Course Notes on
The Logical Structure of Relational Query Languages

The Logical Structure of Relational Query Languages: Topics

- Overview
- First-order logic
- Tuple Relational Calculus (TRC)
- Domain Relational Calculus (DRC)

1

On the Way to SQL: Relational Calculi

- Historically, SQL was a major advance over older database languages (like DL/I of IMS or DDL, DML of CODASYL DBTG) because SQL is far easier to use
- To effectively master and use SQL up to relational completeness, first mastering first-order logic makes things significantly easier

2

Logic as a Basis for Database Languages

- **First-order logic (predicate calculus) is simple** at the level needed for relational languages
- Strong historical **prejudice against logic** (and theory) in the user world
- **Formal definitions have many advantages**
  - ◇ the ultimate reference document
  - ◇ test of language consistency during design
  - ◇ need not be shown to everybody
- **Logic has become a basic formalism in informatics** for e.g.,
  - ◇ assertions in programming
  - ◇ integrity formulation and maintenance in DBMS
  - ◇ data models of DBMS
  - ◇ semantics of programming languages

3

Relational Calculi

- More used than the algebra as a basis for user languages
- Directly based on first-order logic ⇒ regular, systematic structure
- Less **procedural** than the algebra : **what** versus **how**
- **Relational completeness**:
  - ◇ DRC, TRC, and algebra have same expressive power
  - ◇ SQL is slightly more powerful: some computation, ordering, etc.

4

TRC and DRC

- **Domain Relational Calculus (DRC)**
  - ◇ Most similar to logic as a modeling language
  - ◇ Typical modeling formalism in AI and natural-language studies: data is viewed as objects with properties
- **Tuple Relational Calculus (TRC)**
  - ◇ Reflects traditional pre-relational file structures
  - ◇ Closer to a view of relations implemented as files

5

A Simple Introduction to Logic

- General form of first-order logic is not necessary
- Logic is applied to a fixed domain of reference: the DB extension
- Formal system =

$$\begin{cases} \text{formal language (syntax + semantics)} \\ \text{deductive mechanisms} \end{cases}$$

- Here we basically need the syntax of logic, and a simple “applied” semantics linked to the DB extension
- The language of logic is used to combine elementary DB facts

6

- Simple and intuitive introductions to logic:
  - ◇ *Introduction to Logic for Liberal Arts and Business Majors*, by S. Waner and R. Costenoble, http://www.hofstra.edu/ matscw/logicintro.html, July 1996.
  - ◇ *Sweet Reason: A Field Guide to Modern Logic*, by T. Tymoczko and J. Henle, Springer Textbooks in Mathematical Sciences, ISBN 0-287-98930-7, Springer, 2nd ed., 1999

The Structure of First-Order Logic

- The universe of reference is the current database
- **Elementary propositions**: express assertions that are true or false in the universe
- **Propositional connectives** ($\wedge$, $\vee$, $\rightarrow$, $\neg$, $\leftrightarrow$) combine propositions

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$\neg P$	$P \leftrightarrow Q$
T	T	T	T	T	F	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

- Elementary propositions:
  - P1 : Smith was born on 09-Jan-55 is true in the current state of the world (i.e., of the database)
  - P2 : Smith is female is false
- Compound propositions:
  - $P1 \wedge P2$ = Smith was born on 09-JAN-55 $\wedge$ Smith is female is false
  - $\neg P2$ = Smith is not female is true
- Much of the problem with the intuition of logic comes from implication, namely, with the fact that $P \rightarrow Q$ is true when P is false

Relational Schema for the Company Example

Employee

SSN	FName	LName	BDate	Address	Sex	Salary	SuperSSN	DNo

Department

DNumber	DName	MgrSSN	MgrStartDate

DeptLocations

DNumber	DLocation

Project

PNumber	PName	PLocation	DNumber

WorksOn

PNo	ESSN	Hours

Dependent

ESSN	DependentName	Sex	BDate	Relationship

Quantifiers

- Use variables to express more general assertions about the DB:
  F1 : there exists an employee who was born on 09-Jan-55 is true
  F2 : all employees were born on 09-Jan-85 is false, or
  : there is at least one employee who was not born on 09-Jan-85 is true
  F3 : all employees born after 1950 earn more than 40k is false, or
  : there is at least one employee born after 1950 who earns less than 40k is true
- More formally
  F1 : $\exists e$ (e is an employee $\wedge$ e was born on 09-Jan-55)
  F2 : $\neg\, \forall e$ (e is an employee $\rightarrow$ e was born on 09-Jan-55), or
  : $\exists e$ (e is an employee $\wedge$ e was not born on 09-Jan-55)
  F3 : $\neg\, \forall e$ (e is an employee $\wedge$ e was born after 01-Jan-50 $\rightarrow$ e earns more than 40k), or
  : $\exists e$ (e is an employee $\wedge$ e was born after 01-Jan-50 $\wedge$ e earns less than 40k)

- $\forall$ (for all) and $\exists$ (there exists)
- if you cannot do everything ...
  - that does not mean that there is not anything that you can do ...
  - nor that there is anything that you cannot do ...

Queries

- Free variables of logic are used as query variables
- List the employees who were born on 09-Jan-55

$$\{e \mid \text{e is an employee} \wedge \text{e was born on 09-Jan-55}\}$$

- The $\{e \mid P(e)\}$ syntax evokes set theory
- A more fancy syntax for the same expression (see later)

```
SELECT  ...  FROM ...  WHERE  ...
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10

Equivalence Rules

- Allow to replace a formula by another one

$P \rightarrow Q$	is equivalent to	$\neg P \vee Q$
$\neg(P \wedge Q)$		$\neg P \vee \neg Q$
$\neg(P \vee Q)$		$\neg P \wedge \neg Q$
$\forall x\ P(x)$		$\neg(\exists x\ (\neg P(x)))$
$\exists x\ P(x)$		$\neg(\forall x\ (\neg P(x)))$
$\exists x\ (\neg P(x))$		$\neg(\forall x\ P(x))$

- Implication rules for quantifiers

$\forall x\ P(x)$	implies that	$\exists x\ P(x)$
$\neg(\exists x\ P(x))$		$\neg(\forall x\ P(x))$

but not the converse

11

- This is about all the logic that is needed to master languages of traditional relational systems

Tuple Relational Calculus (TRC)

- **Tuple variables**:
  - range on (takes as values) tuples of a relation
  - are explicitly linked to a relation
- List employees who make more than 50k

$$\{t \mid \text{Employee}(t) \wedge t.\text{Salary} > 50k\}$$

  - Employee(t) is a "relation predicate", it links TRC with the DB
  - t.Salary is a term whose value is the value of attribute Salary of tuple t
- List birthdate and address of employees called John Smith

$\{t.\text{BDate}, t.\text{Address} \mid \text{Employee}(t) \wedge t.\text{FName} = \text{'John'} \wedge t.\text{LName} = \text{'Smith'}\}$

12

General Structure of TRC Queries

$$\boxed{\{t_1.A_1, t_2.A_2, \ldots, t_n.A_n \mid F(t_1, \ldots, t_n, t_{n+1}, \ldots, t_m)\}}$$

- $t_1, t_2, \ldots, t_m$: tuple variables each associated in F with a relation through a relation predicate
- A_i: attribute of the relation associated with t_i
- F: logical formula containing variables $t_1, t_2, \ldots, t_m$
- $t_1, t_2, \ldots, t_n$: free variables in F ("query variables")
- $t_{n+1}, \ldots, t_m$: variables quantified in F

13

TRC Semantics

- F is evaluated for all possible values $t_1, t_2, \ldots, t_n$ (= Cartesian product)
- If F is true for a tuple, then the projection $t_1.A_1, t_2.A_2, \ldots, t_n.A_n$ is included in the result
- Result = nameless relation with n attributes; rules must be specified for deciding attribute names (e.g., A_i's if they are all distinct)

Structure of TRC Formulas

- Formula F is defined with the recursive structure of first-order logic
  - $R(t_i)$, where R is a relation name
  - $t_i.A$ comparison $t_j.B$
  - $t_i.A$ comparison constant
  - $\neg F$
  - $F_1 \wedge F_2$
  - $F_1 \vee F_2$
  - $F_1 \rightarrow F_2$
  - $F_1 \leftrightarrow F_2$
  - $\exists t \; F(t)$
  - $\forall t \; F(t)$
- Comparison: $=$, $\neq$, $<$, $>$, $\leq$, $\geq$

14

Join

- List name and address of employees who work for the Research department

$$\{e.\text{LName}, e.\text{Address} \mid \text{Employee}(e) \land \exists d\ (\text{Department}(d) \land d.\text{DName} = \text{`Research'} \land d.\text{DNumber} = e.\text{DNo})\}$$

- "Join term" $d.\text{DNumber} = e.\text{DNo}$ expresses a join between relation Department and relation Employee

15

Relative Procedurality of Languages

- Two different algebraic formulations for the previous example:
  - $\pi_{\text{LName,Address}}(\sigma_{\text{DName=`Research'}}(\text{Employee} \bowtie_{\text{DNo=DNumber}} \text{Department}))$
  - $\pi_{\text{LName,Address}}(\text{Employee} \bowtie_{\text{DNo=DNumber}} (\sigma_{\text{DName=`Research'}}(\text{Department})))$
- Only one TRC formulation
  $\{e.\text{LName}, e.\text{Address} \mid \text{Employee}(e) \land \exists d\ (\text{Department}(d) \land d.\text{DName} = \text{`Research'} \land d.\text{DNumber} = e.\text{DNo})\}$
- The algebra is more procedural than TRC: in TRC, the relative order of join and selection is not an issue
- For casual users, TRC style is simpler than algebra style (less to think about)
- Efficiency is another issue

16

- Efficiency:
  - in most cases, the strategy that evaluates selection before joins is more efficient
  - this is taken care of by the query optimizer of the DBMS

Two Joins

- For every project located in Brussels, list the project number, the controling department number, and the name of the department manager

$$\{p.\text{PNumber}, p.\text{DNum}, m.\text{LName} \mid \text{Project}(p) \land \text{Employee}(m) \land p.\text{Location} = \text{`Brussels'} \land \exists d\ (\text{Department}(d) \land d.\text{DNumber} = p.\text{DNum} \land d.\text{MgrSSN} = m.\text{SSN})\}$$

- Same conclusion about procedurality: algebra is more procedural

17

- In this example, if $p.\text{DNum}$ is replaced by $d.\text{DNumber}$ in the target of the query, then the quantifier $\exists d$ disappears, yielding a more symmetric formulation

$$\{p.\text{PNumber}, d.\text{DNumber}, m.\text{LName} \mid \text{Project}(p) \land \text{Employee}(m) \land \text{Department}(d) \land p.\text{Location} = \text{Brussels} \land d.\text{DNumber} = p.\text{DNum} \land d.\text{MgrSSN} = m.\text{SSN}\}$$

Other Example with two Joins

- List the name of employees who work on some project controled by department number 5

$$\{e.\text{FName}, e.\text{LName} \mid \text{Employee}(e) \wedge \\ \exists p\, \exists w\, (\text{Project}(p) \wedge \text{WorksOn}(w) \wedge \\ p.\text{DNum} = 5 \wedge w.\text{ESSN} = e.\text{SSN} \wedge p.\text{PNumber} = w.\text{PNo})\}$$

- Same conclusion about procedurality: algebra is more procedural

18

A “Complex” Query

- List project names of projects for which an employee whose last name is Smith is a worker or a manager of the department that controls the project

$$\{p.\text{PName} \mid \text{Project}(p) \wedge \\ \exists e\, \exists w\, (\text{Employee}(e) \wedge \text{WorksOn}(w) \wedge \\ w.\text{PNo} = p.\text{PNumber} \wedge w.\text{ESSN} = e.\text{SSN} \wedge e.\text{LName} = \text{‘Smith’}) \\ \vee \\ \exists m\, \exists d\, (\text{Employee}(m) \wedge \text{Department}(d) \wedge \\ p.\text{DNum} = d.\text{DNumber} \wedge d.\text{MgrSSN} = m.\text{SSN} \wedge m.\text{LName} = \text{‘Smith’})\}$$

- Union of two queries in the algebra is expressed in TRC with disjunction

19

- $\{x \mid P(x) \vee Q(x)\} \equiv \{x \mid P(x)\} \cup \{x \mid Q(x)\}$
- Other version: factor out of the disjunction the repeated

$$\exists e\, (\text{Employee}(e) \wedge e.\text{LName} = \text{Smith})$$

Join of a Relation with Itself

- List the first and last name of each employee, and the first and last name of his/her immediate supervisor

$$\{e.\text{FName}, e.\text{LName}, s.\text{FName}, s.\text{LName} \mid \\ \text{Employee}(e) \wedge \text{Employee}(s) \wedge e.\text{SuperSSN} = s.\text{SSN}\}$$

- The attributes of the result relation have to be specified explicitly (if the result is to be used elsewhere, i.e., not just displayed) through some kind of assignment

$$\text{F}(\text{EmpFN}, \text{EmpLN}, \text{MgrFN}, \text{MgrLN}) \leftarrow \{\ldots\}$$

- Syntax is more difficult for the algebra, unless attributes are ordered

20

Other Example of Join of a Relation with Itself

- List the SSN of employees who have both a dependent son and a dependent daughter

$$\begin{aligned}\{e.\text{ESSN} \mid \text{Dependent}(e) \\ \quad \wedge \exists d\ (\text{Dependent}(d) \\ \quad\quad \wedge\ e.\text{ESSN} = d.\text{ESSN} \\ \quad\quad \wedge\ d.\text{Relationship} = \text{`Son'} \\ \quad\quad \wedge\ d.\text{Relationship} = \text{`Daughter'})\}\end{aligned}$$

21

Universal Quantifier

- List the name of employees who work on all projects

$$\begin{aligned}\{e.\text{FName}, e.\text{LName} \mid \text{Employee}(e) \\ \quad \wedge\ \forall p\ \text{Project}(p) \to \\ \quad\quad \exists w\ (\text{WorksOn}(w) \wedge w.\text{PNo} = p.\text{PNumber} \wedge w.\text{ESSN} = e.\text{SSN})\}\end{aligned}$$

- "all projects" are those in relation Project

22

- **Various styles of universal quantification** (for List the employees who work on all projects):
  - logical formulation: $\{e \mid \text{Employee(e)} \wedge \forall p\ (\text{Project(p)} \to \text{Workson(e,p)})\}$
  - logic with range-coupled quantifiers: $\{e \in \text{Employee} \mid \forall p \in \text{Project (Workson(e,p))}\}$
  - towards natural language (where quantification is "infix" rather than "prefix" as in logic, binary predicates are also infix rather than prefix, and variables are seldom used as such):
    - $\{e \in \text{Employee} \mid \text{for all p} \in \text{Project (e Workson p)}\}$
    - $\{e \in \text{Employee} \mid \text{e Workson(all p} \in \text{Project)}\}$
    - {Employee Workson (all Project)}

Universal Quantifier

- List the name of employees who have at least one dependent

$$\begin{aligned}\{e.\text{LName} \mid \text{Employee}(e) \wedge \\ \quad \exists d\ (\text{Dependent}(d)\ \wedge\ e.\text{SSN} = d.\text{ESSN})\}\end{aligned}$$

- List the name of employees who have no dependent

$$\begin{aligned}\{e.\text{LName} \mid \text{Employee}(e) \wedge \\ \quad \neg\exists d\ (\text{Dependent}(d)\ \wedge\ e.\text{SSN} = d.\text{ESSN})\}\end{aligned}$$

$$\begin{aligned}\{e.\text{LName} \mid \text{Employee}(e) \wedge \\ \quad \forall d\ (\text{Dependent}(d)\ \to\ e.\text{SSN} \neq d.\text{ESSN})\}\end{aligned}$$

$$\begin{aligned}\{e.\text{LName} \mid \text{Employee}(e) \wedge \\ \quad \forall d \in \text{Dependent}\ (e.\text{SSN} \neq d.\text{ESSN})\}\end{aligned}$$

23

- Proof of equivalence of the formulations of List the name of employees who have no dependent by applying the equivalence rules of logic:
  - $\neg(\exists d\ P(d)) \equiv \forall d\ (\neg P(d))$
  - $\neg\exists d\ (\text{Dependent}(d) \wedge e.\text{SSN} = d.\text{ESSN})$
  - $\forall d\ \neg(\text{Dependent}(d) \wedge e.\text{SSN} = d.\text{ESSN})$
  - $\forall d\ (\neg\text{Dependent}(d) \vee \neg(e.\text{SSN} = d.\text{ESSN}))$
  - $\forall d\ (\neg\text{Dependent}(d) \vee e.\text{SSN} \neq d.\text{ESSN})$
  - $\forall d\ (\text{Dependent}(d) \to e.\text{SSN} \neq d.\text{ESSN})$
  - $\forall d \in \text{Dependent}\ (e.\text{SSN} \neq d.\text{ESSN})$

Safe Use of Universal Quantification

- Universal quantification must always be associated with implication
- Given relations $\mathrm{Prereq(Course, Pre)}$ and $\mathrm{Took(StudID, Course)}$, give the names of students who took all prerequisites of the course Math210
- Use of $\wedge$ instead of $\rightarrow$

$$\{s.\mathrm{Name} \mid \mathrm{Student}(s)\ \wedge \forall p\ (\mathrm{Prereq}(p) \wedge p.\mathrm{Course} = \text{`Math210'} \wedge \\ \exists t\ \mathrm{Took}(t) \wedge t.\mathrm{StudID} = s.\mathrm{StudID} \wedge t.\mathrm{Course} = p.\mathrm{Pre})\}$$

- If Math210 has no prerequisites, the answer of the above query is always empty
- Correct formulation

$$\{s.\mathrm{Name} \mid \mathrm{Student}(s)\ \wedge \forall p\ (\mathrm{Prereq}(p) \wedge p.\mathrm{Course} = \text{`Math210'} \rightarrow \\ \exists t\ \mathrm{Took}(t) \wedge t.\mathrm{StudID} = s.\mathrm{StudID} \wedge t.\mathrm{Course} = p.\mathrm{Pre})\} \equiv \\ \{s.\mathrm{Name} \mid \mathrm{Student}(s)\ \wedge \forall p\ (\neg(\mathrm{Prereq}(p) \wedge p.\mathrm{Course} = \text{`Math210'}) \vee \\ \exists t\ \mathrm{Took}(t) \wedge t.\mathrm{StudID} = s.\mathrm{StudID} \wedge t.\mathrm{Course} = p.\mathrm{Pre})\}$$

- If Math210 has no prerequisites, the answer will be the names of all students

24

Safe TRC

- Formulas with quantifiers, negation, some comparisons must be restricted so at to be meaningful
- Examples of ill-formed formulas with a comparison, a negation
  - ◇ $\{n \mid n \geq 3\}$
  - ◇ $\{e \mid \neg\mathrm{Employee}(e)\}$
- Existential quantifiers
  - ◇ $\exists t\ F(t)$ must have the form $\exists t\ R(t) \wedge F'(t)$
  - ◇ other notation: $(\exists t \in R)\ F'(t)$
- Universal quantifiers must always be associated with implication
  - ◇ $\forall t\ F(t)$ must have the form $\forall t\ R(t) \rightarrow F'(t)$
  - ◇ other notation: $(\forall t \in R)\ \ F'(t)$

25

- $(\exists t \in R)$ and $(\forall t \in R)$ are called **range-restricted** or **ranged-coupled** quantifiers, where R is a relation predicate that defines and restricts the range of t
- General form of safe use of universal quantifier: $\forall t \in (R(t) \wedge F'(t))\ F''(t)$ ($F'(t)$ and $F''(t)$" are any TRC formulas)
- Intuition: $\forall t\ F(t)$, where F(t) is a conjunction of database or comparison predicates, is meaningless (e.g., $\forall t\ Employee(t)$)

Domain Relational Calculus (DRC)

- **Domain variables** range on (i.e., take as values elements of) DB domains
- Relations are preferably viewed as predicates expressing properties of objects, represented as values
- **Relation predicates** (extensional predicates)
  - ◇ realize the link between DRC and the DB
  - ◇ $R(A_1 : x_1, \ldots, A_n : x_n)$ is associated with relation $R(A_1 : D_1, \ldots, A_n : D_n)$
  - ◇ $R(A_1 : a_1, \ldots, A_n : a_n)$ is true if tuple $\langle A_1 : a_1, \ldots, A_n : a_n \rangle$ belongs to relation R

26

- Predicate WorksOn(ESSN:123456789, PNo:1, Hours:32.5) is true because tuple ⟨ ESSN:123456789, PNo:1, Hours:32.5 ⟩ belongs to relation WorksOn
- In WorksOn(ESSN:123456789, PNo:1, Hours:32.5):
  - ◇ WorksOn(ESSN: , PNo: , Hours:) is the predicate name
  - ◇ 123456789, 1 and 32.5 are the arguments

General Structure of DRC Queries

$$\boxed{\{x_1, x_2, \ldots, x_n \mid F(x_1, \ldots, x_n, x_{n+1}, \ldots, x_m)\}}$$

- where formula F has the structure of first-order logic
  - ◇ $R(A_i : x_i, \ldots, A_j : x_j)$, where R is a relation name
  - ◇ x_i comparison x_j
  - ◇ x_i comparison constant
  - ◇ $\neg F$
  - ◇ $F_1 \wedge F_2$
  - ◇ $F_1 \vee F_2$
  - ◇ $F_1 \rightarrow F_2$
  - ◇ $F_1 \leftrightarrow F_2$
  - ◇ $\exists x\ F(x)$
  - ◇ $\forall x\ F(x)$

27

- As for TRC, the only things specific to DRC are the choice of domain variables and the definition of the relational predicates
- DRC has the structure of logic, applied as a DB query/assertion language
- Restrictions for safety similar to those of TRC for quantified formulas apply to DRC

Simplification of Notation

- List the birth date and address of employees named John Smith

$$\{dn, a \mid \exists fn, m, ln, ssn, sex, sal, ss, d \; \boxed{\begin{array}{r}\text{Employee}(\text{FName}: fn, \text{MInit}: m, \text{LName}: ln, \text{Address}: a, \text{BDate}: dn,\\ \text{ESSN}: ssn, \text{Sex}: sex, \text{Sal}: sal, \text{MgrSSN}: ss, \text{DNo}: d)\end{array}} \wedge fn = \text{'John'} \wedge ln = \text{'Smith'}\}$$

- Many variables! Suppress variables that only appear in a relational predicate under $\exists$

$$\{dn, a \mid \exists fn, ln \; \text{Employee}(\text{FName}: fn, \text{LName}: ln, \text{Address}: a, \text{BDate}: dn) \wedge fn = \text{'John'} \wedge ln = \text{'Smith'}\}$$

- $2^n - 1$ predicates are associated with each relation with n attributes

28

Further Simplification

- Suppress variables that only appear in a relation predicate and in a test for equality with a constant in a conjunction ($\wedge$)

$$\{dn, a \mid \text{Employee}(\text{FName}: \text{'John'}, \text{LName}: \text{'Smith'}, \text{Address}: a, \text{BDate}: dn)\ \}$$

- Corresponds to projection + selection on equality in the algebra
- The rest of DRC has the structure of logic

29

- $P(x) \wedge x = 3 \equiv P(3)$
- TRC formulation of the same example:
$\{t.\text{BDate}, t.\text{Address} \mid \text{Employee}(t) \wedge t.\text{FName} = \text{John} \wedge t.\text{LName} = \text{Smith}\}$

Selection + Projection

List the name of employees with a salary greater than 50k

$$\{fn, ln \mid \exists sal \; (\text{Employee}(\text{FName}: fn, \text{LName}: ln, \text{Salary}: sal) \wedge sal > 50k)\}$$

Could also conceivably be written

$$\{fn, ln \mid \text{Employee}(\text{FName}: fn, \text{LName}: ln, \text{Salary}: > 50k)\}$$

30

Join

- List name and address of employees who work in the Research department

$$\{fn, ln, a \mid \exists d \ (\text{Employee}(\text{FName}: fn, \text{LName}: ln, \text{Address}: a, \text{DNo}: d) \wedge \text{Department}(\text{DName}: \text{`Research'}, \text{DNumber}: d))\}$$

- A join is expressed through the occurrence of the same domain variable in two (or more) relation predicates in a conjunction ($\wedge$)
- In TRC, a join is signaled by an explicit "join condition"

$$\{e.\text{FName}, e.\text{LName}, e.\text{Address} \mid \text{Employee}(e) \wedge \exists d \ (\text{Department}(d) \ \wedge d.\text{DName} = \text{`Research'} \wedge d.\text{DNumber} = e.\text{DNo})\}$$

Double Join

- For every project located in Brussels, list the project number, the controling department number, and the name of the department manager

$$\{pn, d, mfn, mln \mid \exists e \ (\text{Project}(\text{PNumber}: pn, \text{PLocation}: \text{`Brussels'}, \text{DNum}: d) \wedge \text{Department}(\text{MgrSSN}: e, \text{DNumber}: d) \wedge \text{Employee}(\text{SSN}: e, \text{FName}: mfn, \text{LName}: mln))\}$$

"Complex" Query

- List project number of projects for which an employee whose last name is Smith is a worker or a manager of the department that controls the project

$$\{p \mid \text{Project}(\text{PNumber}: p) \ \wedge \exists e \ \text{Employee}(\text{SSN}: e, \text{LName}: \text{`Smith'}) \wedge [\ \text{WorksOn}(\text{ESSN}: e, \text{PNo}: p) \vee \exists d \ (\text{Department}(\text{MgrSSN}: e, \text{DNumber}: d) \wedge \text{Project}(\text{PNumber}: p, \text{DNum}: d))\ ]\ \}$$

- Many variants

Join of a Relation with itself

- List first and last name of employees, and first and last name of their immediate supervisor

$$\{efn, eln, mfn, mln \mid \exists m \ (\text{Employee}(\text{FName}: efn, \text{LName}: eln, \text{SuperSSN}: m) \wedge \text{Employee}(\text{SSN}: m, \text{FName}: mfn, \text{LName}: mln))\}$$

- Like for the algebra and TRC, attribute names for the result have to be explicitly specified through some kind of assertion

$$\text{RES}(\text{EmpFN}, \text{EmpLN}, \text{SupFN}, \text{SupLN}) \leftarrow \{\text{efn}, \text{eln}, \text{mfn}, \text{mln} \mid ...\}$$

Universal Quantifier

- List the name of employees who work on all projects

$$\{fn, ln \mid \exists e \text{ Employee}(\text{FName} : fn, \text{LName} : ln, \text{SSN} : e) \wedge \forall p\ (\text{Project}(\text{PNumber} : p) \rightarrow \text{WorksOn}(\text{PNo} : p, \text{ESSN} : e))\}$$

$$\{fn, ln \mid \exists e \text{ Employee}(\text{FName} : fn, \text{LName} : ln, \text{SSN} : e) \wedge \forall p\ (\text{WorksOn}(\text{PNo} : p) \rightarrow \text{WorksOn}(\text{PNo} : p, \text{ESSN} : e))\}$$

35

Universal Quantifier

- List the name of employees who have no dependent

$$\{\text{name} \mid \exists s\ (\text{Employee}(\text{LName} : name, \text{SSN} : s) \wedge \neg \text{Dependent}(\text{ESSN} : s))\}$$

$$\{\text{name} \mid \exists s\ (\text{Employee}(\text{LName} : name, \text{SSN} : s) \wedge \nexists m\ (\text{Dependent}(\text{ESSN} : m) \wedge m = s))\}$$

$$\{\text{name} \mid \exists s\ (\text{Employee}(\text{LName} : name, \text{SSN} : s) \wedge \forall m\ (\text{Dependent}(\text{ESSN} : m) \rightarrow m \neq s))\}$$

36