELEMENT DECLARATION CONSISTENT CONSTRAINT (EDC)

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XML Schema requires that within the same type definition the same element name must occur with the same type. This is called the element declaration consistent constraint (EDC).

Example

- Legal:
  
  ```
  dealerType → (UsedCars, usedType), (NewCars, newType)
  ```
THE ELEMENT DECLARATION CONSISTENT CONSTRAINT (EDC)

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XML Schema requires that within the same type definition the same element name must occur with the same type. This is called the element declaration consistent constraint (EDC).

Example
- Legal:
  \[\text{dealertype} \rightarrow (\text{UsedCars, usedType}), (\text{NewCars, newType})\]
- Illegal:
  \[\text{dealertype} \rightarrow (\text{Cars, usedType}), (\text{Cars, newType})\]
Definition
XML Schema requires that within the same type definition the same element name must occur with the same type. This is called the element declaration consistent constraint (EDC).

Example
• Legal:
  \[\text{dealertype} \rightarrow (\text{UsedCars, usedType}), (\text{NewCars, newType})\]

• Illegal:
  \[\text{dealertype} \rightarrow (\text{Cars, usedType}), (\text{Cars, newType})\]

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The type assignment is determined by path from element to root

**Theorem** [Martens, Neven, Schwentick, 2006]
The type assignment is determined by path from element to root.

In other words: paths determine types!
Hence if $\in S$ and $\in S$ then $\in S$

We can use this to show that an XML language is not definable in XML Schema.
Part II: Basic Syntax
Definition

Syntactically, an XSDs is a collection of:

- **complex type definitions**: defines content and attributes
- **simple type definitions**: defines a family of legal Unicode text strings
- **element declarations**: associate an element name with a simple or complex type
- **attribute declarations**: associate an attribute name with a simple type
XSDs are written in XML

- All definitions and declarations are put inside an `schema` element

```xml
<schema xmlns="http://www.w3.org/2001/XMLSchema"
    targetNamespace="http://cardealers.org"
    xmlns:b="http://cardealers.org">

    the definitions go here ...

</schema>
```
Example in our made-up syntax:

```
dealerType → (UsedCars, usedType), (NewCars, newType)
```

Real XSD syntax:

```
<complexType name="dealerType">
  <sequence>
    <element name="UsedCars" type="b:usedType"/>
    <element name="NewCars" type="b:newType" />  
  </sequence>
</complexType>
```
Example in our made-up syntax:

```
dealerType   → (UsedCars, usedType), (NewCars, newType)
usedType     → (ad, adType1)*
```

Real XSD syntax:

```
<complexType name="usedType">
  <sequence>
    <element name="ad" type="b:adType1"
      minOccurs="0" maxOccurs ="unbounded"/>
  </sequence>
</complexType>
```
Example in our made-up syntax:

dealerType → (UsedCars, usedType), (NewCars, newType)
usedType → (ad, adType1)*
newType → (ad, adType2)*

Real XSD syntax:
<complexType name="newType">
  <sequence>
    <element name="ad" type="b:adType2"
      minOccurs="0" maxOccurs ="unbounded"/>
  </sequence>
</complexType>
Example in our made-up syntax:

dealerType → (UsedCars, usedType), (NewCars, newType)
usedType → (ad, adType1)*
newType → (ad, adType2)*
adType1 → (model, string), (year, date)

Real XSD syntax:
<complexType name="addType1">
   <sequence>
      <element name="model" type="string" />
      <element name="year" type="date" />
   </sequence>
</complexType>
Example in our made-up syntax:

- **dealerType** → (UsedCars, usedType), (NewCars, newType)
- **usedType** → (ad, adType1)*
- **newType** → (ad, adType2)*
- **adType1** → (model, string), (year, date)
- **adType2** → (model, string)

Real XSD syntax:

```xml
<complexType name="addType2">
  <sequence>
    <element name="model" type="string" />
  </sequence>
</complexType>
```
**XSDS BY EXAMPLE**

Example in our made-up syntax:

```
dealerType → (UsedCars, usedType), (NewCars, newType)
usedType → (ad, adType1)*
newType → (ad, adType2)*
adType1 → (model, string), (year, date)
adType2 → (model, string)
```

Finally we need to declare globally that **Dealer** elements have type **dealerType**:  
`<!– Global element declaration –>`  
`<element name="Dealer" type="b:dealerType"/>`
THE COMPLETE XSD

By means of Online demonstration
Element and attribute declarations

Declarations (global and local):

- `<element name="elem name" type="simple or complex type name"/>`
- `<attribute name="attr name" type="simple type name …"/>`

References:

- `<element ref="elem name"/>`
- `<attribute ref="attr name"/>`
ELEMENT AND ATTRIBUTE DECLARATIONS

**Definition**

Declarations (global and local):

- `<element name="elem name" type="simple or complex type name"/>
- `<attribute name="attr name" type="simple type name ..."/>

**References:**

- `<element ref="elem name"/>
- `<attribute ref="attr name"/>

**Example**

By means of

Online demonstration
Global style:

```xml
<complexType name="newType">
  <sequence>
    <element name="ad" type="b:adType2"
             minOccurs="0" maxOccurs="unbounded"/>
  </sequence>
</complexType>

<complexType name="addType2">
  <sequence>
    <element name="model" type="string"/>
  </sequence>
</complexType>
```
Local style: in-line anonymous type definitions

<complexType name="newType">
    <sequence>
        <element name="ad" minOccurs="0" maxOccurs="unbounded">
            <complexType>
                <sequence>
                    <element name="model" type="string" />
                </sequence>
            </complexType>
        </element>
    </sequence>
</complexType>
An example with global and local declarations:

```xml
<element name="ad" type="adType1">

<complexType name="usedType">
  <sequence>
    <element ref="b:ad" minOccurs="0" maxOccurs ="unbounded"/>
  </sequence>
</complexType>

<complexType name="newType">
  <sequence>
    <element name="ad" type="b:adType2"
      minOccurs="0" maxOccurs ="unbounded"/>
  </sequence>
</complexType>
```

Note that we need at least one local element declaration to distinguish between used car ads and new car ads. This is called overloading.
Definition

- Syntax:
  
  `<complexType name="..."> content model/attributes </complexType>`
COMPLEX TYPE DEFINITIONS

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- Content models are regular expressions with a peculiar syntax
COMPLEX TYPE DEFINITIONS

Definition

• Syntax:

  `<complexType name="..."> content model/attributes </complexType>`

• Content models are regular expressions with a peculiar syntax

  - Element declaration → `<element name="..." type="...">`
  - Element reference → `<element ref="...">`
  - Concatenation → `<sequence> ...</sequence>`
  - Union → `<choice> ...</choice>`
  - All (unordered) → `<all> ...</all>`
  - Element Wildcard → `<any namespace="..." processContents="...">`
  - Cardinalities (*,+) → attributes minOccurs, maxOccurs
  - Mixed content → attribute mixed="true"
COMPLEX TYPE DEFINITIONS

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  - Cardinalities (*,+) → attributes minOccurs, maxOccurs
  - Mixed content → attribute mixed="true"

- Attributes:

  - Attribute declaration → `<attribute name="..." type="..." ...>`
  - Attribute reference → `<attribute ref="..." ...>`
  - Attribute wildcard → `<attribute namespace="...”
                          → processContents="...">`
Example:

```xml
<element name="order" type="n:order_type"/>

<complexType name="order_type" mixed="true">
    <choice>
        <element ref="n:address"/>
        <sequence>
            <element ref="n:email" minOccurs="0" maxOccurs="4"/>
            <element ref="n:phone"/>
        </sequence>
    </choice>

    <attribute ref="n:id" use="required"/>
    <attribute ref="n:email" default="no email address available"/>
    <attribute ref="n:method" fixed="some fixed value"/>
</complexType>
```
XML Schema has a myriad of built-in simple types ...

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>string</td>
<td>any Unicode string</td>
</tr>
<tr>
<td>boolean</td>
<td>true, false, 1, 0</td>
</tr>
<tr>
<td>decimal</td>
<td>3.1415</td>
</tr>
<tr>
<td>float</td>
<td>6.02214199E23</td>
</tr>
<tr>
<td>double</td>
<td>42E970</td>
</tr>
<tr>
<td>dateTime</td>
<td>2004-09-26T16:29:00-05:00</td>
</tr>
<tr>
<td>time</td>
<td>16:29:00-05:00</td>
</tr>
<tr>
<td>date</td>
<td>2004-09-26</td>
</tr>
<tr>
<td>hexBinary</td>
<td>48656c6c6f0a</td>
</tr>
<tr>
<td>base64Binary</td>
<td>SGVsbG8K</td>
</tr>
<tr>
<td>anyURI</td>
<td><a href="http://www.brics.dk/ixwt/">http://www.brics.dk/ixwt/</a></td>
</tr>
<tr>
<td>QName</td>
<td>rcp:recipe, recipe</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>
XML Schema has a myriad of built-in simple types ...  
...but it is also possible to define your own simple types.

<table>
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<tr>
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</tr>
<tr>
<td>QName</td>
<td>rcp:recipe, recipe</td>
</tr>
</tbody>
</table>
New simple types are defined by restricting existing simple types

Example:
To match integers between 0 and 100:

```xml
<simpleType name="score_from_0_to_100">
  <restriction base="integer">
    <minInclusive value="0"/>
    <maxInclusive value="100"/>
  </restriction>
</simpleType>
```
• New simple types are defined by restricting existing simple types

Example:
• To match strings of the form \( N\% \) with \( 0 \leq N \leq 100 \):

```xml
<simpleType name="percentage">
  <restriction base="string">
    <pattern value="([0-9]|[1-9][0-9]|100)\%"/>
  </restriction>
</simpleType>
```
New simple types are defined by restricting existing simple types

By defining lists of simple types

Example:
To match a whitespace-separated list of integers like 1 55 399:

```xml
<simpleType name="score_from_0_to_100">
  <list itemType="integer"/>
</simpleType>
```
New simple types are defined by restricting existing simple types
By defining lists of simple types
Or taking unions of simple types

Example:
To match all booleans and decimals:
```xml
<simpleType name="boolean_or_decimal">
  <union>
    <simpleType>
      <restriction base="boolean"/>
    </simpleType>
    <simpleType>
      <restriction base="decimal"/>
    </simpleType>
  </union>
</simpleType>
```
The things that we restrict by are called **facets**

### Available constraining facets:

- length
- minLength
- maxLength
- pattern
- enumeration
- whiteSpace
- maxInclusive
- maxExclusive
- minInclusive
- minExclusive
- totalDigits
- fractionDigits
XML Schema has a myriad of built-in simple types that are defined by derivation from the primitive simple types.

- normalizedString
- token
- language
- Name
- NCName
- ID
- IDREF
- integer
- nonNegativeInteger
- unsignedLong
- long
- int
- short
- byte
- ...
• Sometimes we want to specify that the content of an element should be of some simple type, but that it can also have some attribute.
• This requires a peculiar syntax

Example:

```xml
<element name="category" type="n:category"/>
<attribute name="class" type="string"/>

<complexType name="category">
  <simpleContent>
    <extension base="integer">
      <attribute ref="n:class"/>
    </extension>
  </simpleContent>
</complexType>
```
Example instance document:

```xml
<b:card xmlns:b="http://businesscard.org"
       xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
       xsi:schemaLocation="http://businesscard.org business_card.xsd">
  <b:name>John Doe</b:name>
  <b:title>CEO, Widget Inc.</b:title>
  <b:email>john.doe@widget.com</b:email>
  <b:phone>(202) 555-1414</b:phone>
  <b:logo b:uri="widget.gif"/>
</b:card>
```

- Only globally declared elements can be starting points for validation!
- The targetNamespace of the Schema, and the nameSpace of the elements in the instance document must match!
- If the XSD does not have a target namespace, use noNameSpaceSchemaLocation instead of schemaLocation
Part III: Advanced Features
Also complex types can be derived by restricting or extending existing complex types.

This is similar to inheritance in object-oriented programming languages.

Example:

Assume given the following complex type:

```xml
<complexType name="basic_card_type">
  <sequence>
    <element name="name" type="string"/>
  </sequence>
</complexType>
```
COMPLEX TYPE DERIVATION

- Also complex types can be derived by restricting or extending existing complex types.
- This is similar to inheritance in object-oriented programming languages.

Example:
- We can extend this type with a title element and optional email elements as follows:

```xml
<complexType name="extended_type">
  <complexContent>
    <extension base="b:basic_card_type">
      <sequence>
        <element ref="b:title"/>
        <element ref="b:email" minOccurs="0"/>
      </sequence>
    </extension>
  </complexContent>
</complexType>
```
COMPLEX TYPE DERIVATION

- Also complex types can be derived by restricting or extending existing complex types.
- This is similar to inheritance in object-oriented programming languages.

Example:
- We can subsequently restrict this type such that email becomes required:

```xml
<complexType name="restricted_type">
  <complexContent>
    <restriction base="b:extended_type">
      <sequence>
        <element name="name" type="string"/>
        <element ref="b:title"/>
        <element ref="b:email"/>
      </sequence>
    </restriction>
  </complexContent>
</complexType>
```
COMPLEX TYPE DERIVATION: SUBSUMPTION

Assume that:

- $T$ is some (complex or simple) type
- $T^-$ is derived from $T$ by restriction
- $T^+$ is derived from $T$ by extension

**Definition**

Subsumption is the principle that whenever a instance of type $T$ is required,

- an instance of type $T^-$ may be used instead (every instance of type $T^-$ is also of type $T$)
- an instance of type $T^+$ may be used instead if the instance has attribute `xsi:type="T+"` with
  
  ```xml
  xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
  ```
Assume that:
- $T$ is some (complex or simple) type
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**Definition**

Subsumption is the principle that whenever an instance of type $T$ is required,
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- an instance of type $T^+$ may be used instead if the instance has attribute `xsi:type="T+"` with

  ```xml
  xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
  ```

**Note:** Derivation, instantiation, and subsumption can be constrained using final, abstract, and block
Prefixes are also used in certain attribute values!

Unqualified Locals:
- if enabled, the name of a locally declared element or attribute in the instance document must have no namespace prefix (i.e. the empty namespace URI)
- such an attribute or element “belongs to” the element declared in the surrounding global definition
- always change the default behavior using elementFormDefault="qualified"
UNIQUENESS, KEYS, REFERENCES

- Keys can be defined by means of `key`
- Keys can be referred to by means of `keyref`
- `unique` functions as `key`, but fields may be absent

Example:

```xml
<element name="w:widget" xmlns:w="http://www.widget.org">
  <complexType> ... </complexType>

  <key name="my_widget_key">
    <selector xpath="w:components/w:part"/>
    <field xpath="@manufacturer"/>
    <field xpath="w:info/@productid"/>
  </key>

  <keyref name="annotation_references" refer="w:my_widget_key">
    <selector xpath=".//w:annotation"/>
    <field xpath="@manu"/>
    <field xpath="@prod"/>
  </keyref>
</element>
```
OTHER FEATURES

- Groups
- Substitution groups (essentially subsumption based on element names, not types)
- Nil values
- Annotations
- Defaults and whitespace
- Modularization

Read the book chapter
By means of
Online demonstration
We had the following problems with the DTD description:

- **calories** should contain a non-negative number; → **FIXED!**
- **protein** should contain a value on the form $N\%$ where $N$ is between 0 and 100; → **FIXED!**
- **comment** should be allowed to appear anywhere in the contents of **recipe**; → **NOT FIXED!**
- **unit** should only be allowed in an elements where **amount** is also present; → **NOT FIXED!**
- nested **ingredient** elements should only be allowed when amount is absent; → **NOT FIXED!**
LIMITATIONS OF XML SCHEMA

- The details are extremely complicated (and the spec is unreadable)
- Declarations are (mostly) context insensitive
- It is impossible to write an XML Schema description of XML Schema
- With mixed content, character data cannot be constrained
- Unqualified local elements are bad practice
- Cannot require specific root element
- Element defaults cannot contain markup
- The type system is overly complicated
- xsi:type is problematic
- Simple type definitions are inflexible
The regular expressions used as element content models must be deterministic, sometimes also called one-unambiguous.

Intuitively:

- Intuitively, a regular expression is deterministic if, when processing the input sequence from left to right, it is always determined which symbol in the expression matches the next input symbol without looking ahead.
- This is supposed to make implementations faster.
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- This is supposed to make implementations faster.

Example:
- $a, a | a, b$ is not deterministic (consider input $aa$)
The regular expressions used as element content models must be deterministic, sometimes also called one-unambiguous.

Intuitively:

- Intuitively, a regular expression is deterministic if, when processing the input sequence from left to right, it is always determined which symbol in the expression matches the next input symbol without looking ahead.
- This is supposed to make implementations faster.

Example:

- \( a, a \mid a, b \) is **not** deterministic (consider input \( aa \))
- \( a, (a \mid b) \) **is** deterministic
The regular expressions used as element content models must be deterministic, sometimes also called one-unambiguous.

Intuitively:
- Intuitively, a regular expression is deterministic if, when processing the input sequence from left to right, it is always determined which symbol in the expression matches the next input symbol without looking ahead.
- This is supposed to make implementations faster.

Example:
- \(a, a \mid a, b\) is **not** deterministic (consider input \(aa\))
- \(a, (a \mid b)\) is deterministic
- Note that these two expressions match the same sequences!
For a regular expression $\alpha$, define $\overline{\alpha}$ to be the regular expression obtained from $\alpha$ by replacing, for each $i$, the $i$-th occurrence of symbol $\sigma$ in $\alpha$ (counting from left to right) by $\sigma_i$

Example:

- $a, a | a, b \rightarrow a_1, a_2 | a_3, b_1$
- $a, (a | b) \rightarrow a_1, (a_2 | b_1)$
- $(a | b)^*, a, c, (b | c)^* \rightarrow (a_1 | b_1)^*, a_2, c_1, (b_2 | c_2)^*$
For a regular expression $\alpha$, define $\overline{\alpha}$ to be the regular expression obtained from $\alpha$ by replacing, for each $i$, the $i$-th occurrence of symbol $\sigma$ in $\alpha$ (counting from left to right) by $\sigma_i$.

So if $\alpha$ is over the alphabet \{a, b, c, \ldots\} then $\overline{\alpha}$ is over the alphabet \{a\_1, a\_2, \ldots, b\_1, b\_2, \ldots, c\_1, c\_2, \ldots\}.

Example:

- $a, a \mid a, b \rightarrow a\_1, a\_2 \mid a\_3, b\_1$
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Definition

- For a regular expression $\alpha$, define $\bar{\alpha}$ to be the regular expression obtained from $\alpha$ by replacing, for each $i$, the $i$-th occurrence of symbol $\sigma$ in $\alpha$ (counting from left to right) by $\sigma_i$.
- So if $\alpha$ is over the alphabet \{a, b, c, \ldots\} then $\bar{\alpha}$ is over the alphabet \{a_1, a_2, \ldots, b_1, b_2, \ldots, c_1, c_2, \ldots\}.
- A regular expression $\alpha$ is deterministic if there are no sequences $wa_i v$ and $wa_j v'$ in $\mathcal{L}(\bar{\alpha})$ with $i \neq j$ ($w$ may be empty).

Example:

- $a, a \mid a, b \rightarrow a_1, a_2 \mid a_3, b_1$
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- A regular expression $\alpha$ is deterministic if there are no sequences $wa_i\nu$ and $wa_j\nu'$ in $L(r)$ with $i \neq j$ ($w$ may be empty).

Example:

- $a, a \mid a, b \rightarrow a_1, a_2 \mid a_3, b_1$ not deterministic: consider $a_1a_2$ and $a_3b_1$
- $a, (a \mid b) \rightarrow a_1, (a_2 \mid b_1)$
- $(a \mid b)^*, a, c, (b \mid c)^* \rightarrow (a_1 \mid b_1)^*, a_2, c_1, (b_2 \mid c_2)^*$
For a regular expression $\alpha$, define $\overline{\alpha}$ to be the regular expression obtained from $\alpha$ by replacing, for each $i$, the $i$-th occurrence of symbol $\sigma$ in $\alpha$ (counting from left to right) by $\sigma_i$.

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A regular expression $\alpha$ is deterministic if there are no sequences $wa_i\nu$ and $wa_j\nu'$ in $L(\overline{\alpha})$ with $i \neq j$ ($w$ may be empty).

Example:

- $a, a \mid a, b \rightarrow a_1, a_2 \mid a_3, b_1$
- $a, (a \mid b) \rightarrow a_1, (a_2 \mid b_1)$ deterministic
- $(a \mid b)^*, a, c, (b \mid c)^* \rightarrow (a_1 \mid b_1)^*, a_2, c_1, (b_2 \mid c_2)^*$
For a regular expression \( \alpha \), define \( \overline{\alpha} \) to be the regular expression obtained from \( \alpha \) by replacing, for each \( i \), the \( i \)-th occurrence of symbol \( \sigma \) in \( \alpha \) (counting from left to right) by \( \sigma_i \).

So if \( \alpha \) is over the alphabet \( \{a, b, c, \ldots\} \) then \( \overline{\alpha} \) is over the alphabet \( \{a_1, a_2, \ldots, b_1, b_2, \ldots, c_1, c_2, \ldots\} \).

A regular expression \( \alpha \) is **deterministic** if there are no sequences \( wa_i v \) and \( wa_j v' \) in \( \mathcal{L}(\overline{\alpha}) \) with \( i \neq j \) (\( w \) may be empty).

**Example:**

- \( a, a \mid a, b \rightarrow a_1, a_2 \mid a_3, b_1 \)
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  deterministic? or not?
Definition

- For a regular expression $\alpha$, define $\overline{\alpha}$ to be the regular expression obtained from $\alpha$ by replacing, for each $i$, the $i$-th occurrence of symbol $\sigma$ in $\alpha$ (counting from left to right) by $\sigma_i$.
- So if $\alpha$ is over the alphabet \{a, b, c, \ldots\} then $\overline{\alpha}$ is over the alphabet \{a$_1$, a$_2$, \ldots, b$_1$, b$_2$, \ldots, c$_1$, c$_2$, \ldots\}.
- A regular expression $\alpha$ is deterministic if there are no sequences $wa_i\nu$ and $wa_j\nu'$ in $L(\overline{r})$ with $i \neq j$ ($w$ may be empty).

Example:

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  not
Can we make every regular expression deterministic?

- \( a, a | a, b \rightarrow a, (a | b) \)
Can we make every regular expression deterministic?

- $a, a | a, b \rightarrow a, (a | b)$

**Theorem**

No there exists regular expressions for which no equivalent deterministic regular expression exists.

So we are limited to a subset of the regular expressions in DTDs and XSDs.