An introduction to Multicriteria Decision Aid: The PROMETHEE & GAIA methods

Prof. Y. De Smet
CoDE-SMG, Université Libre de Bruxelles

EBISS 2012

ULB Short Bio

- Degree in Mathematics (ULB, 1998), consultant in Risk Management (1998→2000), DEA in Applied Sciences (ULB, 2001), PhD in Applied Sciences (ULB, 2005);
- Assistant Professor at the Polytechnic School since 2007;
- Head of the SMG unit (2007), Head of the Computer and Decision Engineering department (2010);
- Research interets:
  - Foundations, properties and applications of the PROMETHEE & GAIA methods
  - Multi-objective optimization (exact methods and heuristics)
  - Trans-disciplinary aspects of MCDA (MCDA & GIS, & Clustering, & Evidence theory, & DEA)
- Currently supervising 6 PhD students;
- Co-founder of the D-SIGHT spin-off: www.d-sight.com

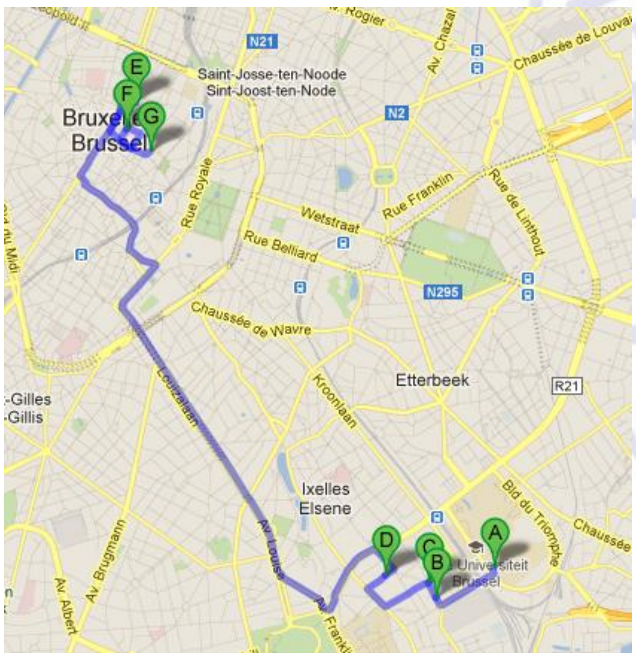
Goals of this talk

- General introduction of MCDA:
  - to master the necessary vocabulary to master further investigations
- Detailed presentation (methodology and software) of a given method: PROMETHEE & GAIA
  - Being able to treat a MCDA problem by your own
- Some current research questions

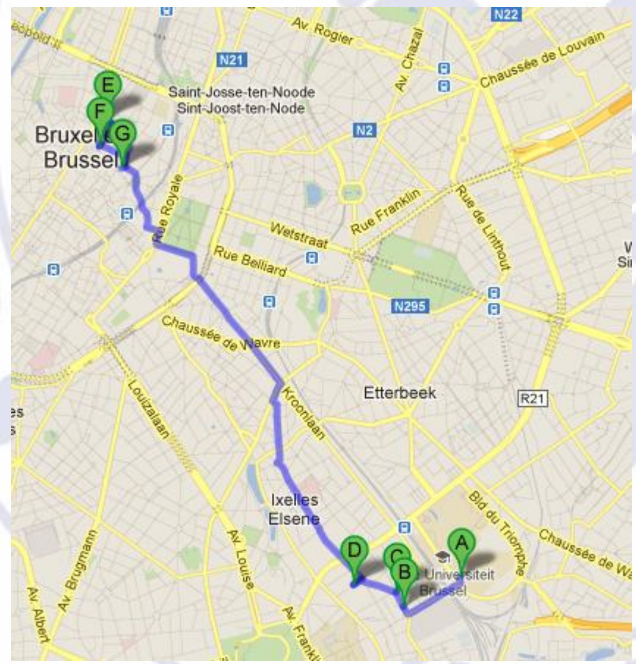
Introduction

Introductory example

- Visiting a few bars in Brussels:
  - Le Waff, Le Gaugain, L’Atelier, Le Corbeau, La Lunette, La Mort Subite.



21 min



1h22

Quantitave decision making

- Aim of Operational Research (OR)
  1. A set of decision variables
  2. A set of constraint
  3. An objective function

$$\max \ f(x)$$

$$A.x \leq b$$

$$x \geq 0$$

Shortest path, travelling salesman, minimum spanning tree, knapsack, flowshop, queueing systems, scheduling, …

ULB

Quantitave decision making

- Main trend in OR: **mono-objective optimization**
- Well-defined from a mathematical point of view !
- Total order ! Optimal solution !
- Realist ? Decisions are only made **on the basis of financial considerations** (minimize cost, maximize profit) !
- All the criteria can not be monetarized (what is the price of a human life ?)

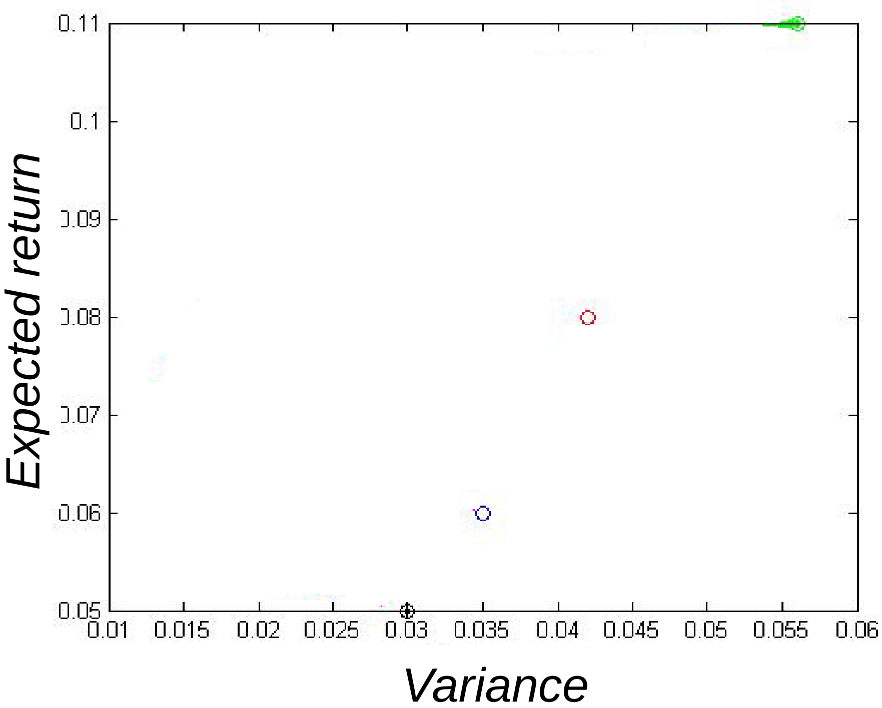
Further examples

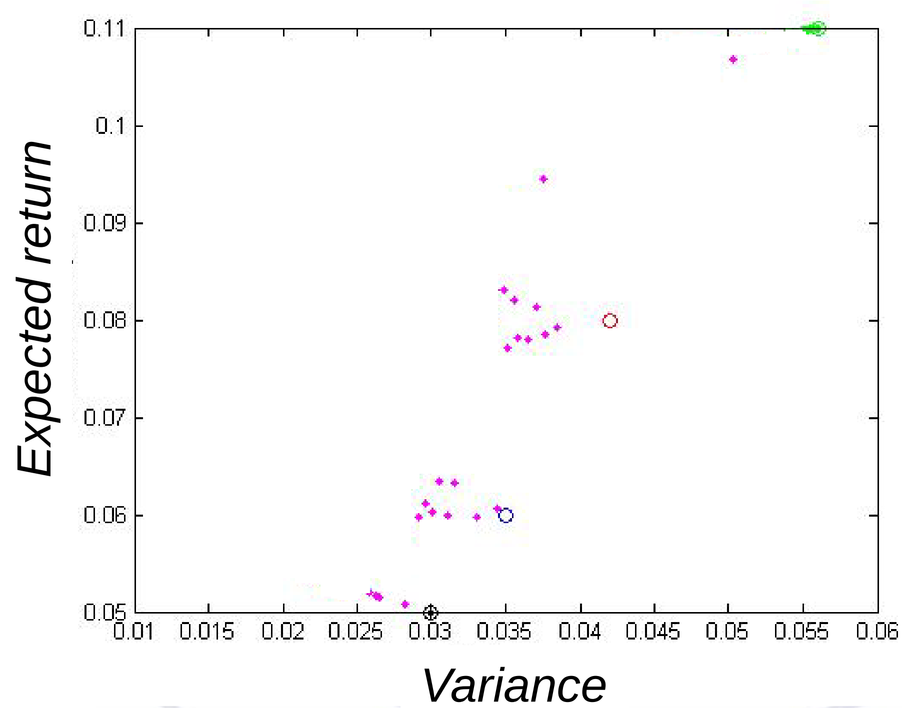
Portfolio management (Markowitz, 1952)

- A capital K to invest in different equities
- **Idea**: « *do not put all your eggs in the same bag* »
- Two (conflicting objectives):
  - Maximize profit (expected return)
  - Minimize risk (variance)

Portfolio management (Markowitz, 1952)

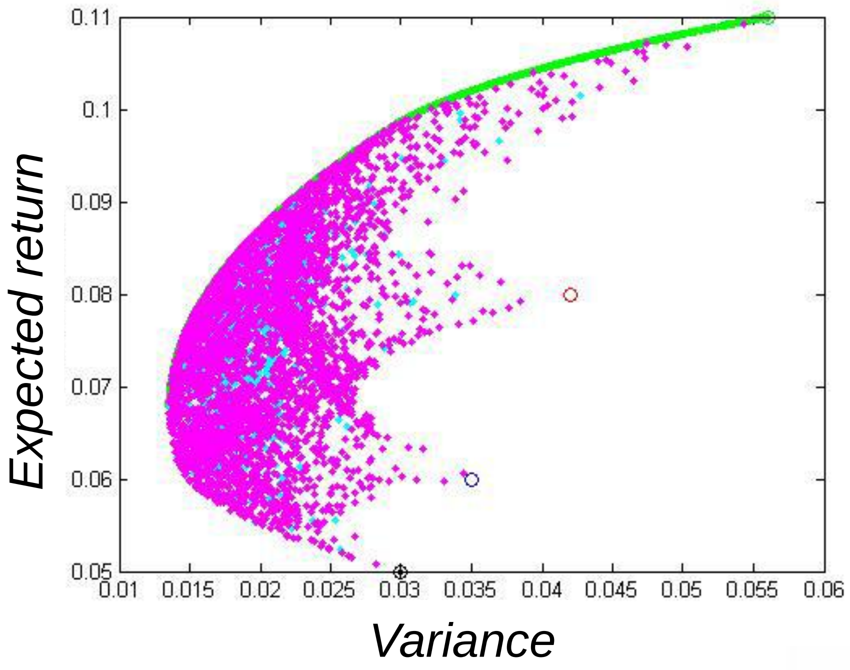
- Example with 4 investments

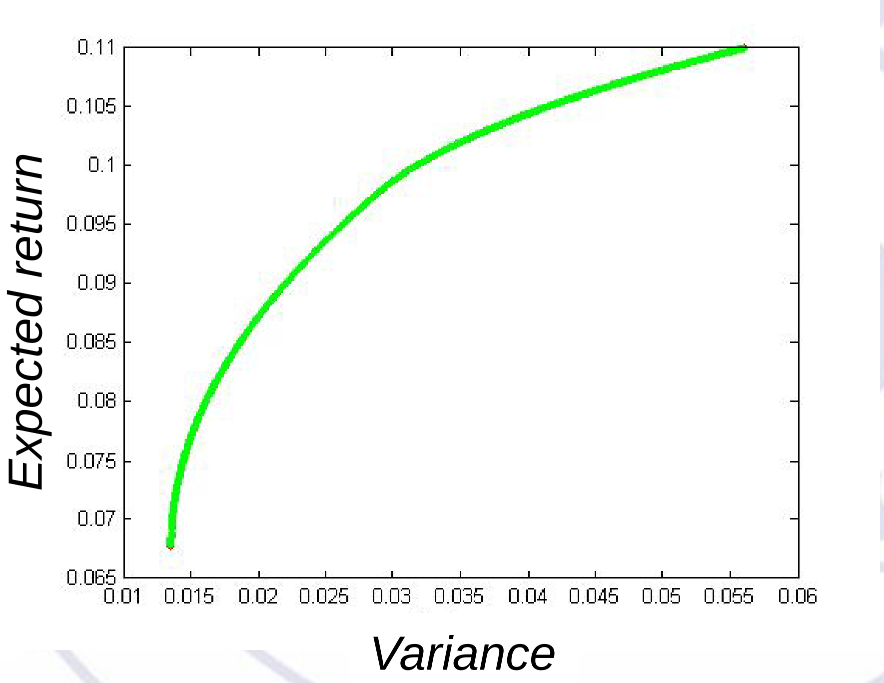




Portfolio management (Markowitz, 1952)

- Example with 4 investments

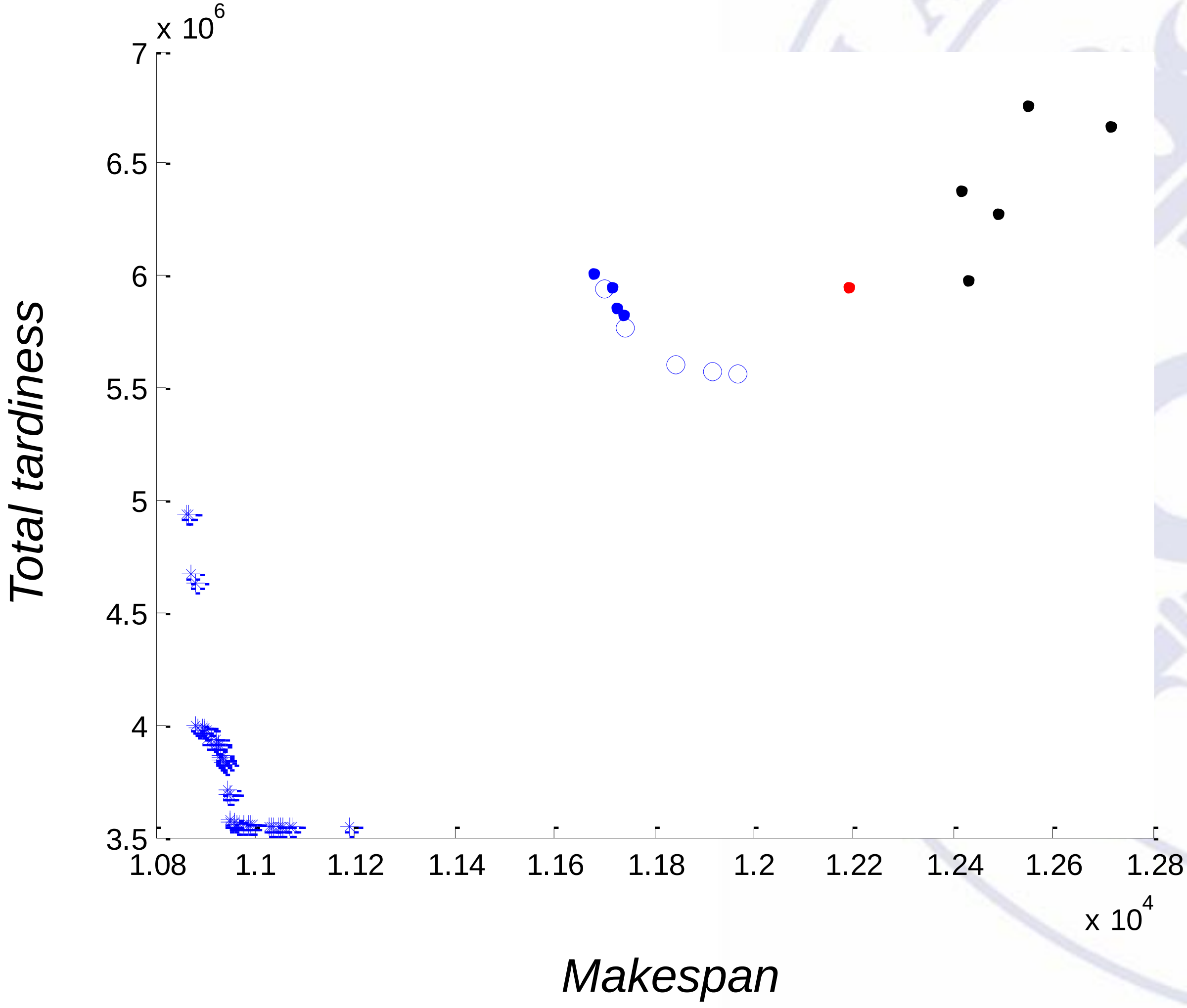




Bi-objective flow shop problems

- n jobs, m machines
- Each job has to pass on every machine (with possibly different delays)
- Each machine process jobs one by one
- Due dates for clients
- Objectives
  - To minimize the makespan;
  - To minimize the total tardiness

Bi-objective flow shop problems



200 jobs,
10 machines

To summarize these 2 examples:

- Bi-objective problems
- Possible alternatives are determined by constraints (impossible to enumerate)

$$\max\{ f_1(x), f_2(x)\}$$

$$G(x) \leq 0$$

$$x \geq 0$$

Electricity grid operator

- Replacement strategies for high voltage equipments
- Items to evaluate: several thousands / different types
- How to assess the severity ?
  - Age (what does-it means ?)
  - Technical features
  - Localisation
  - Experts' evaluation

Electricity grid operator

	Age (unit)	Tech. (unit)	Local. (unit)	Expert (unit)	…
Equip 1					
Equip 2					
Equip 3					
Equip 4					
Equip 5					
…					

Aim: to rank the equipments for the worst to the best ones

Police

- Assess criminilaty in order to allocate (financial, human, material ressources)
- Different « types of criminality): about 20 categories: road accidents, hooliganism, prostitution, drug dealing, vandalism, …
- Criteria: number of victims, financial impact, increasing over the last years, organized crime, social impacts, …

Police

	Nb vict.	Financial impacts	Social impacts	Increasing (5 years)	…
Road accidents					
Hooliganism					
Vandalism					
Drug dealing					
Prostitution					
…					

A first approach:
the WEIGHTED SUM ?

ULB

A first approach: the weighted sum

	f_1	f_2		f_q
a_1				
a_2				
a_n				
	w_1	w_2		w_q

$$V(a) = \sum_{i=1}^{q} w_i . f_i(a)$$

a_i is better than a_j *if $V(a_i) > V(a_j)$*

The weighted sum is simple BUT it induces some effects on decision

The weighted sum: total compensation

- V(a)=4,25
- V(b)=4

	f_1	f_2	f_3	f_4
a	5	5	5	2
b	4	4	4	4
	1/4	1/4	1/4	1/4

The weighted sum: conflicts elimination

- V(a)=V(b)=V(c)=V(d)=5

	f_1	f_2
a	5	5
b	10	0
c	0	10
d	5	5
	1/2	1/2

The weighted sum: meaning of the weights

- « *Production is twice more important than quality* »

	Production (per month)	Quality	Score
a	100	100	100
b	120	80	106.66
Weights	2/3	1/3	

	Production (per week)	Quality	Score
a	25	100	50
b	30	80	46.66
Weights	2/3	1/3	

ULB

Solution ? Normalization ?

	f_1	f_2	f_3
a	2000	500	5
b	1360	440	10
c	1600	375	10
Weights	0.4	0.4	0.2

	f_1	f_2	f_3
a	1700	500	5
b	1360	440	10
c	1600	375	10
Weights	0.4	0.4	0.2

	f_1	f_2	f_3	Score
a	100	100	50	90
b	68	88	100	82.4
c	80	75	100	82

	f_1	f_2	f_3	Score
a	100	100	50	90
b	80	88	100	87.2
c	94	75	100	87.6

MCDA: Altenatives and criteria

(70% of the whole decision process)

Alternatives ?

- **A**
- **Synonisms**: Actions, alternatives, options, items, decisions, …
- Nature of A:
  - *Finite* (Police), *Countable* (Flowshop), *Infinite* (Porfolio)
  - *Stable* (Police), Evolutive (Elia)
  - Fragmented (Flowshop), Globalized (Elia)

Criteria ?

- **Definition**: a criterion is a mapping of A into a totally ordered set

$$g_i: A \longrightarrow E_i$$

- W.l.g. criteria have to be maximized
- Scales ?
  - Nominal
  - Ordinal <,=,>
  - Interval <,=,>,+,-
  - Ratio <,=,>,+,-,/,*

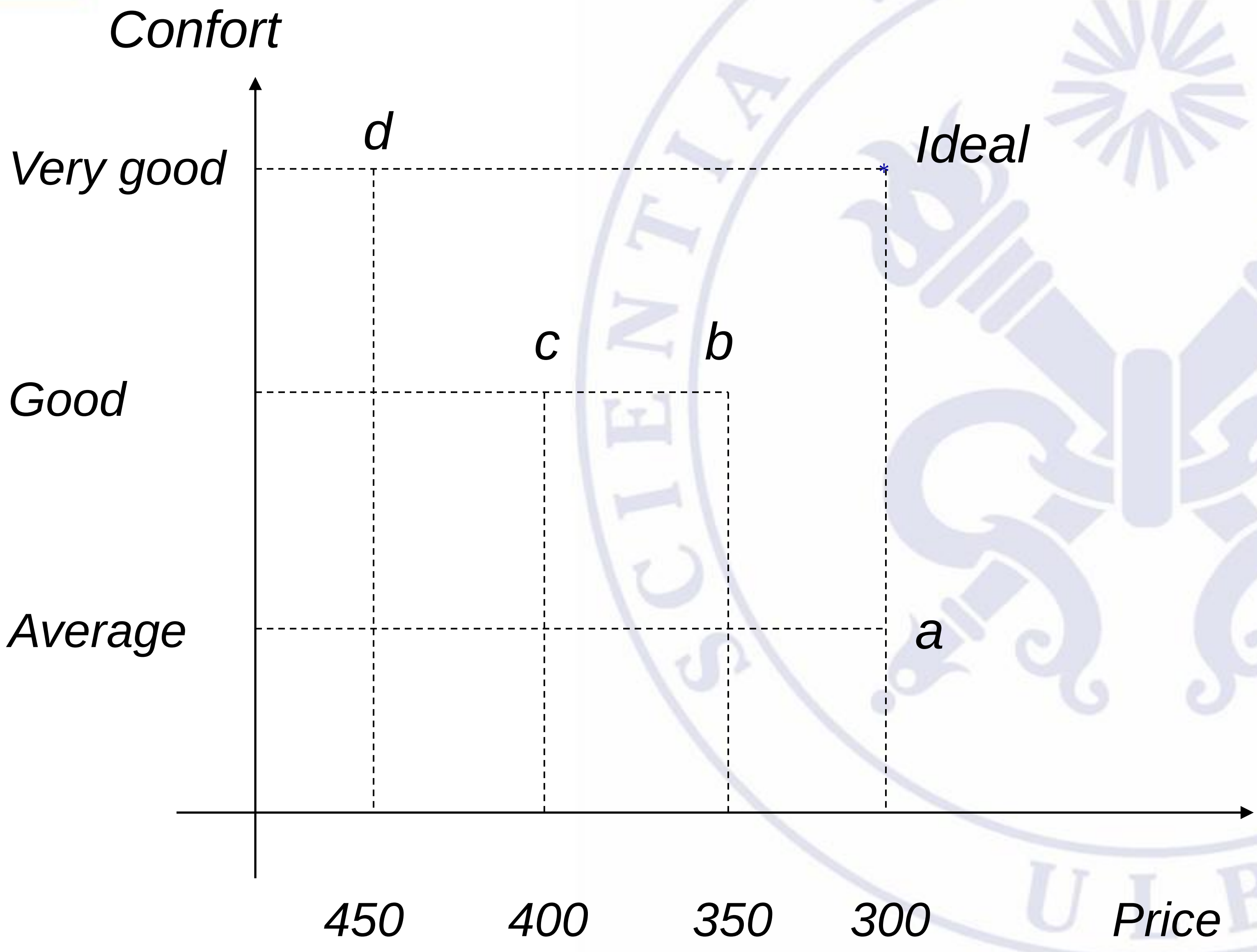
Now that we have an evaluation table …

… we can « objectively » remove not interesting alternatives

Dominance

- Unanimity principle
- **Definition**: a is said to dominate b iff $f_i(a) \geq f_i(b)$ and $\exists j \mid f_j(a) > f_j(b)$
- A = {efficient solutions} ∪ {dominated solutions}
- PO(A) = Pareto optimal set = {efficient solutions}
- Main problem: #PO(A) ≈ #A
- The identification of PO(A) is often a problem itself … (cf Flowshop, Portfolio)

Example (house)



What's next ?

- Once the PO set has been identified, how can we select the best choice;
- There is no unique objective choice !
  - There are plenty of different cars in the streets !

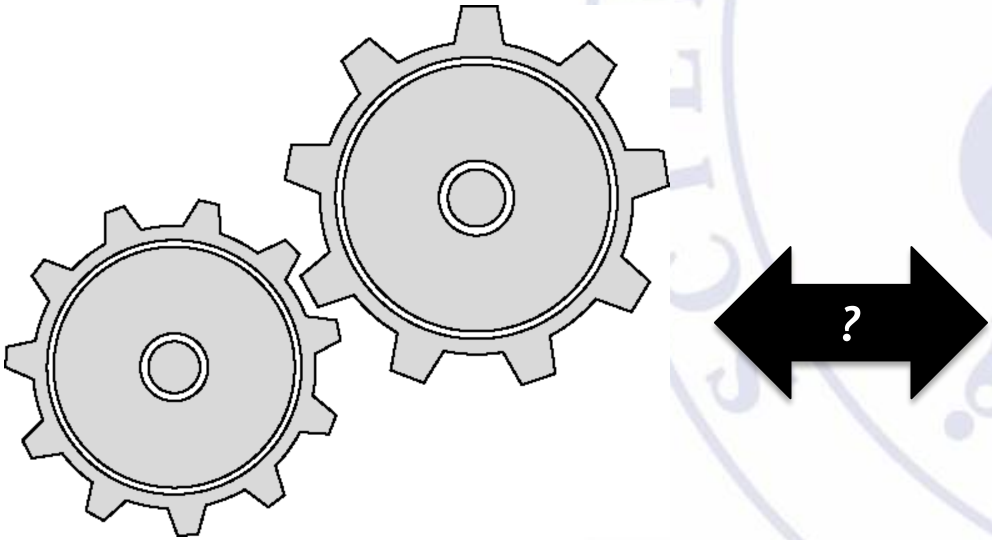
	Price	Power	Cons.	Habitability	Comfort
Moyenne A	18000	75	8,0	3	3
Sport	18500	**110**	9,0	1	2
Moyenne B	17500	85	**7,0**	4	3
Luxury 1	24000	90	8,5	4	**5**
Economic	**12500**	50	7,5	2	1
Luxury 2	22500	85	9,0	**5**	4

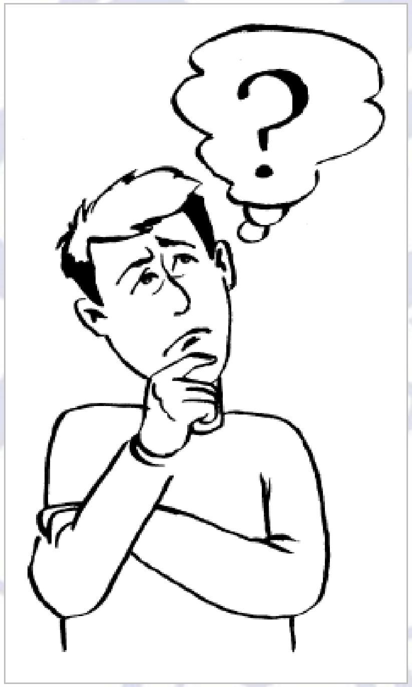
- Central role of the **decision maker**

Central problem:

To parametrize a model that best fit the preferences of a decision maker !

MCDA model *Decision maker (P,I,J)*



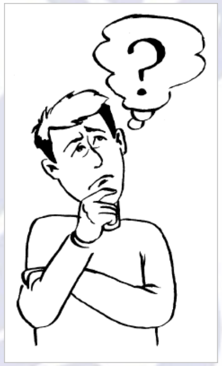


MCDA models

- Plenty of models:
  - Multi-Attribute Utility Theory (Keeney & Raiffa – USA)
  - Analytical Hierachy Process (Saaty et al. – USA-
  - ELECTRE (Roy et al. – France)
  - MACBETH (Van Snick & Bana e Costa – Belgium)
  - PROMETHEE & GAIA (Brans & Mareschal)
  - But also; SMAA, TOPSIS, Interactive Methods, …
- The holy grail of MCDA: to compare these models

Preference structure

- Main assumption: given two alternatives a and b, the decision maker is able to express one of the following statements:
  - a P b (or b P a): preference
  - a I b: indifference
  - a J b: incomparability



- P is **asymmetric**
- I is **reflexive** and **symmetric**
- J is **irreflexive** and **symmetric**
- What about **transitivity** ? Is it a natural property ?

Famous example of Luce (1956)

C_0 C_1 C_2 C_3 ... C_{401}

How to select the criteria ?

- Consistent family of criteria
  - Exhaustivity

    If $f_i(a)=f_i(b)\ \forall\ i=1,..,q \Rightarrow a\ I\ b$
  - Cohesion
  - Non redundancy

Preferential independance (1)

J ⊂ F is preferentially independent within G if ∀ a, b, c, d ∈ A such that

$$f_j(a) = f_j(b), \forall j \in J$$
$$f_j(c) = f_j(d), \forall j \in J$$
$$f_j(a) = f_j(c), \forall j \in \bar{J}$$
$$f_j(b) = f_j(d), \forall j \in \bar{J}$$

We have $a\,P\,b \Leftrightarrow c\,P\,d$

Preferential independance (2)

	Dish	Sauce
a	French fries	Bolognese
b	Spaghetti	Bolognese
c	French fries	Mayonnaise
d	Spaghetti	Mayonnaise

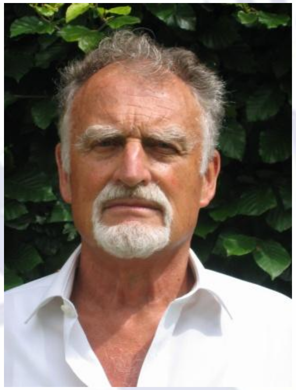
PROMETHEE & GAIA methods

Outline

- **Introduction**
- A **pedagogical** example
- PROMETHEE I & II **rankings**
- **GAIA**
- **Software** demonstration: **D-SIGHT**
- A few words about **rank reversal**
- **Preference** elicitation
- Conclusion

Historical background

Prof. Jean-Pierre Brans
(VUB, Solvay School)



Prof. Philippe Vincke
(ULB, Engineering Faculty)

Prof. Bertrand Mareschal
(ULB, Solvay Brussels School of Economics and Management)

Applications

Behzadian, M.; Kazemzadeh, R.B.; Albadvi, A.; Aghdasi, M. (2010) « ***PROMETHEE: A comprehensive literature review on methodologies and applications*** », *EJOR*, *Vol.200(1),198-215*

- > 200 papers published in > 100 journals
- <u>**Topics**</u>: *Environmental management, hydrology and water management, finance, chemistry, logistics and transportation, energy management, health care, manufactoring and assembly, sports,…*

Let us start with a educational example !

An educational example

- A plant location problem
  - 6 possible locations
  - 6 criteria

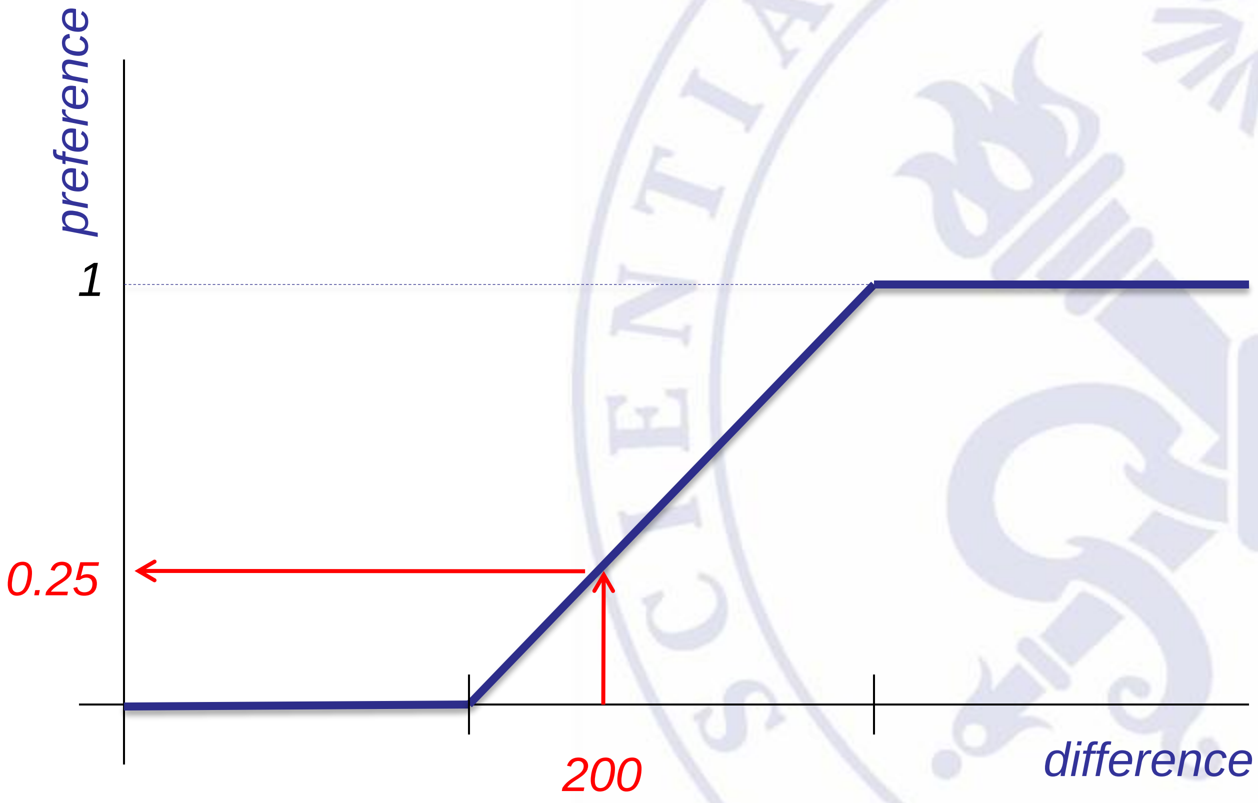
	Engineers	Power	Cost	Maintenance	Village	Security
Italy	75	90	600	5,4	8	5
Belgium	65	58	200	9,7	1	1
Germany	83	60	400	7,2	4	7
Sweden	40	80	1.000	7,5	7	10
Austria	52	72	600	2	3	8
France	94	96	700	3,6	5	6

Main principle: pair-wise comparisons

	Engineers	Power	Cost	Maintenance	Village	Security
Italy	75	90	600	5,4	**8**	5
Belgium	65	**58**	**200**	**9,7**	**1**	**1**
Germany	83	60	**400**	7,2	4	7
Sweden	**40**	80	**1.000**	7,5	7	**10**
Austria	52	72	**600**	**2**	3	8
France	**94**	**96**	700	3,6	5	6

- Concerning the cost, **Germany is better than Austria** !
- How can we **quantify** this advantage ? 200 ?
- What does it **mean** ?

Unicriterion preference function



Step 1: compute unicriterion preference degree for every pair of alternatives

			0.25			
			-200			
Germany	83	60	400	7,2	4	7
	Engineers	Power	Cost	Maintenance	Village	Security
Austria	52	72	600	2	3	8
	-31	**12**		**-5.2**	**-1**	**1**
	1	**0.75**		**1**	**0.3**	**0.63**

Step 2: compute global preference degree for every pair of alternatives

			0.25			
Germany	83	60	400	7,2	4	7
	Engineers	Power	Cost	Maintenance	Village	Security
? **Weights**	**0.2**	**0.2**	**0.2**	**0.1**	**0.15**	**0.15**
Austria	52	72	600	**2**	3	8
	0.5	**0.75**		**1**	**0.3**	**0.63**

$\Pi(\text{Austria, Germany}) = 1 * 0.1 + 0.75 * 0.2 + 1 * 0.1 + 0.3 * 0.15 + 0.63 * 0.15$
$= \mathbf{0.489}$

$\Pi(\text{Germany,Austria}) = 0.25 * 0.4 = \mathbf{0.05}$

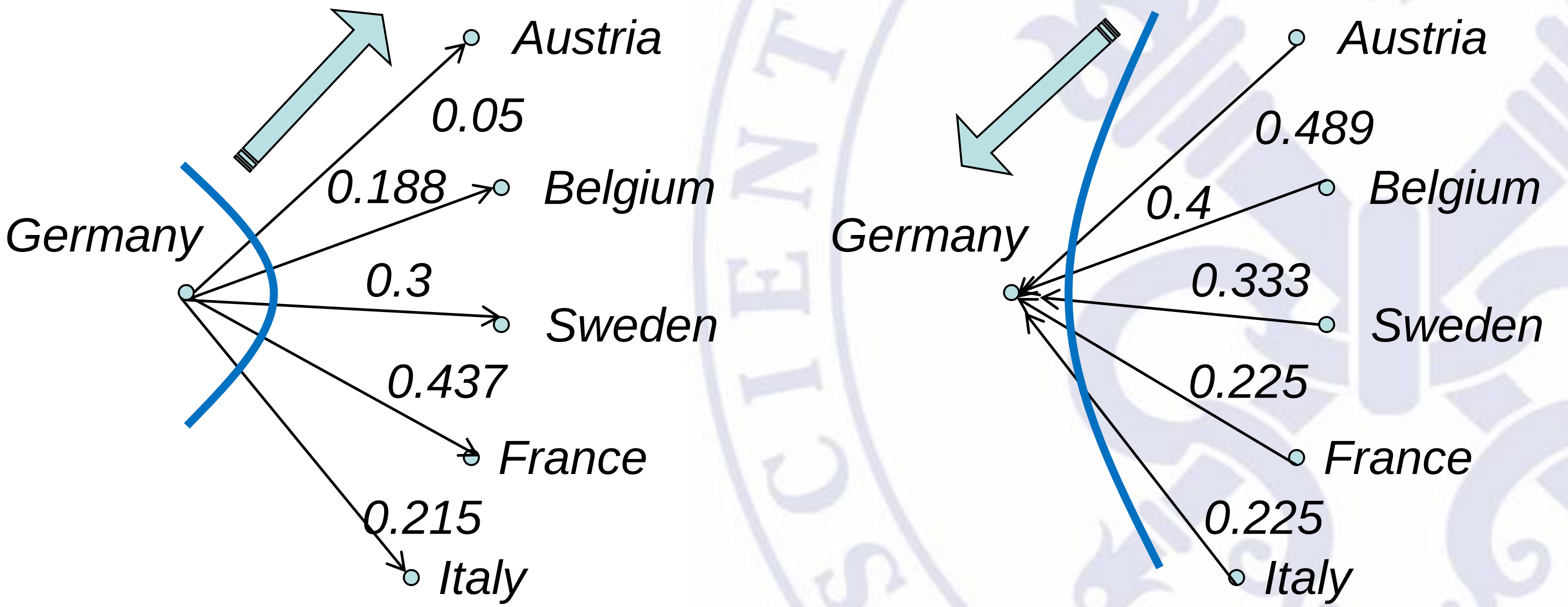
Preference matrix

Preference Degrees

	Italy	Belgium	Germany	Sweden	Austria	France
Italy	0,000	0,280	0,225	0,242	0,090	0,217
Belgium	0,267	0,000	0,400	0,300	0,267	0,500
Germany	0,215	0,188	0,000	0,300	0,050	0,437
Sweden	0,429	0,545	0,333	0,000	0,255	0,255
Austria	0,458	0,545	0,489	0,342	0,000	0,457
France	0,259	0,379	0,225	0,388	0,120	0,000

- How can we **exploit** this matrix ?
- … in order to obtain a **ranking** (complete or partial) ?

Step 3: compute positive, negative and net flow scores



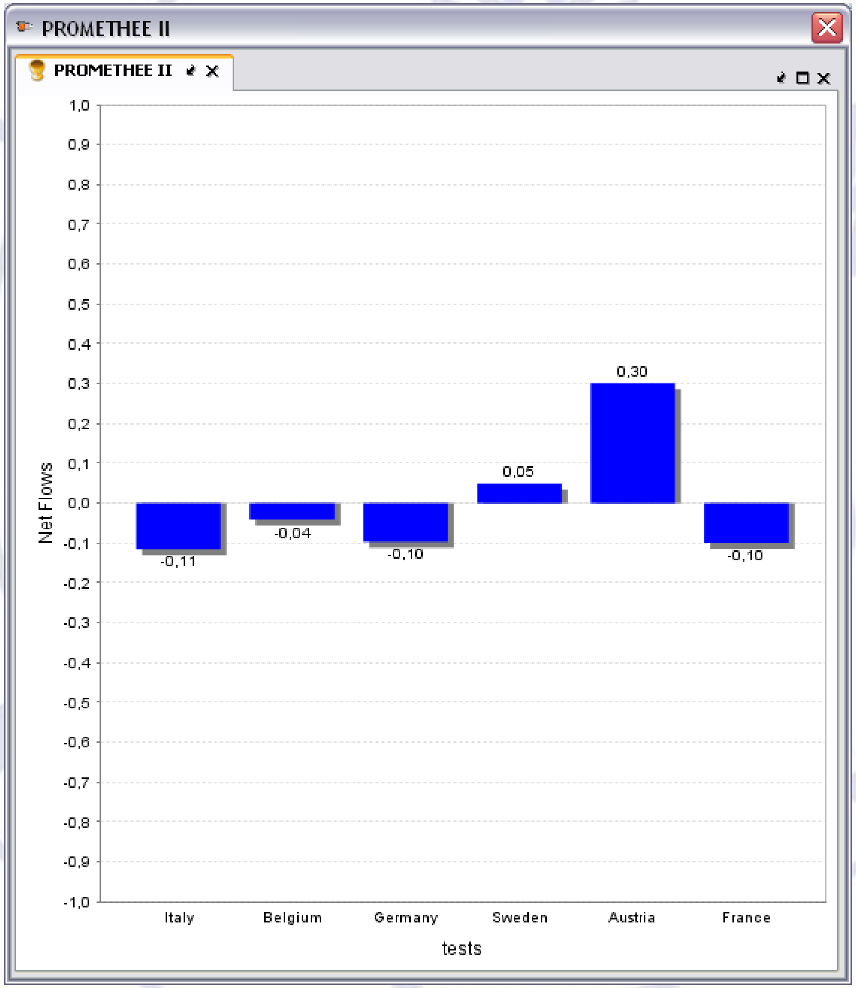
$$\Phi^{+}(Germany) = 0.238 \qquad \Phi^{-}(Germany) = 0.334$$

$$\Phi(Germany) = \Phi^{+}(Germany) - \Phi^{-}(Germany) = -0.1$$

PROMETHEE II

Flows

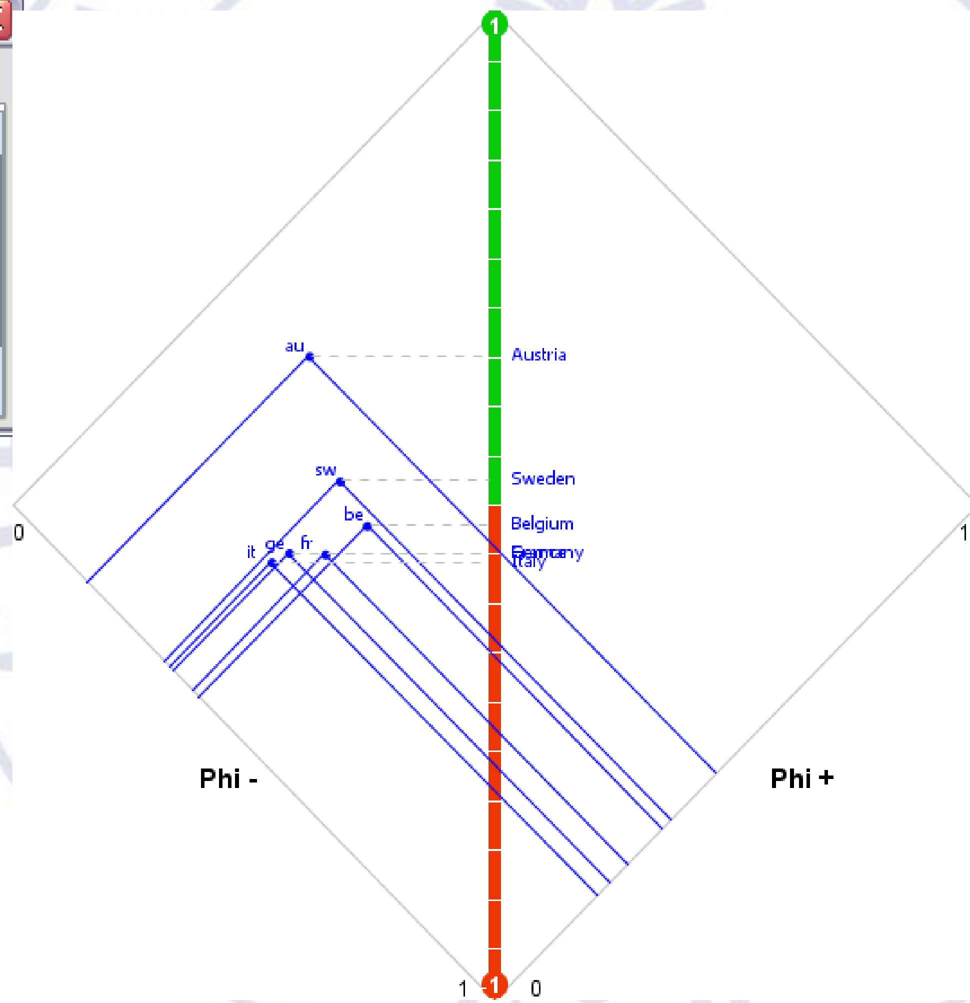
Alternative	Rank	Net Flow	Positive Flow	Negative Flow
Austria	1	0,302	0,458	0,156
Sweden	2	0,049	0,363	0,314
Belgium	3	-0,041	0,347	0,387
Germany	4	-0,096	0,238	0,334
France	5	-0,099	0,274	0,373
Italy	6	-0,115	0,211	0,326



PROMETHEE I

Flows

Alternative	Rank	Net Flow	Positive Flow	Negative Flow
Austria	1	0,302	0,458	0,156
Sweden	2	0,049	0,363	0,314
Belgium	3	-0,041	0,347	0,387
Germany	4	-0,096	0,238	0,334
France	5	-0,099	0,274	0,373
Italy	6	-0,115	0,211	0,326



Formalization

PROMETHEE

Preference Ranking Organisation METHod for Enrichment Evaluations

Formalization

- A **finite** set of alternatives:

$$A=\{a_1,a_2,..,a_n\}$$

- A set of criteria:

$$F=\{f_1,f_2,..,f_q\}$$

- W.l.g. these criteria have to be **maximized**

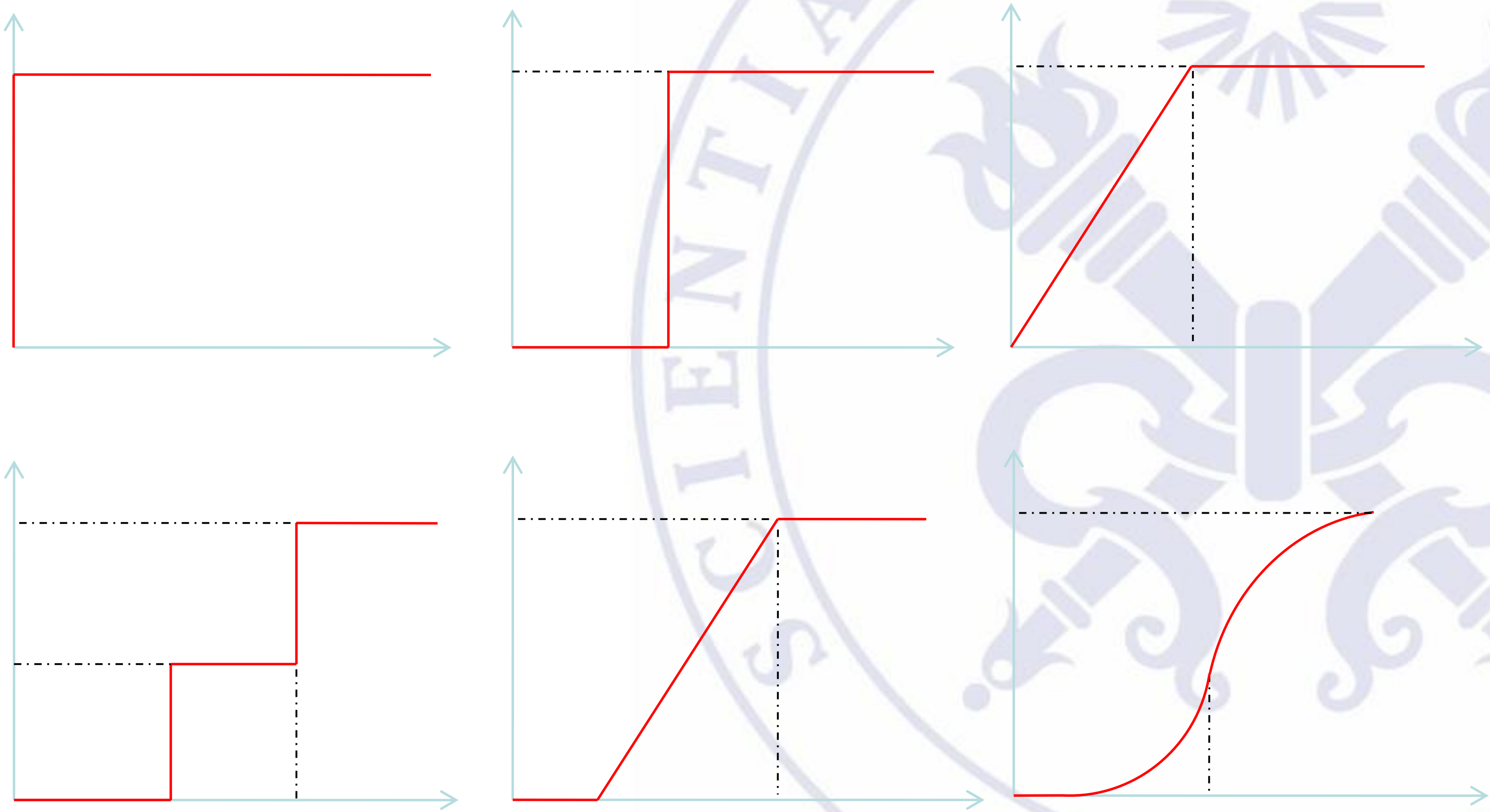
Step 1: uni-criterion preferences

$$\forall a_i, a_j \in A : d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$$

$$\pi_k(a_i, a_j) = P_k[d_k(a_i, a_j)]$$

This operation has to be meaningful:
INTERVAL SCALE

Preference functions



Step 2: Compute preference matrix

$$\forall a_i, a_j \in A : \pi(a_i, a_j) = \sum_{k=1}^{q} w_k \pi_k(a_i, a_j)$$

As a consequence:

$$\pi(a_i, a_i) = 0$$

$$\pi(a_i, a_j) \geq 0$$

$$\pi(a_i, a_j) + \pi(a_j, a_i) \leq 1$$

Step 3: compute flow scores

$$\phi^{+}(a_i) = \frac{1}{n-1}\sum_{b\in A}\pi(a_i,b)$$

$$\phi^{-}(a_i) = \frac{1}{n-1}\sum_{b\in A}\pi(b,a_i)$$

$$\phi(a_i) = \phi^{+}(a_i) - \phi^{-}(a_i)$$

Maximum number of parameters: **3.q-1**

PROMETHEE II

Complete ranking based on the net flow score.

$$a_i P a_j \Leftrightarrow \Phi(a_i) > \Phi(a_j)$$

$$a_i I a_j \Leftrightarrow \Phi(a_i) = \Phi(a_j)$$

PROMETHEE I

Partial ranking based on both the positive and negative flow scores.

$$a_i P a_j \Leftrightarrow [\phi^+(a_i) > \phi^+(a_j)] \wedge [\phi^-(a_i) \leq \phi^-(a_j)]$$

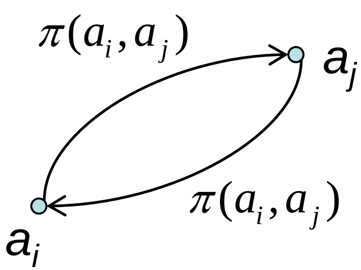
$$a_i P a_j \Leftrightarrow [\phi^+(a_i) \geq \phi^+(a_j)] \wedge [\phi^-(a_i) < \phi^-(a_j)]$$

$$a_i I a_j \Leftrightarrow [\phi^+(a_i) = \phi^+(a_j)] \wedge [\phi^-(a_i) = \phi^-(a_j)]$$

$$a_i J a_j, otherwise$$

The net flow score: a recipe ?

- From **local** to **global** information !



$S_i = S(a_i), S(a_j) = S_j$

ill-defined problem

- One could expect that:

$$\pi_{ij} - \pi_{ji} \approx S_i - S_j$$

Property

Intuition: "The PROMETHEE multicriteria net flow $\phi(a_i)$ is the centred score s_i (i=1,…,n) that minimizes the sum of the squared deviations from the pair-wise comparisons of the actions"

$$Q = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\; s_i - s_j \; - \; \pi_{ij} - \pi_{ji} \; \right]^2$$

Proof (1):

$$L\left(s_1,\ldots,s_n,\lambda\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(s_i-s_j\right)-\left(\pi_{ij}-\pi_{ji}\right)\right]^2-\lambda\sum_{i=1}^{n}s_i$$

$$\frac{\partial L\left(s_1,\ldots,s_n,\lambda\right)}{\partial s_i}=0$$

$$\frac{\partial L\left(s_1,\ldots,s_n,\lambda\right)}{\partial \lambda}=0$$

Proof (2):

$$L\ \ s_1,\ldots,s_n,\lambda\ \ = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\ s_i - s_j\ \ -\ \ \pi_{ij} - \pi_{ji}\ \right]^2 - \lambda\sum_{i=1}^{n} s_i$$

$$\frac{\partial L}{\partial s_i} = 4.\sum_{j=1,\, j\neq i}^{n}[(s_i - s_j) - (\pi_{ij} - \pi_{ji})] - \lambda$$

$$= 4.[(n-1).s_i - \sum_{j=1,\, j\neq i}^{n} s_j - \sum_{j=1,\, j\neq i}^{n}(\pi_{ij} - \pi_{ji})] - \lambda$$

$$= 4[n.s_i - \sum_{j=1,\, j\neq i}^{n}(\pi_{ij} - \pi_{ji})] - \lambda$$

$$s_i = \frac{1}{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(\pi_{ij} - \pi_{ji}) = \frac{n-1}{n}\phi(a_i)$$

Prerential independance (3)

$$f_j(a) = f_j(b), \forall j \in J$$
$$f_j(c) = f_j(d), \forall j \in J$$
$$f_j(a) = f_j(c), \forall j \in \bar{J}$$
$$f_j(b) = f_j(d), \forall j \in \bar{J}$$

$$aPb \Leftrightarrow \phi(a) > \phi(b)$$

$$\phi(a) = \sum_{j=1}^{q} w_j.\phi_j(a) = \sum_{j \in J} w_j.\phi_j(a) + \sum_{j \in \bar{J}} w_j.\phi_j(a)$$

$$= \sum_{j \in J} w_j.\phi_j(b) + \sum_{j \in \bar{J}} w_j.\phi_j(a) > \sum_{j \in J} w_j.\phi_j(b) + \sum_{j \in \bar{J}} w_j.\phi_j(b) = \phi(b)$$

$$\sum_{j \in \bar{J}} w_j.\phi_j(a) > \sum_{j \in \bar{J}} w_j.\phi_j(b)$$

Prerential independance (5)

$$f_j(a) = f_j(b), \forall j \in J$$
$$f_j(c) = f_j(d), \forall j \in J$$
$$f_j(a) = f_j(c), \forall j \in \bar{J}$$
$$f_j(b) = f_j(d), \forall j \in \bar{J}$$

$$\sum_{j \in \bar{J}} w_j.\phi_j(a) > \sum_{j \in \bar{J}} w_j.\phi_j(b)$$

$$\phi(c) = \sum_{j=1}^{q} w_j.\phi_j(c) = \sum_{j \in J} w_j.\phi_j(c) + \sum_{j \in \bar{J}} w_j.\phi_j(c)$$
$$= \sum_{j \in J} w_j.\phi_j(d) + \sum_{j \in \bar{J}} w_j.\phi_j(a) > \sum_{j \in J} w_j.\phi_j(d) + \sum_{j \in \bar{J}} w_j.\phi_j(b)$$
$$= \sum_{j \in J} w_j.\phi_j(d) + \sum_{j \in \bar{J}} w_j.\phi_j(d) = \phi(d)$$

GAIA

Geometrical Analysis for Interactive Assistance

ULB GAIA (1)

- We have:

$$\Phi(a_i) = \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(a_i,b) - \frac{1}{n-1}\sum_{b\in A}\sum_{k=1}^{q} w_k.\pi_k(b,a_i)$$

$$= \sum_{k=1}^{q} w_k . \frac{1}{n-1}\sum_{b\in A}\pi_k(a_i,b) - \pi_k(b,a_i) = \sum_{k=1}^{q} w_k.\phi_k(a_i)$$

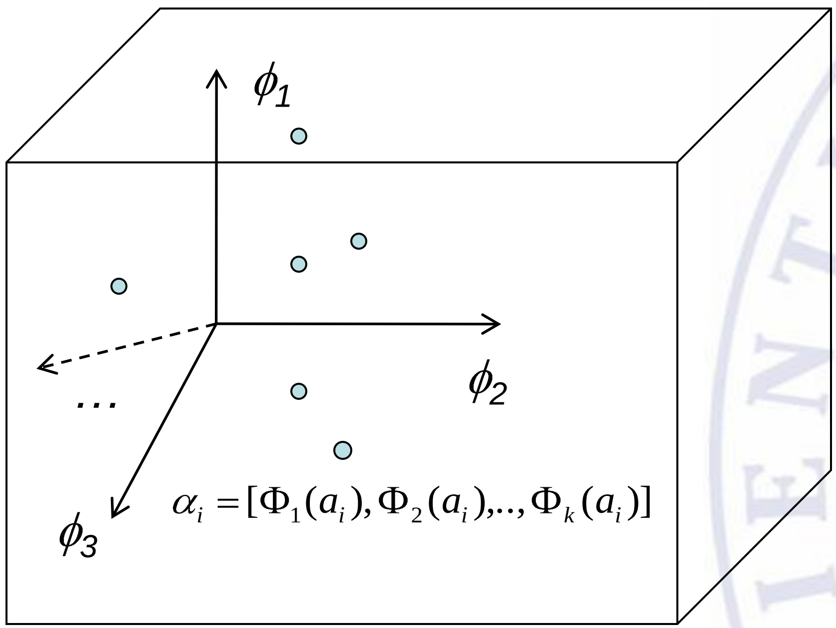
- Where

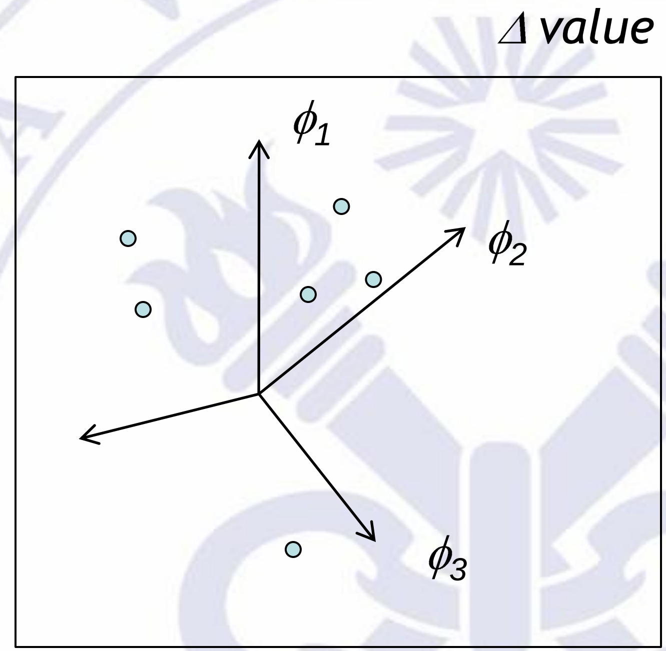
$$\Phi_k(a_i) = \sum_{b\in A}\pi_k(a_i,b) - \pi_k(b,a_i)$$

- In other words, every alternative can be represented by a vector:

$$\alpha_i = [\Phi_1(a_i), \Phi_2(a_i), .., \Phi_k(a_i)]$$

GAIA (2)





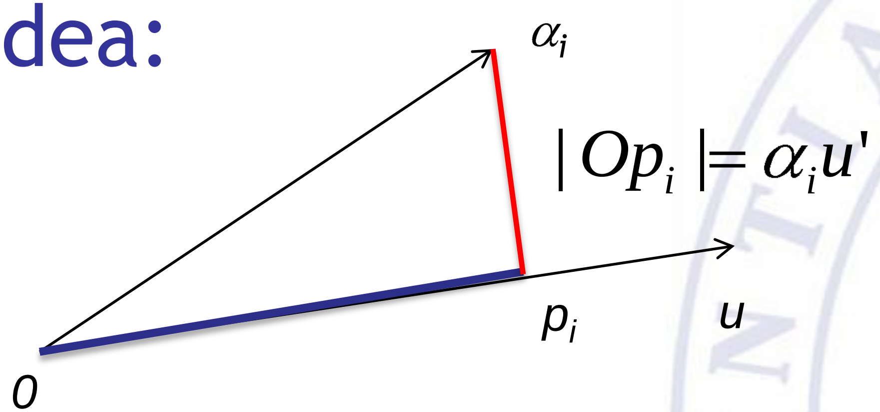
q dimensions

2 dimensions

Principal component analysis

GAIA (3)

- Idea:



$$Min \sum_{i=1}^{n} |\alpha_i p_i|^2 = Max \sum_{i=1}^{n} |Op_i|^2 \Longleftrightarrow Max \;\; uCu'$$

$$u.u' = 1$$

$$\Phi = \begin{pmatrix} \Phi_1(a_i) & \dots & \Phi_k(a_i) \\ \dots & \dots & \dots \\ \Phi_1(a_n) & \dots & \Phi_k(a_n) \end{pmatrix}$$

$$where \;\; nC = \Phi\Phi'$$

GAIA (4)

$$Max \quad uCu'$$
$$u.u' = 1$$

$\Longleftrightarrow$

$$L(u,\lambda) = uCu' - \lambda.(u.u' - 1)$$

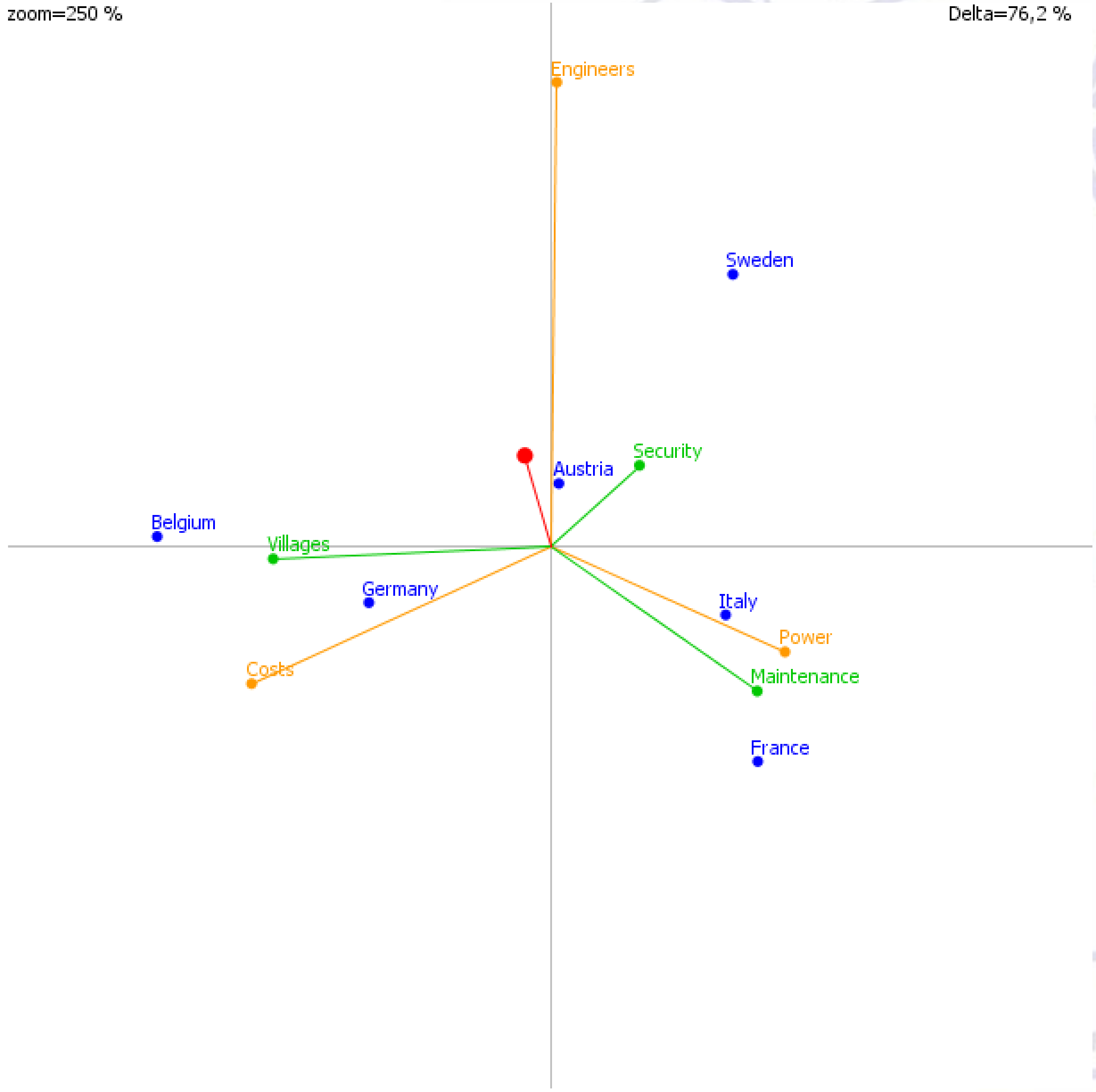
$\Downarrow$

$$\begin{cases} C.u = \lambda.u \\ u.u' = 1 \end{cases}$$

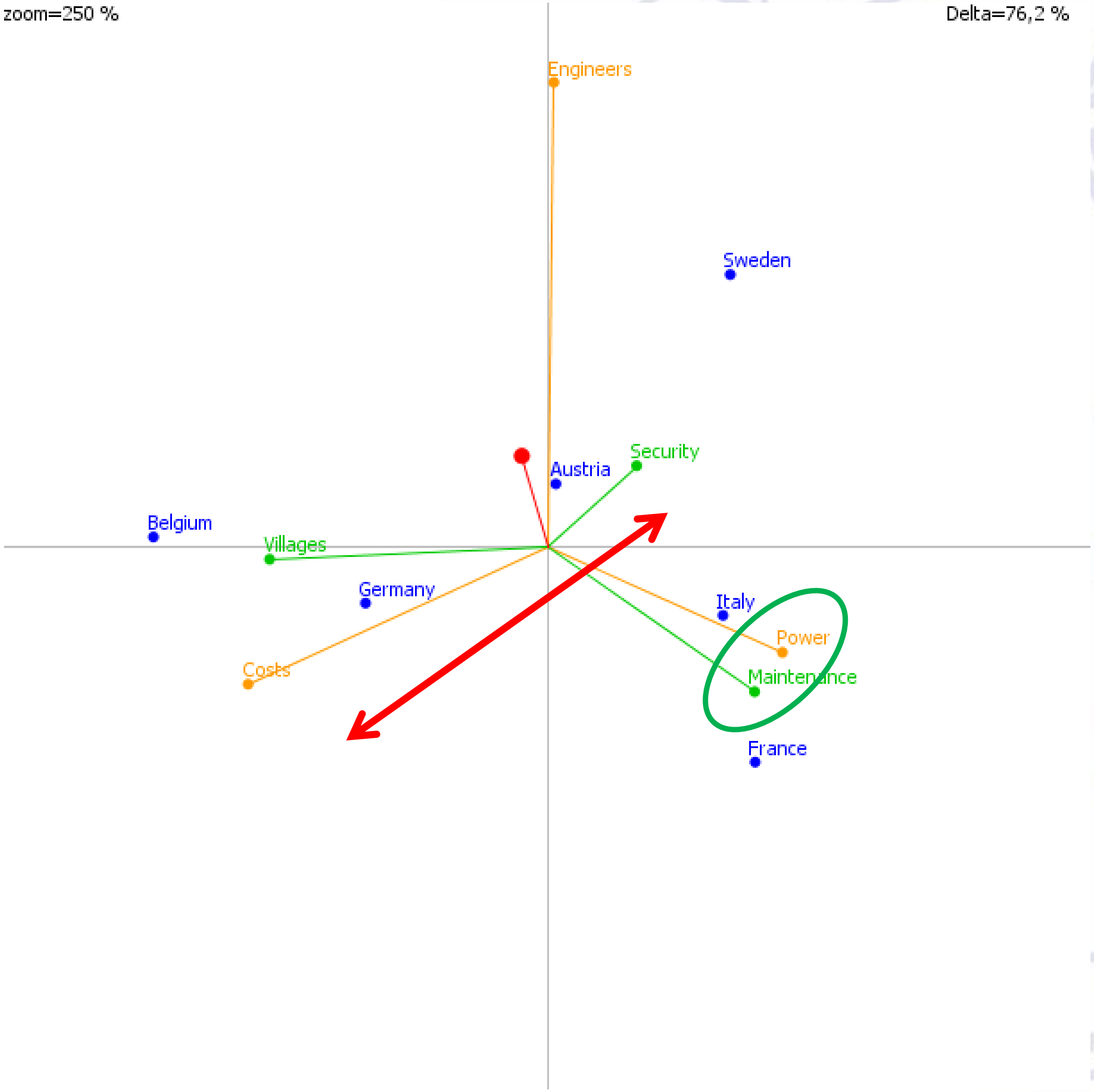
$\Downarrow$

$$Max \quad uCu' = \lambda_1.u$$

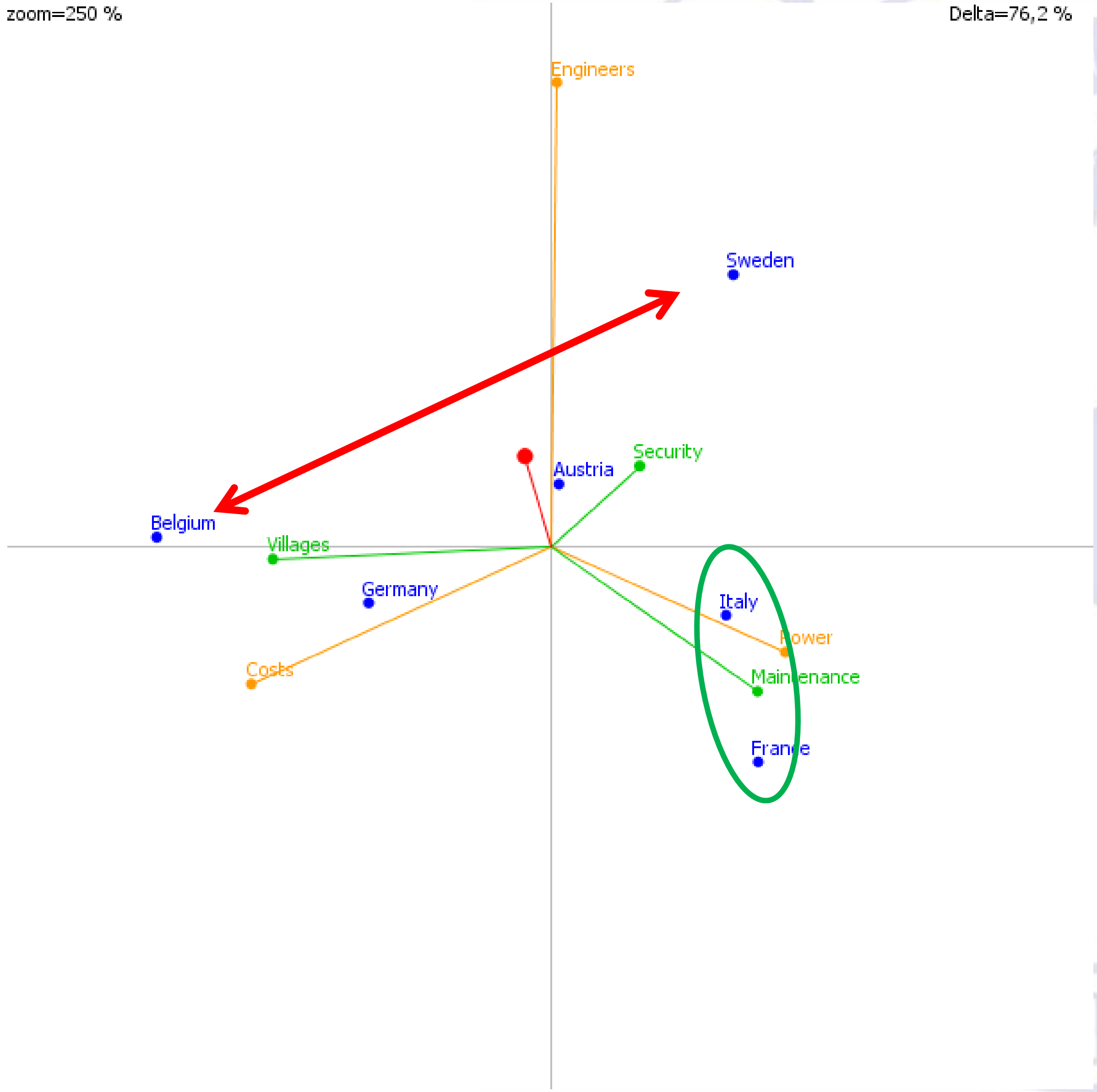
GAIA(3)


zoom=250 %
Delta=76,2 %
Engineers
Sweden
Security
Austria
Belgium
Villages
Germany
Italy
Power
Costs
Maintenance
France

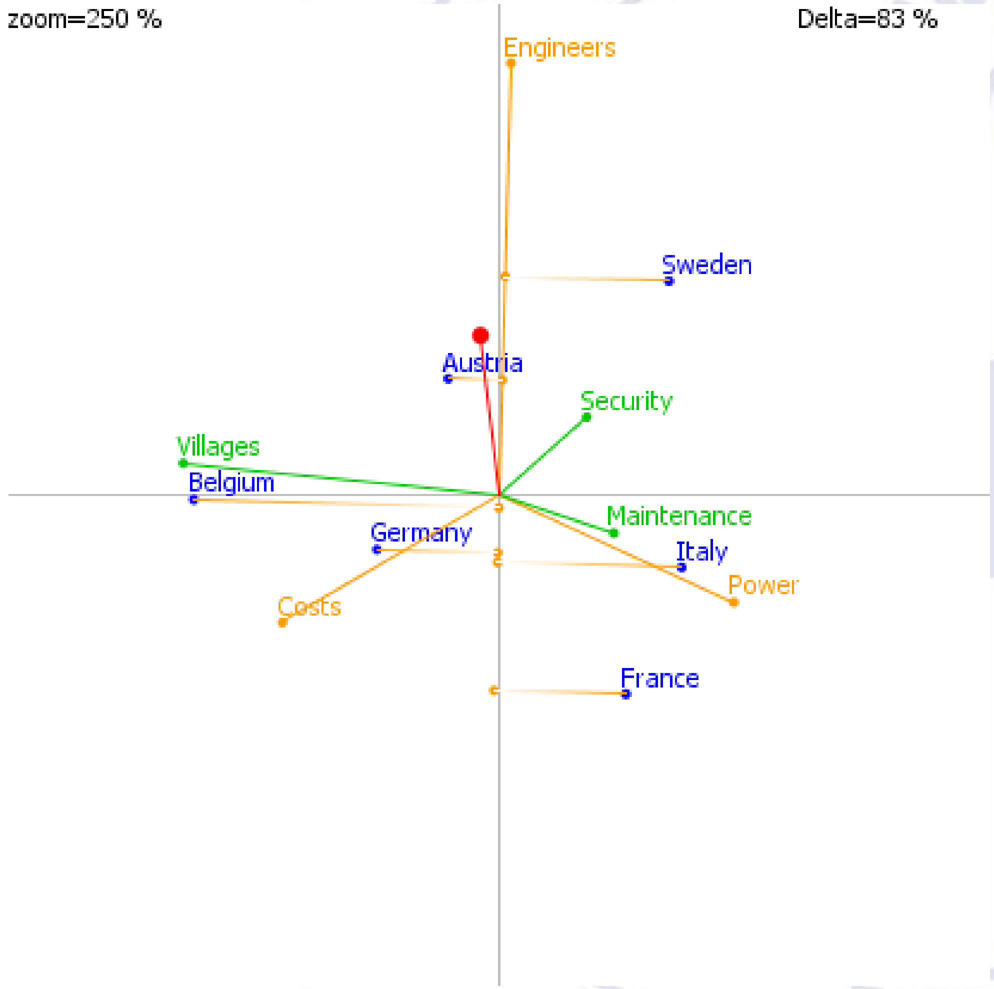
GAIA(4): criteria



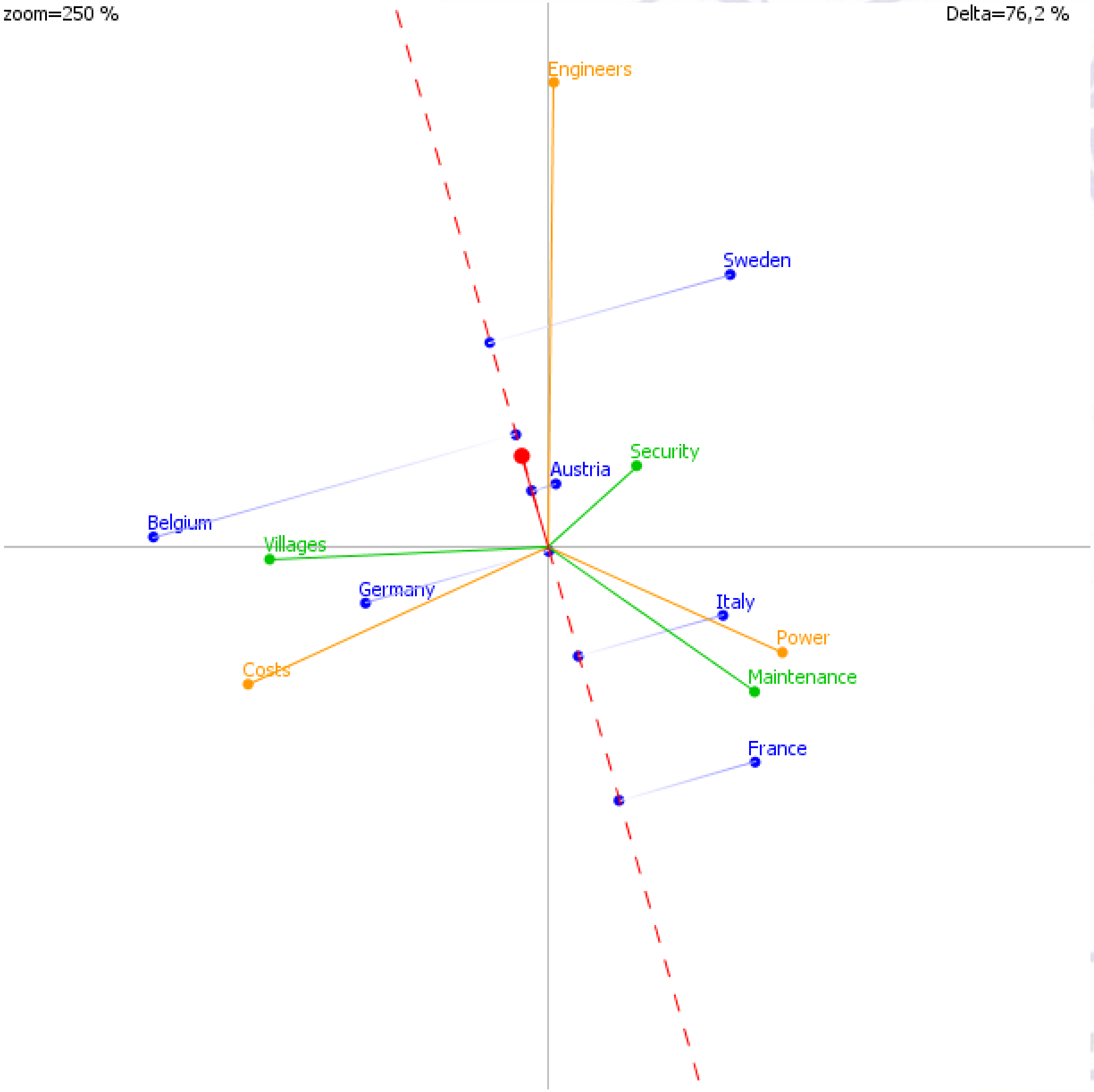
GAIA(5): alternatives



GAIA(6): alternatives / criteria



GAIA(7): Decision stick



Software demonstration

A few words about rank reversal

Rank reversal

- We could have:

$$\pi_{ij} \geq \pi_{ji} \wedge \phi(a_i) \leq \phi(a_j)$$

- In other words: a pairwise rank reversal …
- This opens a discussion about rank reversal …
  - AHP: *Belton and Gear (1983), Saaty and Vargas (1984), Triantaphyllou (2001), Wang and Elhag (2006), Wijnmalen and Wedley (2009)*
  - ELECTRE: *Wang and Triantaphyllou (2005)*
  - PROMETHEE: *De Keyser and Peeters (1996)*
- **The concept of rank reversal is not fully formalized** (*add a copy of an alternative, deletion of a non discriminating criterion, deletion of an alternative, …)*
- *A direct consequence of Arrow's theorem*

Deletion of a non discriminating criterion

$$\phi_k(a_i) = 0 \forall a_i \in A$$

$$\phi(a_i) = \sum_{j=1}^{q} w_j \phi_j(a_i)$$

$$= \sum_{j=1, j\neq k}^{q} w_j \phi_j(a_i)$$

$$= W_k \sum_{j=1, j\neq k}^{q} \frac{w_j}{W_k} \phi_j(a_i)$$

$$= W_k \sum_{j=1, j\neq k}^{q} w'_j \phi_j(a_i)$$

$$= W_k \phi'(a_i)$$

	f_1	f_2	…	f_k	…	f_q
a_1	$f_1(a_1)$	$f_2(a_1)$	…	α	…	$f_q(a_1)$
a_2	$f_1(a_2)$	$f_2(a_2)$	…	α	…	$f_q(a_2)$
…	…	…	…	…	…	…
a_n	$f_1(a_n)$	$f_2(a_n)$	…	α	…	$f_q(a_n)$

where $W_k = \sum_{j=1, j\neq k}^{q} w_j$ *and* $w'_j = \frac{w_j}{W_k}$

$$\phi(a_i) > \phi(a_j) \Leftrightarrow \phi'(a_i) > \phi'(a_j)$$

Dominance

- Let us assume that:

$$f_k(a_i) \geq f_k(a_j), \forall k = 1,..q$$

- Then:

$$\phi(a_i) = \frac{1}{n-1} \sum_{k=1}^{q} w_k \sum_{b \in A} \pi_k(a_i, b) - \pi_k(b, a_i)$$

$$\geq \frac{1}{n-1} \sum_{k=1}^{q} w_k \sum_{b \in A} \pi_k(a_j, b) - \pi_k(b, a_j) = \phi(a_j)$$

This result holds for any set A such that a_i, $a_j \in A$

More general result (1)

Notations: $A_x = A \backslash \{x\}$, $\Phi_x(a)$

No RR $\Leftrightarrow \big(\Phi(a) - \Phi(b)\big)\big(\Phi_x(a) - \Phi_x(b)\big) > 0$

$$\textit{if} \quad \Phi(a) - \Phi(b) > \frac{[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$$

No RR (for any action removed) if

$$\Phi(a) - \Phi(b) > \frac{\max_x[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}$$

More general result (2)

➔ *RR can only occur if*

$$\Phi(a) - \Phi(b) < \underbrace{\frac{\max_x[(\pi_{ax} - \pi_{xa}) - (\pi_{bx} - \pi_{xb})]}{n-1}}_{\text{refined threshold (depends on the sample and (a,b))}} \leq \underbrace{\frac{2}{n-1}}_{\text{rough threshold (constant)}}$$

refined threshold
(depends on the sample and (a,b))

rough threshold (constant)

Generalization: when k actions are removed

No RR if $\Phi(a) - \Phi(b) > \frac{2k}{n-1}$

Mareschal, B., De Smet, Y. and Nemery, P. « *Rank Reversal in the PROMETHEE II Method : Some New Results* », proceedings of de IEEE 2008 International Conference on Industrial Engineering and Engineering Management, Singapore, (2008): 959-963

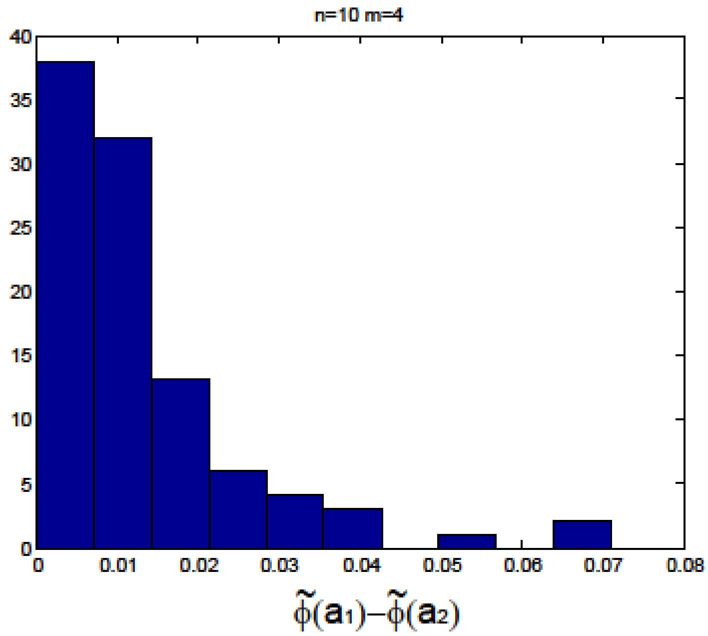
More general result (3)

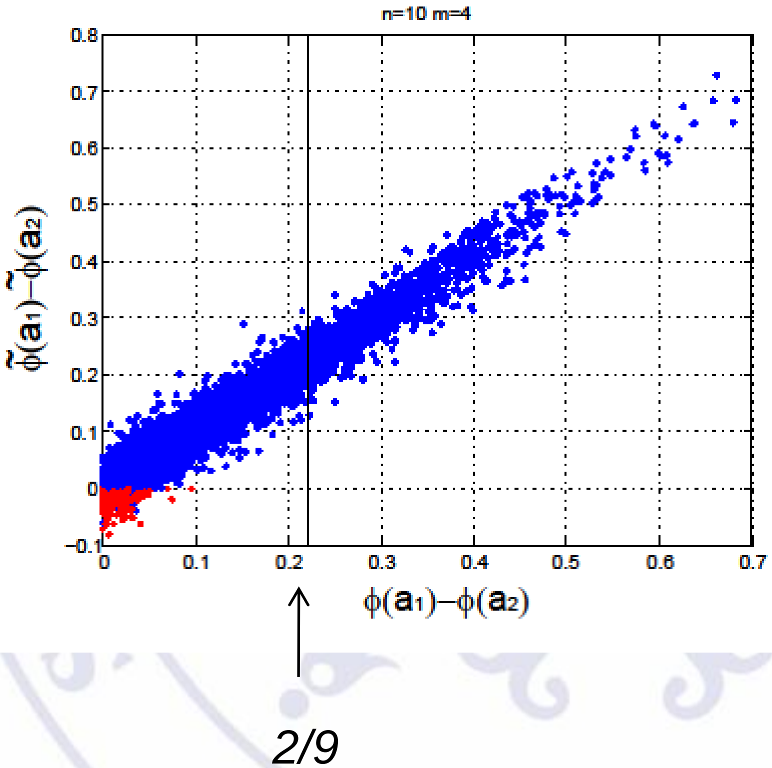
Statistical results relative to the «rough threshold» (for q = 2, DA=Unif)

n	nb RR	$b = \frac{2}{n-1}$	nb $\Delta\Phi \leq b$	nb RR $\mid \Delta\Phi \leq b$
5	2,20 %	0,50	47,4 %	4,6 %
10	0,98 %	0,22	33,5 %	2,9 %
15	0,66 %	0,14	24,7 %	2,6 %
20	0,45 %	0,10	19,9 %	2,2 %
50	0,18 %	0,04	9 %	1,9 %

Conclusion: *The number of RR occurences is really small.*

More general result (4)





Verly, C. and De Smet, Y « Some considerations about rank reversal occurrences in the PROMETHEE methods » to appear in the International Journal of Multicriteria Decision Making.

Related works for PROMETHEE I

- No rank reversal will happen between a_i and a_j if

$$|\phi^+(a_i)-\phi^+(a_j)|\geq \frac{1}{n-1}$$

$$|\phi^-(a_i)-\phi^-(a_j)|\geq \frac{1}{n-1}$$

ULB

Rank reversal = risk of manipulation

- **Joint work** with **Julien Roland** and **Céline Verly** (to appear in the proceedings of the IPMU 2012 conference)
- **Aim**: to quantify the likelihood of manipulation in an simplified version of the PROMETHEE II ranking:
  - Usual preference function and equal weights
  - Copeland scores
- **More formaly:**
  - A given decision maker has a perfect information on the evaluation table;
  - He may propose new alternatives in order to make alternative a_i the first one;
  - **Question**: how many alternatives are necessary ?

ULB Linear mathematical program

$$\max \sum_{a \in A \cup C} y(a_s, a)$$

$$
\begin{aligned}
\text{subject to: } (P_j(a_i, a_j) - 1)\overline{g_k} &< g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C, \ \forall k \in K \\
P_k(a_i, a_j)\overline{g_k} &\geqslant g_k(a_i) - g_k(a_j), \forall a_i, a_j \in A \cup C, \ \forall k \in K \\
\pi(a_i, a_j) &= \frac{1}{q} \sum_{k \in K} P_k(a_i, a_j), \forall a_i, a_j \in A \cup C \\
\phi(a) &= \frac{1}{n+m-1} \sum_{x \in A \cup C} \pi(a, x) - \pi(x, a), \forall a \in A \cup C \\
g_k(a) &\leqslant \overline{g_k}, \forall a \in C \\
g_k(a) &\geqslant 0, \forall a \in C \\
2(y(a_s, a) - 1) &\leqslant \phi(a_s) - \phi(a), \forall a \in A \cup C \\
2y(a_s, a) &> \phi(a_s) - \phi(a), \forall a \in A \cup C
\end{aligned}
$$

Results for 10 alternatives and 3 criteria

Table 1. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives.

$j \backslash m$	1	2	3	4	5	6	7	8	9
2	7	3	0	0	0	0	0	0	0
3	37	13	7	0	0	0	0	0	0
4	57	33	17	3	0	0	0	0	0
5	83	63	40	13	0	0	0	0	0
6	90	83	60	23	0	0	0	0	0
7	90	90	77	43	10	0	0	0	0
8	100	100	87	70	37	7	0	0	0
9	100	100	97	83	63	33	3	0	0
10	100	100	97	93	83	63	33	3	0

Comparison with the bound

Table 3. Percentage of instances (with 10 alternatives and 3 criteria) where it was not possible to bring the alternative ranked at the j-th place to the top when adding m well-chosen artificial alternatives while the Mareschal's bound is not reached.

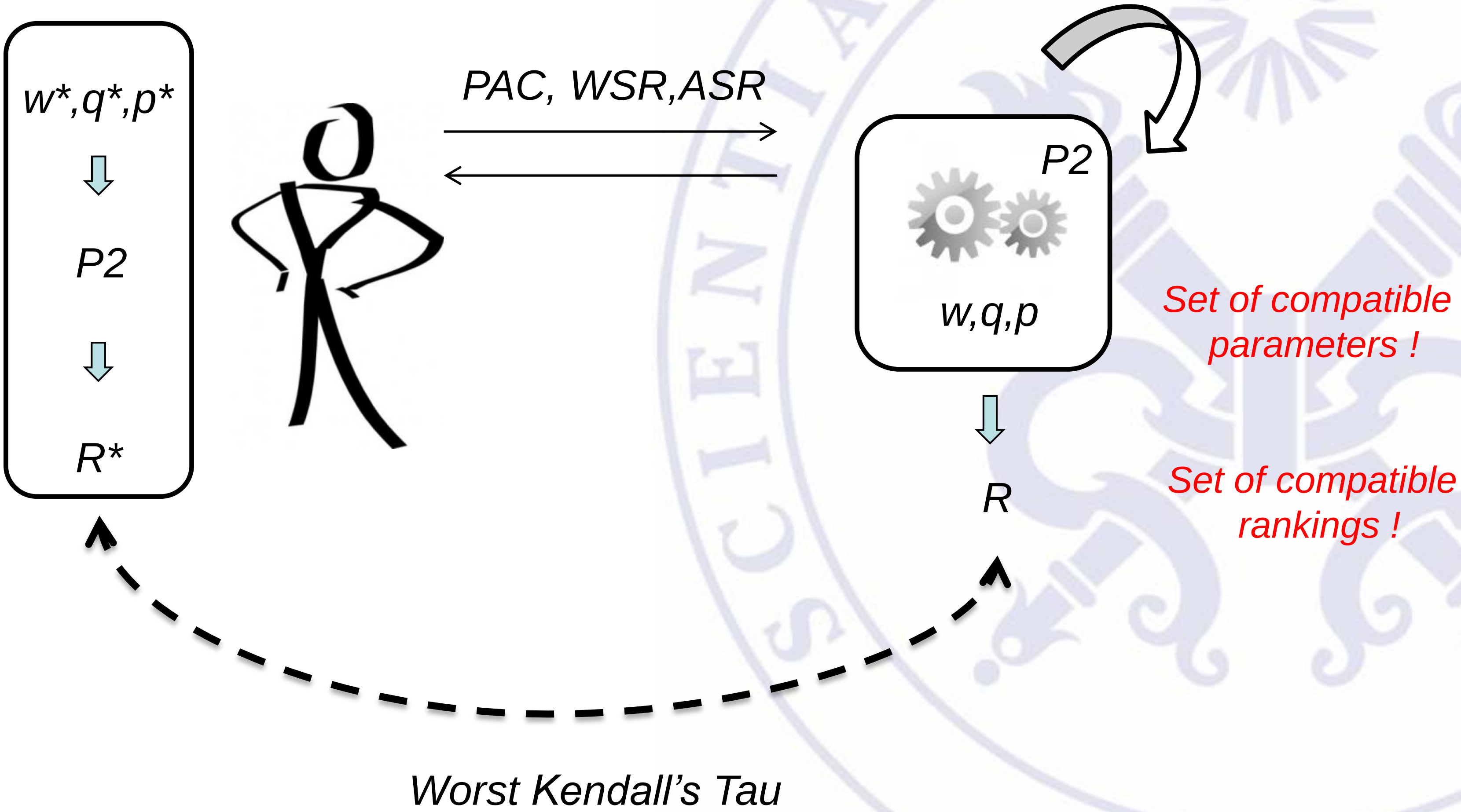
$j \backslash m$	1	2
2	7	3
3	30	13
4	27	33
5	17	63
6	7	83
7	0	90
8	7	73
9	3	40
10	0	20

A few words about preferences elicitation

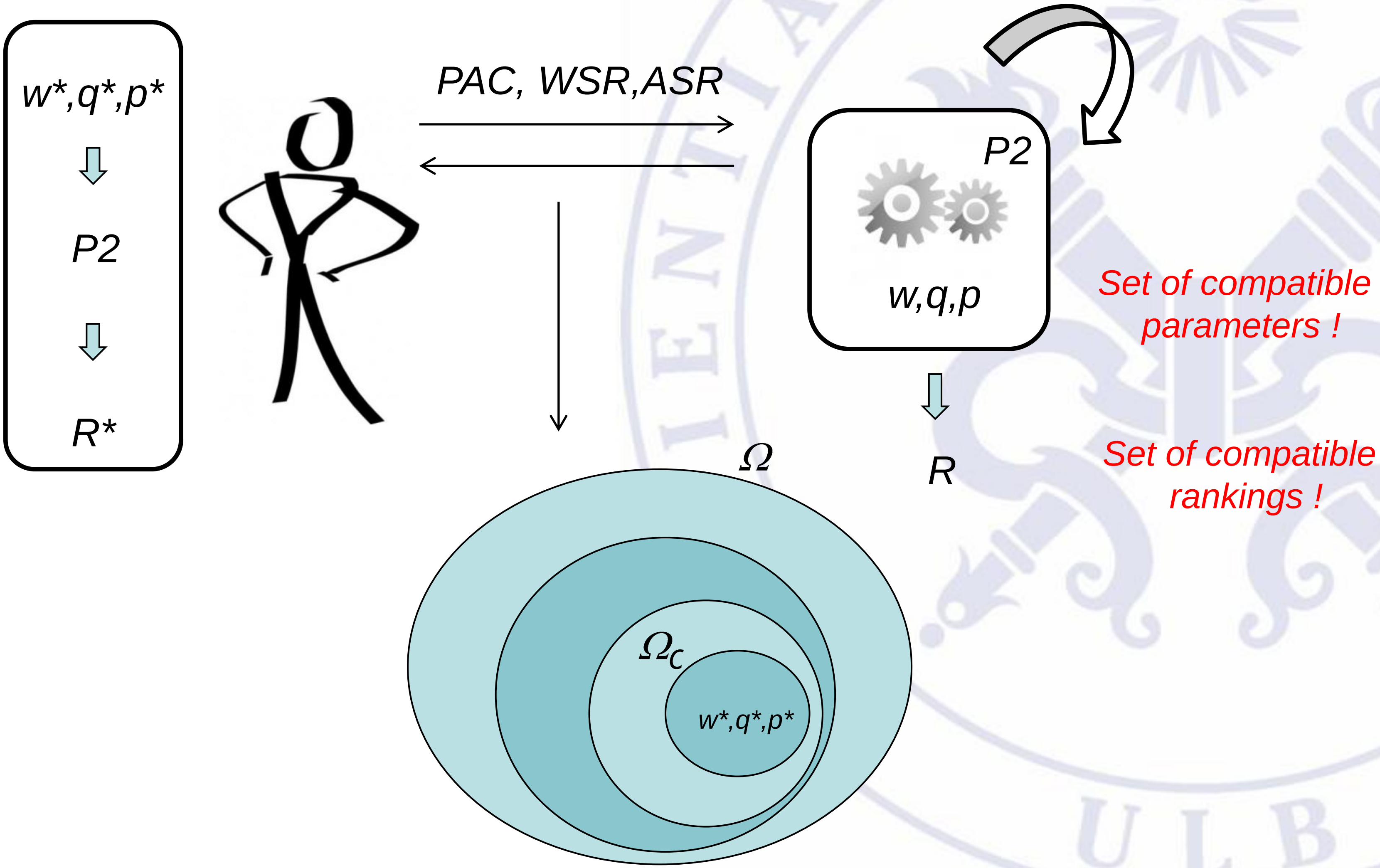
To our knowledge, only a few contributions:

- Özerol, G., Karasakal, E. (2007) « Interactive outranking approaches for multicriteria decision-making problems with imprecise information » JORS, 59, 1253-1268
- Sun, Z. and Han, M. (2010) « Multi-criteria decision making based on PROMETHEE method », in Proceedings of the 2010 international Conference on Computing, Control and Industrial Engineering, 416-418
- Frikha, H., Chabchoub, H., Martel, J.-M. (2010) « *Inferring criteria's relative importance coefficients in PROMETHEE II* », International Journal of Operational Research 7(2), 357-275
- Eppe, S., De Smet, Y., Stützle, T. (2011) « *A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method* » Proceedings of ADT (2011), 56-66
- Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to appear in the proceddings of the IPMU 2012 conference
- Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (1)

General idea (1)



General idea (2)



Eppe, S., De Smet, Y., Stützle, T. (2011) « *A bi-objective optimization model to eliciting decision maker's preferences for the PROMETHEE II method* » Proceedings of ADT (2011), 56-66

- **Main idea**: quality and robustness
- **Distinctive feature:** the DM may communicate mistakes

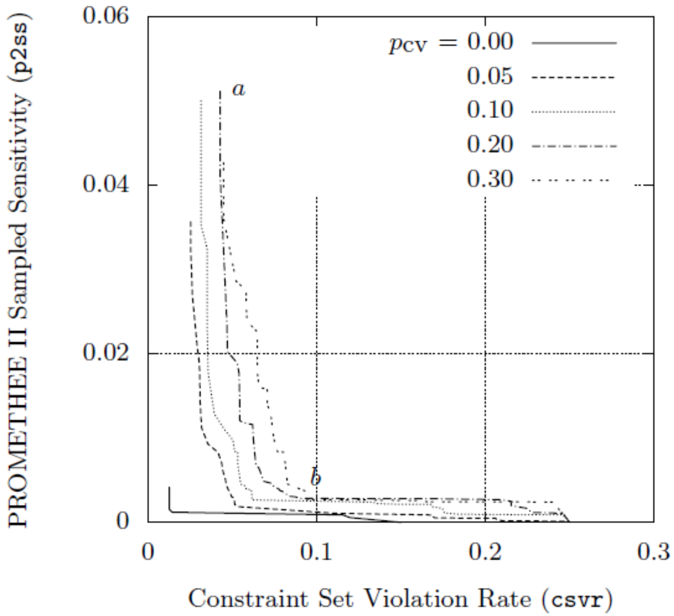


Fig. 4. This plot represents the approximated Pareto frontiers in the objective space, for 20 constraints and several values of the constraint violation rate p_{cv}, i.e., the proportion of inconsistent constraints with respect to the total number of constraints. As expected, increasing the value of p_{cv} has the effect of deteriorating the quality of the solution set both in terms of constraint violation rate and PROMETHEE II sampled sensitivity.

(n=100,q=2)

NSGA-II

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to appear in the proceddings of the IPMU 2012 conference

- **Main idea**: quantify information infrastructure …

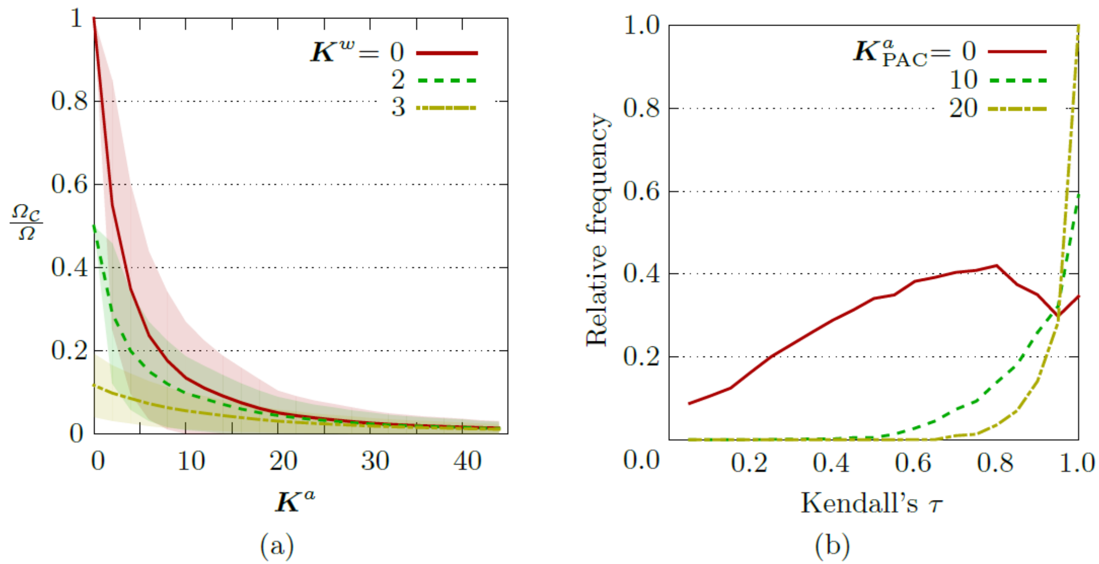


Fig. 3. **(a)** Evolution of the ratio of compatible weight domain area $\frac{\Omega_C}{\Omega}$ with respect to the whole domain of possible weights Ω, depending on the number of action constraints K^a, for different numbers of weight constraints K^w. Results are shown for 1000 randomly generated action sets ($n = 10$, $m = 3$) and PAC constraint sets. — **(b)** Distribution of all values of Kendall's τ in the compatible weights domain Ω_C, for respectively $K^a_{PAC} = 0$, 10, and 20 pairwise action comparisons. No constraints on the weights relative importance are given here ($K^w = 0$).

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to appear in the proceddings of the IPMU 2012 conference

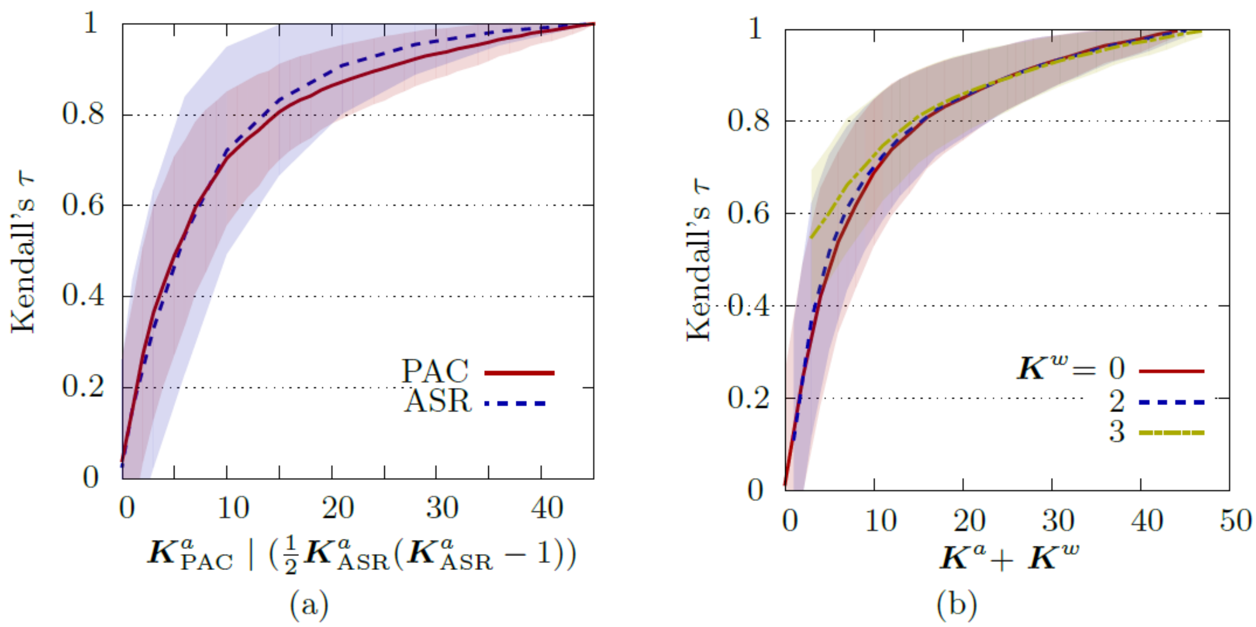


Fig. 4. **(a)** Evolution of worst Kendall's τ for two different information types: pairwise action comparisons (PAC) and action sub-ranking (ASR). — **(b)** Impact of "weight constraints" on the reachable quality for pairwise action comparisons (PAC). Note that the x-axis represents the sum of action *and* weight constraints, i.e. $K^a + K^w$.

Eppe, S., De Smet, Y. (2012) « *Studying the impact of information structure in the PROMTHEE II preference elicitation process: A simulation based approach* » to appear in the proceddings of the IPMU 2012 conference

Table 1. Number of pairwise action comparisons that have to be given by a DM to reach the desired level of quality $\underline{\boldsymbol{w}}$, assuming that $\boldsymbol{K}^w$ weight constraints have already been provided. The results are shown for randomly generated 3-criteria action sets with a uniform distribution.

	$\underline{\boldsymbol{w}}$						
$\boldsymbol{K}^w$	0.50	0.60	0.70	0.80	0.90	0.95	0.99
0	6	8	10	15	25	34	43
2	4	6	9	14	24	33	43
3	0	1	5	11	22	32	42

- **Main idea**: to overcome the limitations of the previous approach; pairwise comparisons have to be « well-chosen »
- **q-Eval**

Table 2 Maximum number of queries that could be generated on average with *q-Eval* for 30 requested queries for 50 randomly generated instances.

	m			
n	3	5	7	10
10	8.2	12.7	15.4	17.4
20	12.3	20.5	26.3	29.5
50	15.88	28.46	30.0	30.0
100	17.26	29.46	30.0	30.0
200	17.96	29.84	30.0	30.0

Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (2)

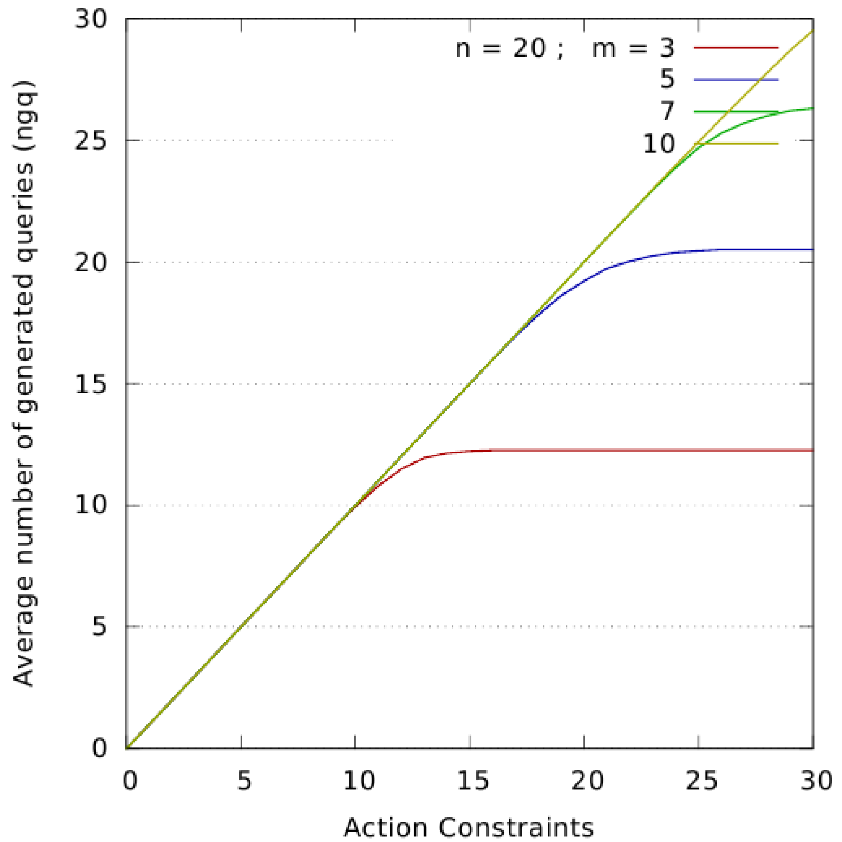


Figure 2 Evolution of the average number of constraints that can be generated with the *q-Eval* method depending on the instance size (number m of alternatives) for a given number of alternatives. Plots for a constant value of m and different number of alternatives n are very similar.

Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (3)

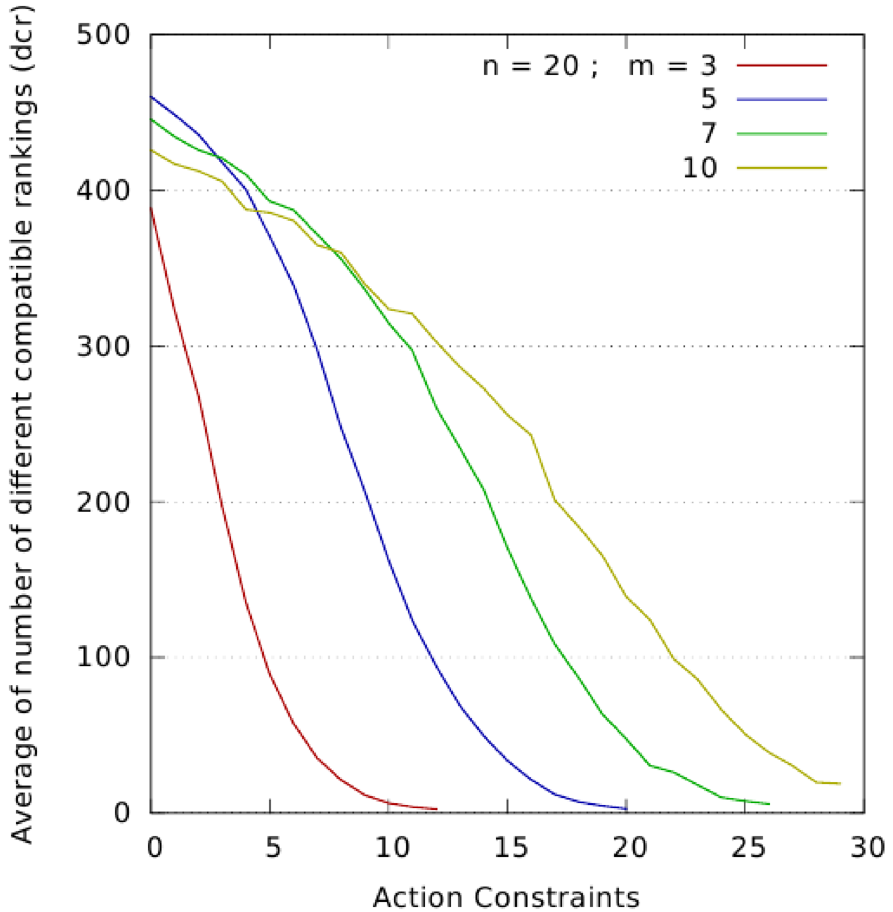


Figure 3 For $n = 20$ actions, shows the average evolution of the number of different rankings found in the sampling of the compatible parameter domain Ω, for different numbers m of criteria.

Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (4)

Table 3 This table represents the average value of the worst Kendall's correlation coefficient τ_G for the sampling of the compatible parameter domain Ω for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

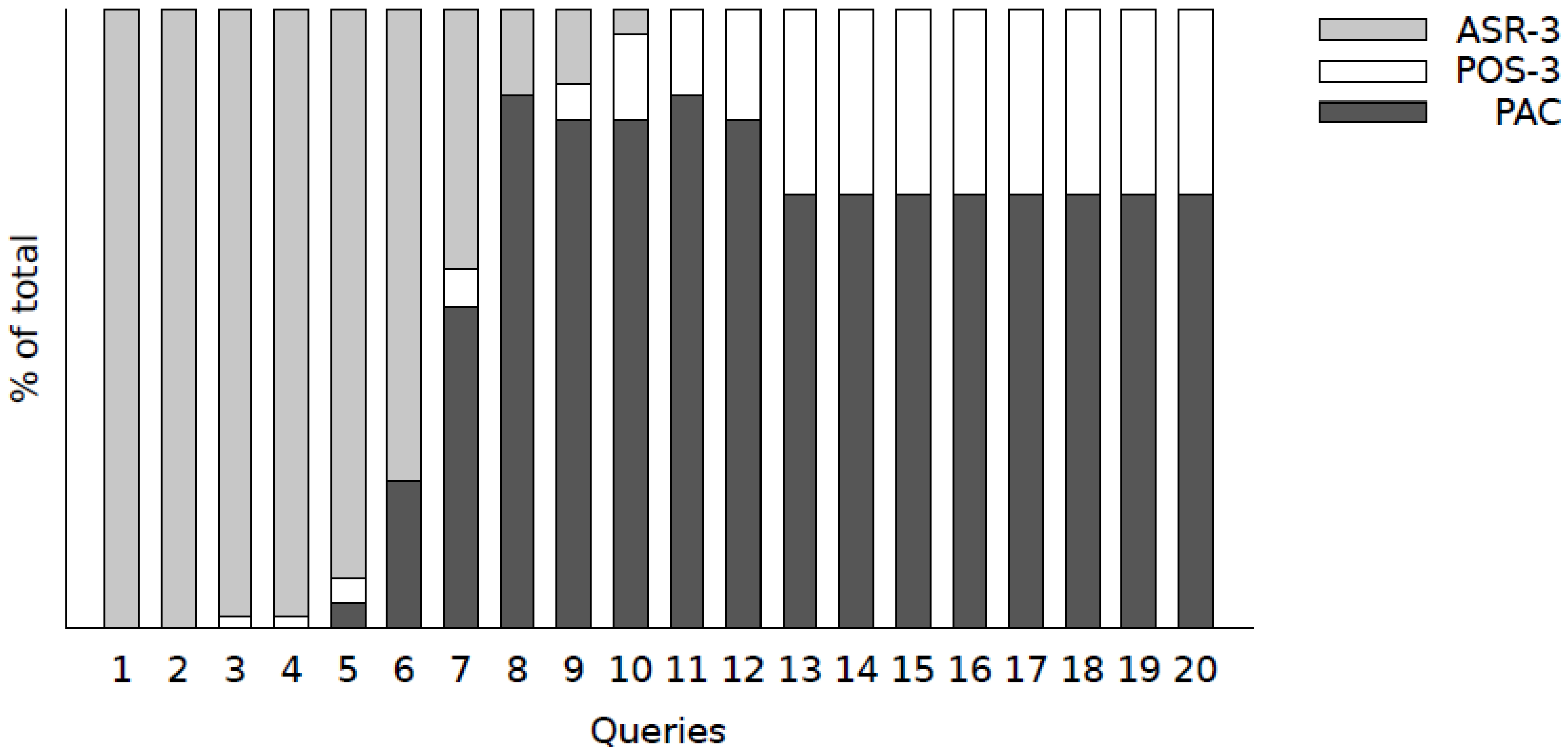
	m			
n	3	5	7	10
10	0.999	0.998	0.988	0.988
20	1.000	1.000	0.999	0.994
50	1.000	1.000	0.998	0.987
100	1.000	1.000	0.997	0.983
200	1.000	1.000	0.998	0.982

Table 4 This table represents the average number of different rankings in the samples for each trial after the last query. Note that the sample size of $n_S = 500$ has a clear impact on the result, in particular in the upper range of values. The actual number of different rankings inside the compatible domain is probably higher in those cases.

	m			
n	3	5	7	10
10	1.1	1.1	2.4	3.9
20	1.0	1.5	2.2	14.1
50	4.8	18.6	204.4	358.8
100	44.6	226.7	430.8	465.9
200	187.9	432.1	477.8	490.5

Eppe, S., De Smet, Y. « *An experimental parameter space analysis of the PROMETHEE II outranking method* » ongoing work (4)

		Worst generalized Kendall's τ_G						
		0.50	0.60	0.70	0.80	0.90	0.95	0.99
random	PAC	13	17	21	25	35	47	
Q-Eval	PAC	4	5	6	7	9	11	17
	POS-3	2	3	4	4	6	8	
	ASR-3	2	3	4	4	4	5	
	adaptive	2	2	3	3	4	5	10


% of total
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Queries
ASR-3
POS-3
PAC

Directions for future research

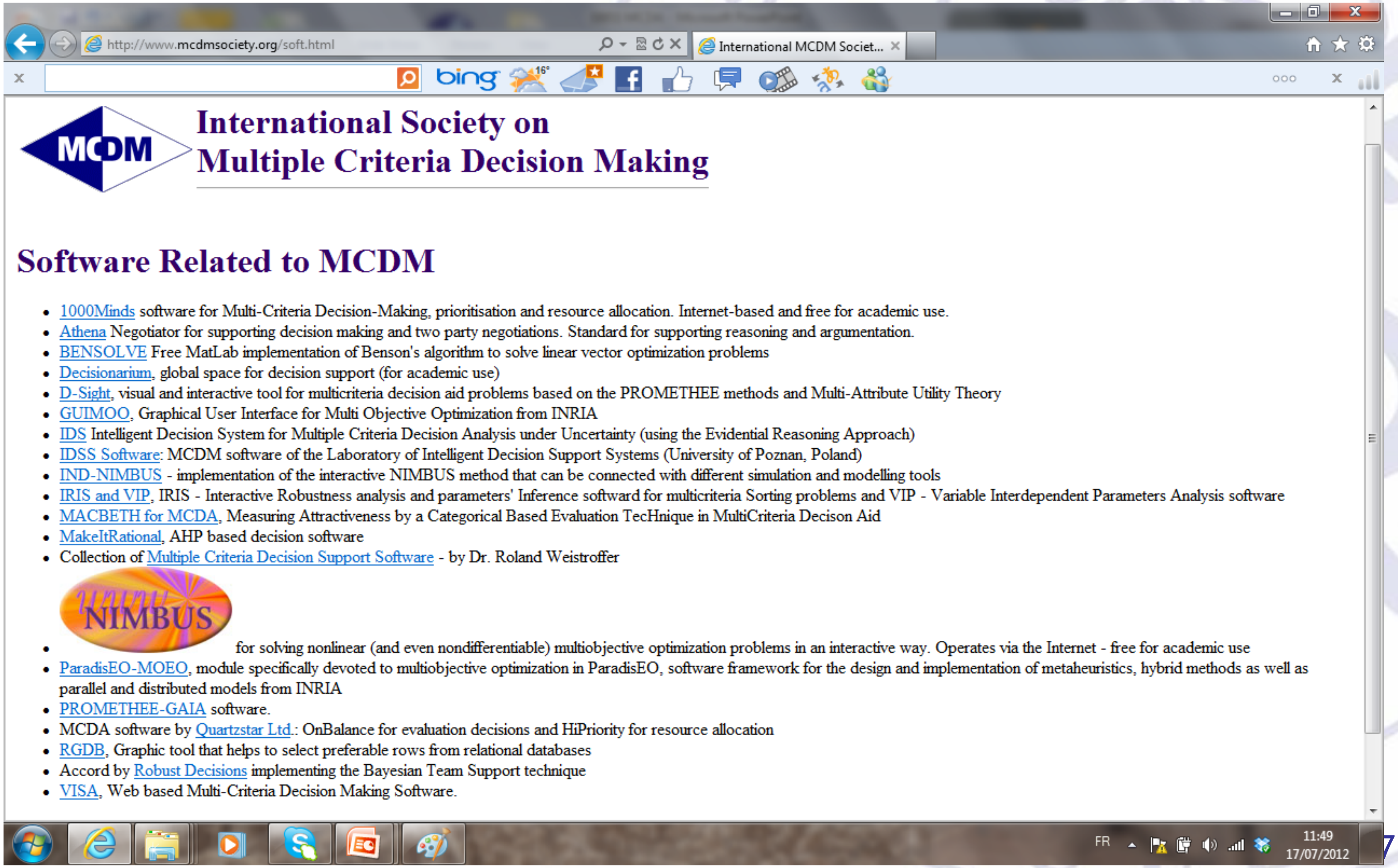
1. Theoretical foundations of the PROMETHEE & GAIA methods
   - Rank reversal
2. Preferences' elicitation
3. Current directions
   - GIS, DEA, Uncertainty (see Mareschal et al.), Sorting (see Nemery et al.), …

To continue your investigations…

- EURO Working Group on MCDA
  - http://www.cs.put.poznan.pl/ewgmcda/
- International Society on Multiple Criteria Decision Making
  - http://www.mcdmsociety.org/
- Journals:
  - *Journal of Multicriteria Decision Analysis*
  - *International Journal of Multicriteria decision making*
  - Top journals in OR and Management Sciences

Software

Decision Deck, M-MACBETH, Expert Choice



http://www.mcdmsociety.org/soft.html

International Society on Multiple Criteria Decision Making

Software Related to MCDM

- 1000Minds software for Multi-Criteria Decision-Making, prioritisation and resource allocation. Internet-based and free for academic use.
- Athena Negotiator for supporting decision making and two party negotiations. Standard for supporting reasoning and argumentation.
- BENSOLVE Free MatLab implementation of Benson's algorithm to solve linear vector optimization problems
- Decisionarium, global space for decision support (for academic use)
- D-Sight, visual and interactive tool for multicriteria decision aid problems based on the PROMETHEE methods and Multi-Attribute Utility Theory
- GUIMOO, Graphical User Interface for Multi Objective Optimization from INRIA
- IDS Intelligent Decision System for Multiple Criteria Decision Analysis under Uncertainty (using the Evidential Reasoning Approach)
- IDSS Software: MCDM software of the Laboratory of Intelligent Decision Support Systems (University of Poznan, Poland)
- IND-NIMBUS - implementation of the interactive NIMBUS method that can be connected with different simulation and modelling tools
- IRIS and VIP, IRIS - Interactive Robustness analysis and parameters' Inference softward for multicriteria Sorting problems and VIP - Variable Interdependent Parameters Analysis software
- MACBETH for MCDA, Measuring Attractiveness by a Categorical Based Evaluation TecHnique in MultiCriteria Decison Aid
- MakeItRational, AHP based decision software
- Collection of Multiple Criteria Decision Support Software - by Dr. Roland Weistroffer
- NIMBUS for solving nonlinear (and even nondifferentiable) multiobjective optimization problems in an interactive way. Operates via the Internet - free for academic use
- ParadisEO-MOEO, module specifically devoted to multiobjective optimization in ParadisEO, software framework for the design and implementation of metaheuristics, hybrid methods as well as parallel and distributed models from INRIA
- PROMETHEE-GAIA software.
- MCDA software by Quartzstar Ltd.: OnBalance for evaluation decisions and HiPriority for resource allocation
- RGDB, Graphic tool that helps to select preferable rows from relational databases
- Accord by Robust Decisions implementing the Bayesian Team Support technique
- VISA, Web based Multi-Criteria Decision Making Software.