A summary of recent contributions in the PROMETHEE methods (from members of the SMG unit)

Prof. Y. De Smet
CoDE-SMG, Université libre de Bruxelles (Belgium)

International MCDA workshop on PROMETHEE: Research and case studies
Outline

- Brief reminder about PROMETHEE and GAIA
- Main topics developed since 2007 (SMG);
  - Software implementations
  - Preference elicitation
  - Sorting and clustering
  - Visual representations (application to GIS)
  - Rank reversal
- Directions for future research
PROMETHEE in a nutshell (1)

- A finite set of alternatives:
  \[ A = \{a_1, a_2, \ldots, a_n\} \]
- A set of criteria:
  \[ F = \{f_1, f_2, \ldots, f_q\} \]
- **W.l.g.** these criteria have to be maximized
- **Step 1**: computation of unicriterion preferences degrees

\[
\forall a_i, a_j \in A: d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)
\]

\[
\pi_k(a_i, a_j) = P_k[d_k(a_i, a_j)]
\]
PROMETHEE in a nutshell (2)

- **Step 2**: Computation of preference degrees:

\[ \forall a_i, a_j \in A : \pi(a_i, a_j) = \sum_{k=1}^{q} w_k \pi_k(a_i, a_j) \]

- **Step 3**: Computation of flow scores:

\[ \phi^+(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(a_i, b) \]

\[ \phi^-(a_i) = \frac{1}{n-1} \sum_{b \in A} \pi(b, a_i) \]

\[ \phi(a_i) = \phi^+(a_i) - \phi^-(a_i) \]

More formally:

\[ \phi_A^+(a_i), \phi_A^-(a_i), \phi_A(a_i) \]
GAIA in a nutshell (1)

- We have:

\[
\Phi(a_i) = \frac{1}{n-1} \sum_{b \in A} \sum_{k=1}^{q} w_k \cdot \pi_k(a_i, b) - \frac{1}{n-1} \sum_{b \in A} \sum_{k=1}^{q} w_k \cdot \pi_k(b, a_i)
\]

\[
= \sum_{k=1}^{q} w_k \cdot \frac{1}{n-1} \sum_{b \in A} \pi_k(a_i, b) - \pi_k(b, a_i) = \sum_{k=1}^{q} w_k \cdot \Phi_k(a_i)
\]

- Where

\[
\Phi_k(a_i) = \sum_{b \in A} \pi_k(a_i, b) - \pi_k(b, a_i)
\]

- In other words, every alternative can be represented by a vector:

\[
\alpha_i = [\Phi_1(a_i), \Phi_2(a_i), \ldots, \Phi_k(a_i)]
\]
GAIA in a nutshell (2)

\[ \Phi_1, \Phi_2, \Phi_3, \ldots, \Phi_q(a_i) \]

q dimensions

2 dimensions

Principal component analysis
Software implementations
Software implementations


- Long story: PROMCALC, DECISION LAB 2000
- Spin-off project funded by the Walloon Region (2007-2010)
- D-SIGHT desktop
- D-SIGHT web Session C - 2nd presentation
- Innoviris award in 2012 « Jeune entreprise innovante »
- But also Decision Deck, Smart Picker, Visual Promethee, ...
Areas of Expertise

D-Sight has been developing specific expertise in certain industries over the past years. We are proud of having excellent customers, which help us developing and improving our software solutions.

- Environmental Analysis
- Projects Prioritization
- Vendor Selection

Academia: 50 universities in 27 countries
Preference elicitation
Contributions:


General idea (1)

$w^*, q^*, p^*$

$P2$

$R^*$

$PAC, WSR, ASR$

$w, q, p$

$P2$

$R$

Set of compatible parameters!

Set of compatible rankings!

Worst Kendall’s Tau
General idea (2)

Set of compatible parameters!
Set of compatible rankings!

$w^*, q^*, p^*$

$P2$

$w, q, p$

$\Omega$

$\Omega_C$

$\omega^*, \omega^*, \omega^*$

- **Main idea**: quality and robustness
- **Distinctive feature**: the DM may communicate mistakes

Fig. 4. This plot represents the approximated Pareto frontiers in the objective space, for 20 constraints and several values of the constraint violation rate $p_{cv}$, i.e., the proportion of inconsistent constraints with respect to the total number of constraints. As expected, increasing the value of $p_{cv}$ has the effect of deteriorating the quality of the solution set both in terms of constraint violation rate and PROMETHEE II sampled sensitivity.

$(n=100,q=2)$

*NSGA-II*

• **Main idea**: quantify information infrastructure ...

![Graphs showing the ratio of compatible weight domain area](image)

**Fig. 3.** (a) Evolution of the ratio of compatible weight domain area \(\frac{\Omega_c}{\Omega}\) with respect to the whole domain of possible weights \(\Omega\), depending on the number of action constraints \(K^a\), for different numbers of weight constraints \(K^w\). Results are shown for 1000 randomly generated action sets \((n = 10, m = 3)\) and PAC constraint sets. — (b) Distribution of all values of Kendall’s \(\tau\) in the compatible weights domain \(\Omega_c\), for respectively \(K^a_{PAC} = 0, 10,\) and 20 pairwise action comparisons. No constraints on the weights relative importance are given here \((K^w = 0)\).

![Graphs showing evolution of Kendall's τ](Image)

**Fig. 4.** (a) Evolution of worst Kendall’s τ for two different information types: pairwise action comparisons (PAC) and action sub-ranking (ASR). — (b) Impact of “weight constraints” on the reachable quality for pairwise action comparisons (PAC). Note that the x-axis represents the sum of action and weight constraints, i.e. $K^a + K^w$. 

Table 1. Number of pairwise action comparisons that have to be given by a DM to reach the desired level of quality $w$, assuming that $K^w$ weight constraints have already been provided. The results are shown for randomly generated 3-criteria action sets with a uniform distribution.

<table>
<thead>
<tr>
<th>$K^w$</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>0.95</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>34</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>14</td>
<td>24</td>
<td>33</td>
<td>43</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>22</td>
<td>32</td>
<td>42</td>
</tr>
</tbody>
</table>
Main idea: to overcome the limitations of the previous approach; pairwise comparisons have to be « well-chosen »

q-Eval

Table 2 Maximum number of queries that could be generated on average with q-Eval for 30 requested queries for 50 randomly generated instances.

<table>
<thead>
<tr>
<th>n</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8.2</td>
<td>12.7</td>
<td>15.4</td>
<td>17.4</td>
</tr>
<tr>
<td>20</td>
<td>12.3</td>
<td>20.5</td>
<td>26.3</td>
<td>29.5</td>
</tr>
<tr>
<td>50</td>
<td>15.88</td>
<td>28.46</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>100</td>
<td>17.26</td>
<td>29.46</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>200</td>
<td>17.96</td>
<td>29.84</td>
<td>30.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>
Figure 2  Evolution of the average number of constraints that can be generated with the \textit{q-Eval} method depending on the instance size (number \( m \) of alternatives) for a given number of alternatives. Plots for a constant value of \( m \) and different number of alternatives \( n \) are very similar.
Eppe, S., De Smet, Y. « An adaptative questioning procedure for eliciting PROMETHEE II’s weight parameters » to appear in the International of Multicriteria decision making.

Figure 3  For $n = 20$ actions, shows the average evolution of the number of different rankings found in the sampling of the compatible parameter domain $\Omega$, for different numbers $m$ of criteria.
Eppe, S., De Smet, Y. «An adaptative questioning procedure for eliciting PROMETHEE II’s weight parameters» to appear in the International of Multicriteria decision making.

### Table 3
This table represents the average value of the worst Kendall’s correlation coefficient \( \tau_{\text{C}} \) for the sampling of the compatible parameter domain \( \Omega \) for each trial after the last query. Note that the sample size of \( n_S = 500 \) has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

<table>
<thead>
<tr>
<th>( n )</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.999</td>
<td>0.988</td>
<td>0.988</td>
<td>0.988</td>
</tr>
<tr>
<td>20</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.994</td>
</tr>
<tr>
<td>50</td>
<td>1.000</td>
<td>1.000</td>
<td>0.998</td>
<td>0.987</td>
</tr>
<tr>
<td>100</td>
<td>1.000</td>
<td>1.000</td>
<td>0.997</td>
<td>0.983</td>
</tr>
<tr>
<td>200</td>
<td>1.000</td>
<td>1.000</td>
<td>0.998</td>
<td>0.982</td>
</tr>
</tbody>
</table>

### Table 4
This table represents the average number of different rankings in the samples for each trial after the last query. Note that the sample size of \( n_S = 500 \) has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

<table>
<thead>
<tr>
<th>( n )</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.1</td>
<td>1.1</td>
<td>2.4</td>
<td>3.9</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>1.5</td>
<td>2.2</td>
<td>14.1</td>
</tr>
<tr>
<td>50</td>
<td>4.8</td>
<td>18.6</td>
<td>204.4</td>
<td>358.8</td>
</tr>
<tr>
<td>100</td>
<td>44.6</td>
<td>228.7</td>
<td>430.8</td>
<td>465.9</td>
</tr>
<tr>
<td>200</td>
<td>187.9</td>
<td>432.1</td>
<td>477.8</td>
<td>490.5</td>
</tr>
</tbody>
</table>
Eppe, S., De Smet, Y. « An adaptative questioning procedure for eliciting PROMETHEE II’s weight parameters » to appear in the International of Multicriteria decision making

Table 12  Number of queries that has to be answered by a DM to reach the desired level of quality $\tau_G$ for randomly generated action sets ($n = 20$ actions and $m = 5$ criteria)

<table>
<thead>
<tr>
<th></th>
<th>$\tau_G$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
<td>0.80</td>
<td>0.90</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>Random</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PAC</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>25</td>
<td>35</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Q-Eval</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PAC</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>POS-3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>ASR-3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Adaptive</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: The extended Q-Eval method significantly outperforms the bottom-line approach (Eppe and De Smet, 2012), presented in the first row, and the original Q-Eval, shown in the second row.
Eppe, S., De Smet, Y. « **An adaptative questioning procedure for eliciting PROMETHEE II’s weight parameters** » to appear in the International of Multicriteria decision making.

**Figure 9** Evolution of the mean ratio of each Q-Eval query-type when applying the *adaptive query selection scheme*.

Note: Each type of query plays a role and has its usefulness in the process at different stages of the eliciting process.
Sorting and clustering
Contributions

De Smet, Y. et Gilbart, F. « A class definition method for country risk problem », IS-MG 2001/13


Flowsort - main idea

- Let us consider a set of limit (or central profiles)
  \[ R = \{ r_1, r_2, \ldots, r_{K+1} \} \]
  \[ R_i = R \cup \{ a_i \} \]

- Each actions \( a_i \) is sorted according to its relative position based with respect to the profiles according to
  \[ \phi_{R_i}(a_i) \]

- Complete or partial sorting
- Idea behind P2CLUST Session A1 - 1\textsuperscript{st} presentation
PROMETHEE and GIS (Karim Lidouh PhD)

Session B2 - 4th presentation
Rank reversal
Session A1 - 5th presentation
Rank reversal (1)

- Rank reversal ...
  - **ELECTRE:** Wang and Triantaphyllou (2005)
  - **PROMETHEE:** De Keyser and Peeters (1996)

- The concept of rank reversal is not fully formalized (*add a copy of an alternative, deletion of a non discriminating criterion, deletion of an alternative, ...*)

- A direct consequence of Arrow’s theorem

- Positive results:
  - Dominance
  - Non discriminating criterion
Contributions


Roland, J., De Smet, Y. and Verly, C. “Rank reversal as a source of uncertainty and manipulation in the PROMETHEE II ranking: a first investigation” to appear in the proceedings of the IPMU 2012 conference (LNAI)

Verly, C. and De Smet, Y. “Some considerations about rank reversal occurrences in the PROMETHEE methods” accepted for publication in the International Journal of Multicriteria Decision Making.
More general result (1)

Notations: \[ A_x = A \setminus \{x\} \], \[ \Phi_x(a) \]

No RR \[ \Leftrightarrow (\Phi(a) - \Phi(b))(\Phi_x(a) - \Phi_x(b)) > 0 \]

if \[ \Phi(a) - \Phi(b) > \frac{\left[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})\right]}{n - 1} \]

No RR (for any action removed) if

\[ \Phi(a) - \Phi(b) > \frac{\max_x \left[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})\right]}{n - 1} \]
More general result (2)

$\Rightarrow RR$ can only occur if

$$\Phi(a) - \Phi(b) < \frac{\max_x \left[ (\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb}) \right]}{n - 1} \leq \frac{2}{n - 1}$$

Refined threshold (depends on the sample and $(a,b)$)

Rough threshold (constant)

Generalization: when $k$ actions are removed

No RR if

$$\Phi(a) - \Phi(b) > \frac{2k}{n - 1}$$

More general result (3)

Statistical results relative to the «rough threshold» (for $q = 2$, $DA=\text{Unif}$)

| $n$ | nb RR  | $b = \frac{2}{n-1}$ | nb $\Delta \Phi \leq b$ | nb RR $| \Delta \Phi \leq b$ |
|-----|--------|---------------------|-------------------------|---------------------------|
| 5   | 2,20%  | 0,50                | 47,4%                   | 4,6%                      |
| 10  | 0,98%  | 0,22                | 33,5%                   | 2,9%                      |
| 15  | 0,66%  | 0,14                | 24,7%                   | 2,6%                      |
| 20  | 0,45%  | 0,10                | 19,9%                   | 2,2%                      |
| 50  | 0,18%  | 0,04                | 9%                      | 1,9%                      |

**Conclusion:** The number of RR occurrences is really small.
More general result (4)
Related works for PROMETHEE I

- No rank reversal will happen between $a_i$ and $a_j$ if

$$|\phi^+(a_i) - \phi^+(a_j)| \geq \frac{1}{n-1}$$

$$|\phi^-(a_i) - \phi^-(a_j)| \geq \frac{1}{n-1}$$
Rank reversal = risk of manipulation

- Joint work with Julien Roland and Céline Verly (to appear in the proceedings of the IPMU 2012 conference)
- Aim: to quantify the likelihood of manipulation in a simplified version of the PROMETHEE II ranking:
  - Usual preference function and equal weights
  - Copeland scores
- More formally:
  - A given decision maker has a perfect information on the evaluation table;
  - He may propose new alternatives in order to make alternative $a_i$ the first one;
  - Question: how many alternatives are necessary?
Linear mathematical program

\[
\begin{align*}
\max & \quad \sum_{a \in A \cup C} y(a_s, a) \\
\text{subject to: } & (P_j(a_i, a_j) - 1)\overline{g_k} < g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C, \forall k \in K \\
& P_k(a_i, a_j)\overline{g_k} \geq g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C, \forall k \in K \\
& \pi(a_i, a_j) = \frac{1}{q} \sum_{k \in K} P_k(a_i, a_j), \forall a_i, a_j \in A \cup C \\
& \phi(a) = \frac{1}{n + m - 1} \sum_{x \in A \cup C} \pi(a, x) - \pi(x, a), \forall a \in A \cup C \\
& g_k(a) \leq \overline{g_k}, \forall a \in C \\
& g_k(a) \geq 0, \forall a \in C \\
& 2(y(a_s, a) - 1) \leq \phi(a_s) - \phi(a), \forall a \in A \cup C \\
& 2y(a_s, a) > \phi(a_s) - \phi(a), \forall a \in A \cup C
\end{align*}
\]
Results for 10 alternatives and 3 criteria

Table 1. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives.

<table>
<thead>
<tr>
<th>j \ m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>13</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>57</td>
<td>33</td>
<td>17</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>83</td>
<td>63</td>
<td>40</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>83</td>
<td>60</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>90</td>
<td>77</td>
<td>43</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>100</td>
<td>87</td>
<td>70</td>
<td>37</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100</td>
<td>97</td>
<td>83</td>
<td>63</td>
<td>33</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>100</td>
<td>97</td>
<td>93</td>
<td>83</td>
<td>63</td>
<td>33</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Comparison with the bound

Table 3. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding \( m \) well-chosen artificial alternatives while the Mareschal’s bound is not reached.

<table>
<thead>
<tr>
<th>( j )</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>63</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>83</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>73</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>
Current applications

- 3D Integrated circuits (PhD A.V. Doan) Session B1 - 4th presentation

- Sustainable security in road design (PhD R. Sarrazin) Session A2 - 2nd presentation
Future/current researches

- Rank reversal: exact conditions;
- Management of missing values;
- Synergies with Data Envelopement Analysis; (PhD Bagherikavari) Session A2 - 2nd presentation
- Extension of PROMETHEE to temporal evaluations; (PhD I. Benamar).
Thank you for your attention ;-)