Task:

Consider the following relational schema:

- Emp(eid, did, sal, hobby)
- Dept(did, dname, floor, phone)
- Finance(did, budget, sales, expenses)

For the following SQL statement:

- 1. Translate the query into the relational algebra.
- 2. Remove redundant joins from the select-project-join subexpressions in the obtained logical query plan.
- 3. By means of the algebraic laws, further optimize the obtained expression.

Task (continued)

```
SELECT D.floor
FROM Dept D, Emp E
WHERE
(D.floor = 1
    OR D.floor IN
    ( SELECT D2.floor FROM Dept D2, Finance F1
        WHERE F1.budget > 150 AND D2.did = F1.did)
)
AND E.did = D.did
AND E.did IN (SELECT F2.did FROM Finance F2, Emp E2
        WHERE F2.did = E.did AND E2.did = D.did
        AND E2.eid = E.eid AND F2.expenses = 300)
```

Solution: translation into the relational algebra

First, we normalize the query to a form with only EXISTS and NOT EXISTS subqueries:

```
SELECT D.floor
FROM Dept D, Emp E
WHERE
(D.floor = 1 OR EXIST
  ( SELECT D2.floor FROM Dept D2, Finance F1
    WHERE F1.budget > 150 AND D2.did = F1.did
    AND D2.floor = D.floor) )
AND E.did = D.did
AND E.did = D.did
AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2
    WHERE F2.did = E.did AND E2.did = D.did
    AND E2.eid = E.eid AND F2.expenses = 300
    AND E.did = F2.did)
```

Conjunctive Normal Form

```
SELECT D.floor
FROM Dept D, Emp E
WHERE ( D.floor = 1
 AND E.did = D.did
  AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2
   WHERE F2.did = E.did AND E2.did = D.did
  AND E2.eid = E.eid AND F2.expenses = 300 AND E.did = F2.did)
) OR (
 EXIST ( SELECT D2.floor FROM Dept D2, Finance F1
   WHERE F1.budget > 150 AND D2.did = F1.did
   AND D2.floor = D.floor)
  AND E.did = D.did
  AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2)
   WHERE F2.did = E.did AND E2.did = D.did
   AND E2.eid = E.eid AND F2.expenses = 300 AND E.did = F2.did) )
```

Normalize to UNION

```
Q1 = SELECT D.floor

FROM Dept D, Emp E

WHERE D.floor = 1

AND E.did = D.did

AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2

WHERE F2.did = E.did AND E2.did = D.did

AND E2.eid = E.eid AND F2.expenses = 300

AND E.did = F2.did)
```

Normalize to UNION

```
Q2 = SELECT D.floor
FROM Dept D, Emp E
WHERE
EXIST ( SELECT D2.floor FROM Dept D2, Finance F1
WHERE F1.budget > 150 AND D2.did = F1.did
AND D2.floor = D.floor)
AND E.did = D.did
AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2
WHERE F2.did = E.did AND E2.did = D.did
AND E2.eid = E.eid AND F2.expenses = 300
AND E.did = F2.did)
```

The new query is Q1 UNION Q2.

Translation of the innermost subqueries

SELECT F2.did FROM Finance F2, Emp E2
WHERE F2.did = E.did AND E2.did = D.did
AND E2.eid = E.eid AND F2.expenses = 300
AND E.did = F2.did

This subquery is translated as follows:

 $e_1 = \pi_{F_2.\operatorname{did}, E.*, D.*} \sigma_{F_2.\operatorname{did}=E.\operatorname{did}\wedge E_2.\operatorname{did}=D.\operatorname{did}\wedge E_2.\operatorname{eid}=E.\operatorname{eid}}$ $\sigma_{F_2.\operatorname{expenses}=300\wedge E.\operatorname{did}=F_2.\operatorname{did}}(\rho_D(\operatorname{Dept}) \times \rho_E(\operatorname{Emp}) \times \rho_{F2}(\operatorname{Finance}) \times \rho_{E2}(\operatorname{Emp}))$

Translation of the innermost subqueries

SELECT D2.floor FROM Dept D2, Finance F1
WHERE F1.budget > 150 AND D2.did = F1.did
AND D2.floor = D.floor

This subquery is translated as follows:

 $e_2 = \boldsymbol{\pi}_{D_2.\texttt{floor}, D.*} \boldsymbol{\sigma}_{F_1.\texttt{budget} > 150 \land D_2.\texttt{did} = F_1.\texttt{did}} \\ \boldsymbol{\sigma}_{D_2.\texttt{floor} = D.\texttt{floor}} (\boldsymbol{\rho}_D(\texttt{Dept}) \times \boldsymbol{\rho}_{D2}(\texttt{Dept}) \times \boldsymbol{\rho}_{F1}(\texttt{Finance}))$

Translation of the Middle Queries

The translation of the from part gives

$$e_3 = (oldsymbol{
ho}_D({ t Dept}) imes oldsymbol{
ho}_E({ t Emp}))$$

To de-correlate we compute:

$$f = \hat{e_3} \bowtie \boldsymbol{\pi}_{D.*,E.*}(e_1)$$

Note that $\hat{e_3}$ is empty and hence

$$f = \boldsymbol{\pi}_{D.*,E.*}(e_1)$$

To this expression we add the WHERE and SELECT clause:

$$e_4 = \boldsymbol{\pi}_{D.\texttt{floor}}(\boldsymbol{\sigma}_{D.\texttt{floor}=1 \land E.did=D.did}(\boldsymbol{\pi}_{D.*,E.*}(e_1))$$

Translation of the Middle Queries

```
Q2 = SELECT D.floor
FROM Dept D, Emp E
WHERE
EXIST ( SELECT D2.floor FROM Dept D2, Finance F1
WHERE F1.budget > 150 AND D2.did = F1.did
AND D2.floor = D.floor)
AND E.did = D.did
AND EXISTS (SELECT F2.did FROM Finance F2, Emp E2
WHERE F2.did = E.did AND E2.did = D.did
AND E2.eid = E.eid AND F2.expenses = 300
AND E.did = F2.did)
```

The translation of the from part gives

$$e_5 = (\boldsymbol{\rho}_D(\texttt{Dept}) \times \boldsymbol{\rho}_E(\texttt{Emp}))$$

To de-correlate we compute:

$$f' = \hat{e_5} \bowtie (\pi_{D.*,E.*}(e_1) \bowtie \pi_{D.*}(e_2)) = (\pi_{D.*,E.*}(e_1) \bowtie \pi_{D.*}(e_2))$$

To this expression we add the WHERE and SELECT clause:

$$e_6 = \boldsymbol{\pi}_{D.\texttt{floor}} \boldsymbol{\sigma}_{E.did=D.did}(\boldsymbol{\pi}_{D.*,E.*}(e_1) \bowtie \boldsymbol{\pi}_{D.*}(e_2))$$

Translation of the Whole Query

Q1 UNION Q2

Since the schemas of e_4 and e_6 are the same, the union is straightforward:

$$e = e_4 \cup e_6$$

Written in full:

$$\begin{split} e &= \boldsymbol{\pi}_{D.\text{floor}} \boldsymbol{\sigma}_{D.\text{floor}=1 \land E.did=D.did} \\ \boldsymbol{\pi}_{D.*,E.*} \boldsymbol{\sigma}_{F_2.\text{did}=E.\text{did} \land E_2.\text{did}=D.\text{did} \land E_2.\text{eid}=E.\text{eid} \land F_2.\text{expenses}=300 \land E.\text{did}=F_2.\text{did}} \\ (\boldsymbol{\rho}_D(\text{Dept}) \times \boldsymbol{\rho}_E(\text{Emp}) \times \boldsymbol{\rho}_{F2}(\text{Finance}) \times \boldsymbol{\rho}_{E2}(\text{Emp})) \\ \cup \\ \boldsymbol{\pi}_{D.\text{floor}} \boldsymbol{\sigma}_{E.did=D.did} (\\ [\boldsymbol{\pi}_{D.*,E.*} \boldsymbol{\sigma}_{F_2.\text{did}=E.\text{did} \land E_2.\text{did}=D.\text{did} \land E_2.\text{eid}=E.\text{eid} \land F_2.\text{expenses}=300 \land E.\text{did}=F_2.\text{did}} \\ (\boldsymbol{\rho}_D(\text{Dept}) \times \boldsymbol{\rho}_E(\text{Emp}) \times \boldsymbol{\rho}_{F2}(\text{Finance}) \times \boldsymbol{\rho}_{E2}(\text{Emp}))] \\ & \bowtie [\boldsymbol{\pi}_{D.*} \boldsymbol{\sigma}_{F_1.\text{budget}>150 \land D_2.\text{did}=F_1.\text{did} \land D_2.\text{floor}=D.\text{floor}} \\ (\boldsymbol{\rho}_D(\text{Dept}) \times \boldsymbol{\rho}_{D2}(\text{Dept}) \times \boldsymbol{\rho}_{F1}(\text{Finance}))]) \end{split}$$

Redundant Joins Removal

The query comprises the following maximal select-project-join subexpressions:

- $\pi_{D.floor}\sigma_{D.floor=1 \land E.did=D.did}\pi_{D.*,E.*}\sigma_{...}(\rho_D(Dept) \times \rho_E(Emp) \times \rho_{F2}(Finance) \times \rho_{E2}(Emp))$
- $[\boldsymbol{\pi}_{D.*,E.*}\boldsymbol{\sigma}_{...}(\boldsymbol{\rho}_{D}(\texttt{Dept}) \times \boldsymbol{\rho}_{E}(\texttt{Emp}) \times \boldsymbol{\rho}_{F2}(\texttt{Finance}) \times \boldsymbol{\rho}_{E2}(\texttt{Emp}))]$
- $(\boldsymbol{\rho}_D(\texttt{Dept}) \times \boldsymbol{\rho}_{D2}(\texttt{Dept}) \times \boldsymbol{\rho}_{F1}(\texttt{Finance}))$

Note that " F_1 .budget > 150" cannot be included in a select-project-join expression. Also note that the third expression does not contain redundant joins (Why?).

Redundant Joins Removal

The first expression corresponds to:

$$Q_1("1") \leftarrow \texttt{Dept}(a_1, a_2, "1", a_4), \texttt{Emp}(b_1, a_1, b_3, b_4), \texttt{Finance}(a_1, c_2, c_3, "300"), \\ \texttt{Emp}(b_1, a_1, d_3, d_4)$$

The first and third atoms cannot be removed (Why?)

We check whether we can remove the second atom: $Q_2("1") \leftarrow \text{Dept}(a_1, a_2, "1", a_4), \text{Finance}(a_1, c_2, c_3, "300"), \text{Emp}(b_1, a_1, d_3, d_4)$ The corresponding canonical database: $D_2("1") =$ $\{\text{Dept}(\dot{a_1}, \dot{a_2}, "1", \dot{a_4}), \text{Finance}(\dot{a_1}, \dot{c_2}, \dot{c_3}, "300"), \text{Emp}(\dot{b_1}, \dot{a_1}, \dot{d_3}, \dot{d_4})\}$ Clearly $("1") \in Q_1(D_2)$ because of the matching

Redundant Joins Removal

No other atom can be removed (Why?).

The optimal query is hence $Q_2(``1") \leftarrow \text{Dept}(a_1, a_2, ``1", a_4), \text{Finance}(a_1, c_2, c_3, ``300"), \text{Emp}(b_1, a_1, d_3, d_4)$ The optimal query is hence

Translating this query back to the relational algebra, we obtain:

$$\begin{aligned} \boldsymbol{\pi}_{D.\texttt{floor}}([\boldsymbol{\sigma}_{D.\texttt{floor}=1 \land E_2.did=D.did \land F_2.\texttt{did}=E_2.\texttt{did} \land E_2.\texttt{did}=D.\texttt{did} \land F_2.\texttt{expenses}=300 \\ (\boldsymbol{\rho}_D(\texttt{Dept}) \times \boldsymbol{\rho}_{F2}(\texttt{Finance}) \times \boldsymbol{\rho}_{E2}(\texttt{Emp}))]) \end{aligned}$$

Redundant Joins Removal

The second expression is:

 $[\boldsymbol{\pi}_{D.*,E.*}\boldsymbol{\sigma}_{F_2.\texttt{did}=E.\texttt{did}\wedge E_2.\texttt{did}=D.\texttt{did}\wedge E_2.\texttt{eid}=E.\texttt{eid}\wedge F_2.\texttt{expenses}=300\wedge E.\texttt{did}=F_2.\texttt{did}} \\ (\boldsymbol{\rho}_D(\texttt{Dept}) \times \boldsymbol{\rho}_E(\texttt{Emp}) \times \boldsymbol{\rho}_{F2}(\texttt{Finance}) \times \boldsymbol{\rho}_{E2}(\texttt{Emp}))]$

Translated:

$$\begin{aligned} Q_3(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4) \leftarrow \texttt{Dept}(a_1, a_2, a_3, a_4), \texttt{Emp}(b_1, b_2, b_3, b_4), \\ \texttt{Finance}(a_1, c_2, c_3, ``300"), \texttt{Emp}(b_1, a_1, d_3, d_4) \end{aligned}$$

We cannot remove the second atom, this time (why?)

Redundant Joins Removal

Let us try to remove the fourth atom, let

$$\begin{aligned} Q_4(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4) \leftarrow \texttt{Dept}(a_1, a_2, a_3, a_4), \texttt{Emp}(b_1, b_2, b_3, b_4), \\ \texttt{Finance}(a_1, c_2, c_3, ``300'') \end{aligned}$$

By evaluating Q_3 on the canonical database of Q_4 we see that $Q_4 \not\subseteq Q_3$. Hence $Q_3 \not\equiv Q_4$ and the fourth atom can hence not be removed. This original SPJ expression was hence optimal.

Redundant Joins Removal

The third expression is:

$$(\boldsymbol{\rho}_D(\texttt{Dept}) imes \boldsymbol{\rho}_{D2}(\texttt{Dept}) imes \boldsymbol{\rho}_{F1}(\texttt{Finance}))$$

Translated:

$$Q_5(a_1, \dots, a_4, b_1, \dots, b_4, c_1, \dots, c_4) \leftarrow \texttt{Dept}(a_1, a_2, a_3, a_4), \texttt{Dept}(b_1, b_2, b_3, b_4), \\ \texttt{Finance}(c_1, c_2, c_3, c_4)$$

No atoms can be removed (why?)

Redundant Joins Removal

The optimized expression is therefore:

$$e = \pi_{D.floor}([\sigma_{D.floor=1 \land E_2.did=D.did \land F_2.did=E_2.did \land E_2.did=D.did \land F_2.expenses=300} (\rho_D(\text{Dept}) \times \rho_{F2}(\text{Finance}) \times \rho_{E2}(\text{Emp}))]) \cup \\ \cup \\ \pi_{D.floor}\sigma_{E.did=D.did}([\pi_{D.*,E.*}\sigma_{F_2.did=E.did \land E_2.did=D.did \land E_2.eid=E.eid \land F_2.expenses=300 \land E.did=F_2.did} (\rho_D(\text{Dept}) \times \rho_E(\text{Emp}) \times \rho_{F2}(\text{Finance}) \times \rho_{E2}(\text{Emp}))] \\ \bowtie [\pi_{D.*}\sigma_{F_1.budget>150 \land D_2.did=F_1.did \land D_2.floor=D.floor} (\rho_D(\text{Dept}) \times \rho_{D2}(\text{Dept}) \times \rho_{F1}(\text{Finance}))])$$

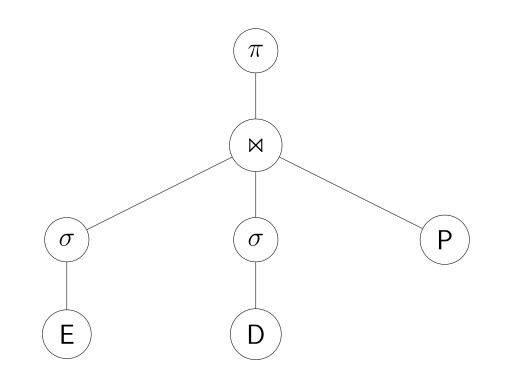
Task

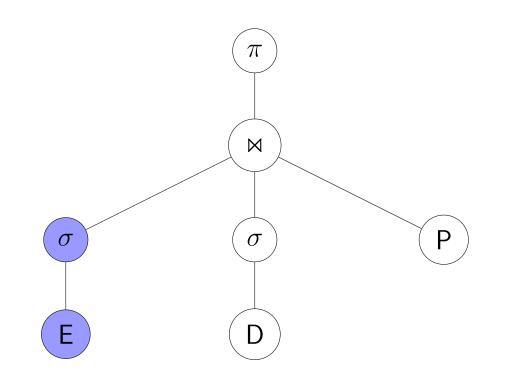
```
(refer to the handouts for the full exercise)
```

```
Construct a sufficiently optimal physical query plan for:
```

```
\pi_{\texttt{E.eid},\texttt{D.did},\texttt{P.pid}}\sigma_{\texttt{E.sal}=\texttt{50000}}(E) \Join \sigma_{\texttt{D.budget} \geq 20000}(D) \Join P
```

Assume that employee salaries are uniformly distributed over the range [10009, 110008] and that project budgets are uniformly distributed over [10000, 30000]. There are clustered indexes available on E.sal, D.did and P.pid.





Solution

Subexpression:

 $\sigma_{\rm E.sal=50000}(E)$

First possibility: we use the clustered index on E.sal to get the records such that E.sal = 50000.

The number of tuples that satisfy the salary requirement is estimated to:

$$\left[\frac{1}{110008 - 10009 + 1} \text{ selectivity } \times 20000 \text{ employees } \right] = 1 \text{ tuples}$$

Hence, the result can be stored in 1 block, which is also the cost of the index scan:

$$\left| \frac{20 \text{ bytes}}{4000 \text{ bytes/block}} \right| = 1 \text{ block}$$

Solution

Subexpression:

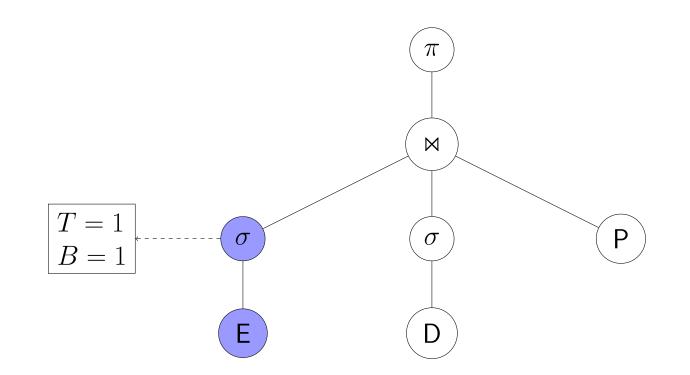
 $\sigma_{\rm E.sal=50000}(E)$

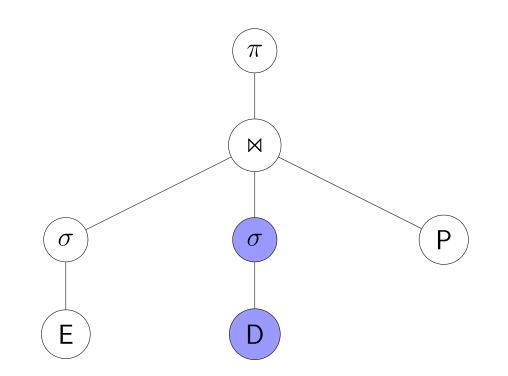
Second possibility: we use a table scan, which costs

$$\frac{20000 \text{ tuples}}{\left\lfloor \frac{4000 \text{ bytes/block}}{20 \text{ bytes/tuple}} \right\rfloor} = 100 \text{ block I/Os}$$

Clearly, we prefer the index scan.

Pipelining: since selections can be pipelined, we (greedily) decide to pipeline this seclection





Solution

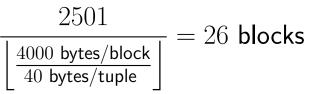
Subexpression:

 $\sigma_{\texttt{D.budget} \geq 20000}(D)$

The number of tuples returned is estimated to 2501:

 $\left[\frac{30000 - 20000 + 1}{30000 - 10000 + 1} \text{ selectivity } \times 5000 \text{ departments} \right] = 2501 \text{ tuples}$

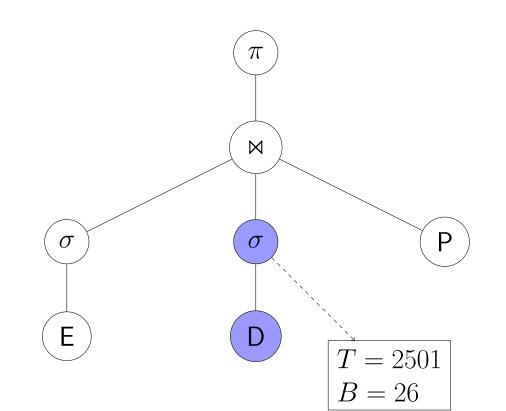
This corresponds to 26 Blocks:

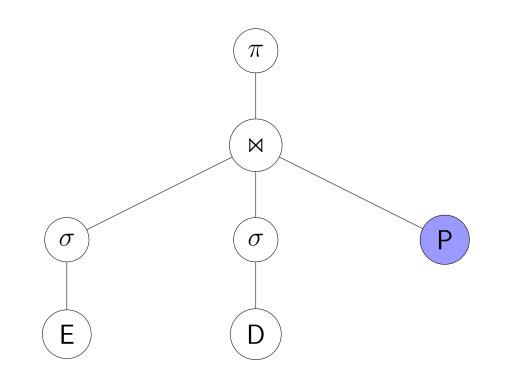


Since no index is available, a table scan is our only possibility:

 $\frac{5000}{\left\lfloor\frac{4000 \text{ bytes/block}}{40 \text{ bytes/tuple}}\right\rfloor} = 50 \text{ blocks}$

Pipelining: since selections can be pipelined, we (greedily) decide to pipeline this seclection





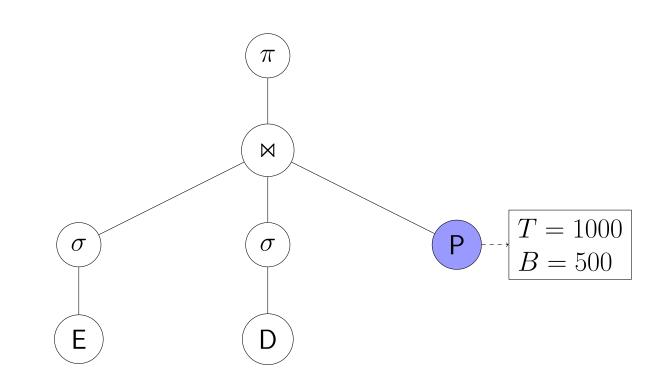
Solution

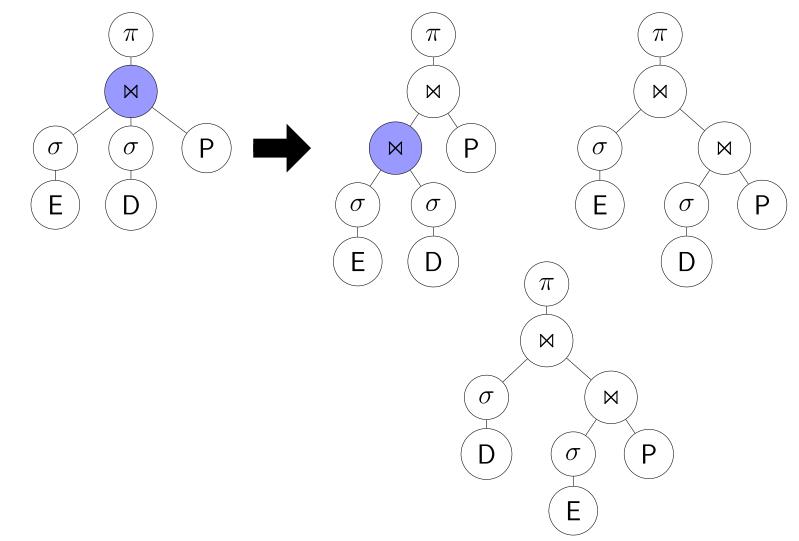
Subexpression:

P

A table scan on P requires 500 block I/O's. This is also the estimated number of blocks returned:

$$\frac{1000 \text{ tuples}}{\left\lfloor \frac{4000 \text{ bytes/block}}{2000 \text{ bytes/tuple}} \right\rfloor} = 500 \text{ blocks}$$





Solution

Now, we must determine an ordering for the joins. We consider all pairs of joins and keep the one with the smallest cost.

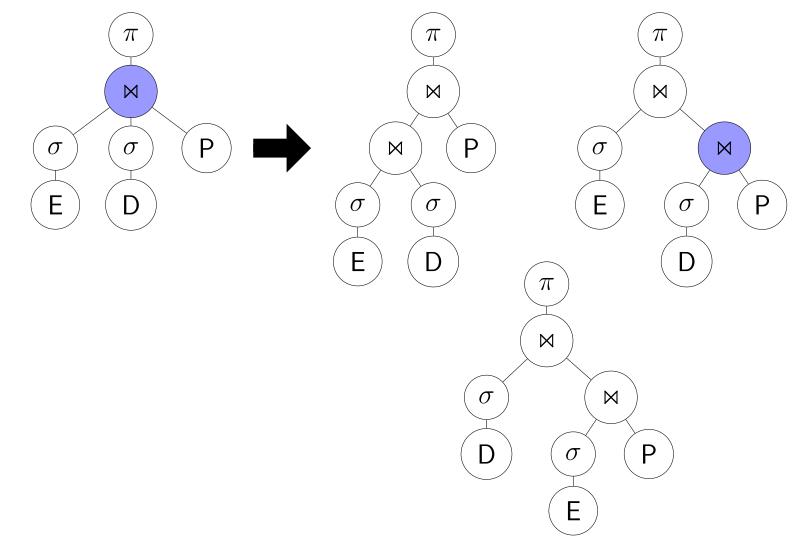
$$\underbrace{\sigma_{\texttt{e.sal}=50000}(E)}_{e_1} \text{ and } \underbrace{\sigma_{\texttt{d.budget} \ge 20000}(D)}_{e_2}$$

The selection on each side requires one buffer to execute, leaving only $10 \ {\rm buffers}$ for the join.

The output of e_1 contains only 1 tuples, and can therefore be computed in 1 block. Since $1 = B(e_1) \le M = 10$, we can apply the one-pass join algorithm. Its cost is

$$B(e_1) + B(e_2) = 1 + 26 = 27 \text{ I/O's}$$

An index-join cannot be used on e_2 since it is not a base relation. All other join methods always cost more than one-pass join. Hence the one-pass join is preferred.



Solution

The second join pair is:

$$\underbrace{\sigma_{\text{D.budget} \geq 20000}(D)}_{e_2} \text{ and } P$$

We have 11 buffers at our disposal, given that we need 1 buffer to perform the selection in e_2 . It is not possible to use a one-pass join, since $26 = B(e_2) \ge M = 11$ and $500 = B(P) \ge M = 11$.

A block-based nested-loop join costs:

$$B(e_2) + \left\lceil \frac{B(e_2)}{M-1} \right\rceil \times B(P) = 26 + \left\lceil \frac{26}{10} \right\rceil \times 500 = 1526 \text{ I/Os}$$

Solution

The second join pair is:

$$\underbrace{\sigma_{\mathtt{D.budget} \geq 20000}(D)}_{e_2} \text{ and } P$$

We have enough memory to perform an optimized sort-merge join:

$$8 = \left\lceil \frac{B(e_2)}{M^{\lceil \log_M B(e_2) \rceil - 1}} \right\rceil + \left\lceil \frac{B(P)}{M^{\lceil \log_M B(P) \rceil - 1}} \right\rceil \le M = 11 \text{ available buffers}$$

This optimized sort-merge join has a cost of:

$$2B(e_2) \lceil \log_M B(e_2) \rceil + 2B(P) \lceil \log_M B(P) \rceil - B(e_2) - B(P) = 2 \times 26 \times 2 + 2 \times 500 \times 3 - 26 - 500 = 2578 \text{ I/O's}$$

Solution

Assuming that the clustered index on P.pid is a *BTree*, it ensues that P is already sorted on this join attribute. Given that we then only need to sort e_2 , the cost of a non-optimized sort-merge join is:

$$2B(e_2) \left\lceil \log_M B(e_2) \right\rceil + B(e_2) + B(P)$$

Futhermore, we can optimize the last merge:

4 necessary buffers =
$$\left\lceil \frac{B(e_2)}{M} \right\rceil + 1 \le M = 11$$
 available buffers

The cost thereof is:

$$2B(e_2)(\lceil \log_M B(e_2) \rceil - 1) + B(e_2) + B(P)$$

= 2 × 26 × 1 + 26 + 500
= 578 I/Os

Solution

The cost of an hash-join is:

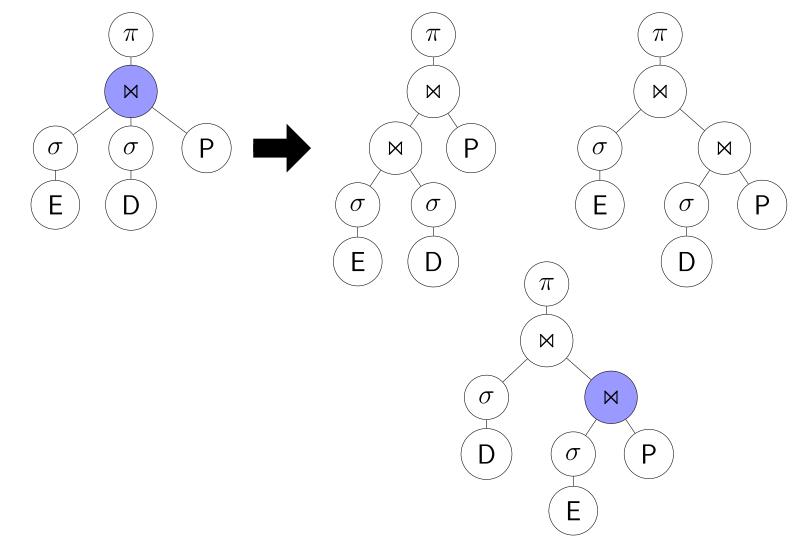
$$2B(e_2) \lceil \log_{M-1} B(e_2) - 1 \rceil + 2B(P) \lceil \log_{M-1} B(e_2) - 1 \rceil + B(e_2) + B(P)$$

= 2 × 26 × 1 + 2 × 500 × 1 + 26 + 500
= 1578 I/O's

It is also possible to use an index-join, using the clustered index on P.pid. This method has a cost of:

$$B(e_2) + T(e_2) \times \left[\frac{B(P)}{V(P, pid)} \right] = 26 + 2501 \times 1 = 2527 \text{ I/O's}$$

Hence, we assume that the index on P.pid is a BTree, and sorting P is not necessary. In that case the optimized sort-merge join that only sorts e_2 is preferred.



Solution

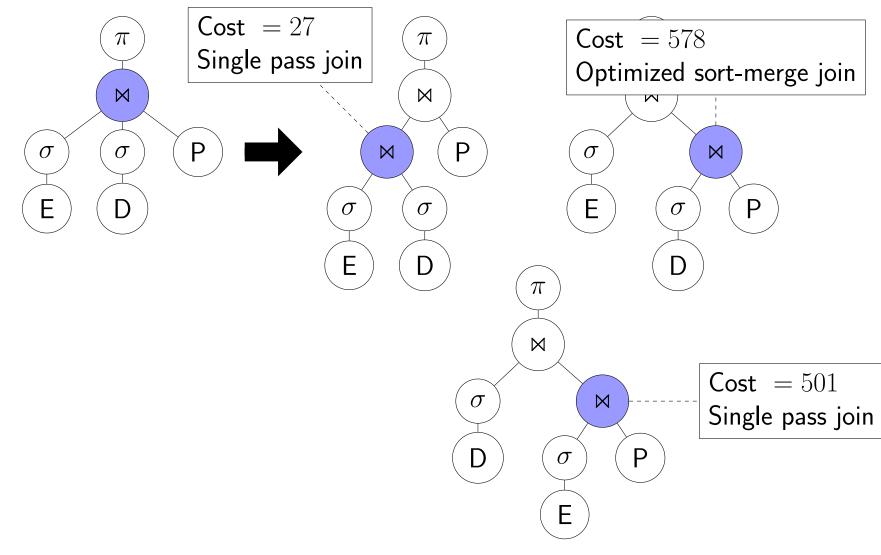
The third join pair is:

$$\underbrace{\sigma_{\mathrm{E.sal}=50000}(E)}_{e_1}$$
 and P

Note that this join is a full cartesian product. A one-pass join is available at the following cost:

$$B(e_1) + B(P) = 1 + 500 = 501 \text{ I/O's}$$

No index can help up for this join, and the one-pass join algorithm gives the best cost.



Solution

The join-pair with the least cost is therefore:

$$\underbrace{\sigma_{\texttt{e.sal}=50000}(E)}_{e_1} \text{ and } \underbrace{\sigma_{\texttt{D.budget} \ge 20000}(D)}_{e_2}$$

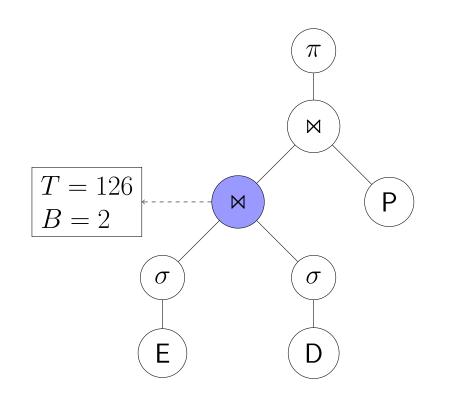
Where an one-pass join on E.did is used. This one-pass join *computes in-memory* the result of e_1 , and iterates block-by block over the results of e_2 . Therefore, only 2 buffers are necessary (why?).

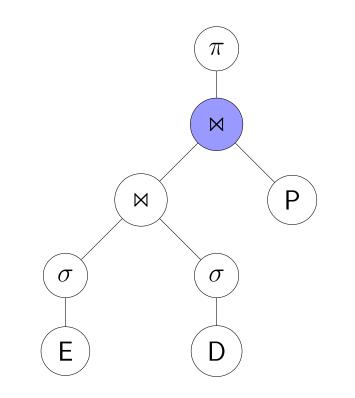
We greedily decide that the output of this join is pipelined to the next operator.

The estimated number of tuples in the output of this join is:

$$\frac{T(e_1) \times T(e_2)}{\max(V(e_1, \text{did}), V(e_2, \text{did}))} = \frac{1 \times 2501}{20} = 126$$

These records are 60 bytes long and can be stored in 2 blocks





Solution

We still need to find the best way to join the whole expression

$$\underbrace{\sigma_{\texttt{e.sal}=\texttt{50000}}(E) \Join \sigma_{\texttt{D.budget} \geq 20000}(D)}_{e_3} \text{ and } P$$

We expect to have 12 - 2 = 10 main memory buffers available.

The output of e_3 fits in 2 blocks. Given that $2 = B(e_3) \le M = 10$, a one-pass join is possible. The cost thereof is:

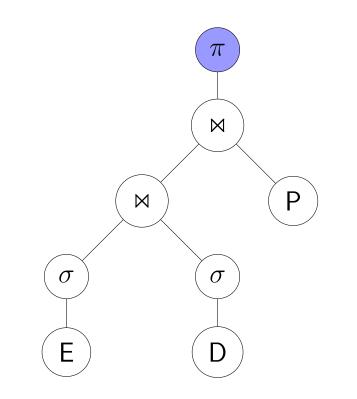
$$B(e_3) + B(P) = 2 + 500 = 502$$

This join can also be performed by means of an index-join, using the clustered index on P.pid.

$$B(e_3) + T(e_3) \times \left\lceil \frac{B(P)}{V(P, pid)} \right\rceil = 2 + 125 \times 1 = 127 \text{ I/O's}$$

Hence, the index-join is preferred.

We greedily decide to pipeline this join.



Solution

The projection $\pi_{\text{E.eid,D.did,P.pid}}$ can be performed on the fly at the same time as the last join.

Notice that we did not need to materialize any of the intermediate results.

