Graph Mining & Community Detection
An Introduction to Social Networks Data Analysis

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Graph Mining and SNDA. Why?

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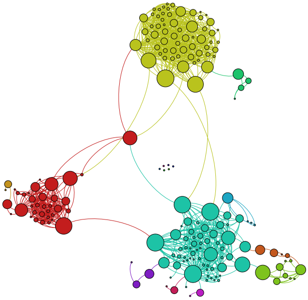
My Facebook's friends set reduced in 3 main communities.



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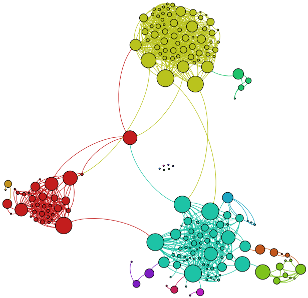

What you don't tell, your network can tell it for you (MIT's Gaydar experience).

A talk with a minimum of formulas?

Stephen Hawking

My editor told me that each equation I included in the book would halve sales.

"A Brief History of Time"

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But...min $>$ 0 ! (Argh! A first formula!)

Outline

Plan

The Graph Theory start

How to walk through the city that would cross each bridge once and only once ? (Typically mathematician's game).

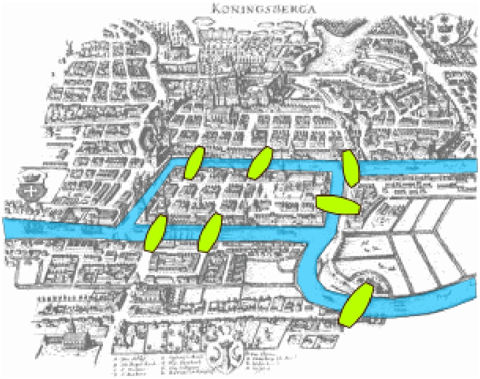

Figure: source: wikipedia

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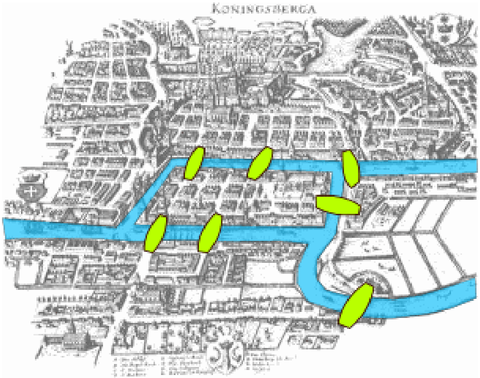

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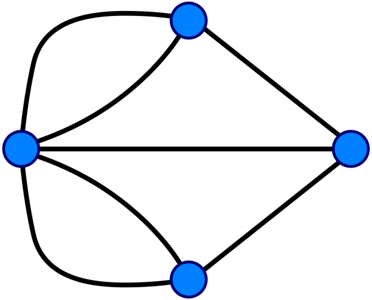

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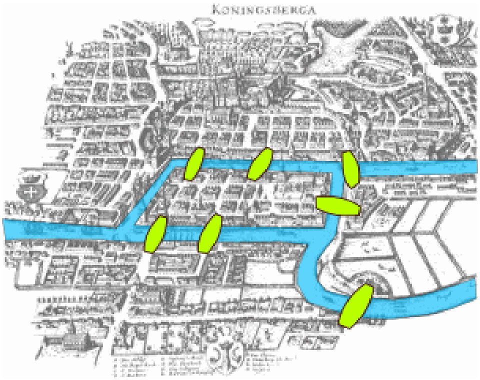

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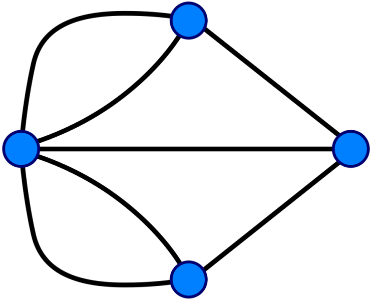

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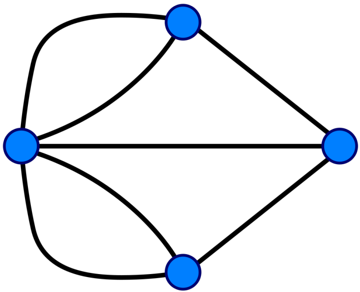
Using graphs Euler proved that the problem has no solution.

The Graph Theory Paradigm

A graph is denoted $G = (V, E)$ with

- V the set of vertices,
- E the set of edges.

$\{v, w\}$ is the edge connecting vertices v and w.



$$A = \left\{ \begin{array}{ccc} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{array} \right\}$$

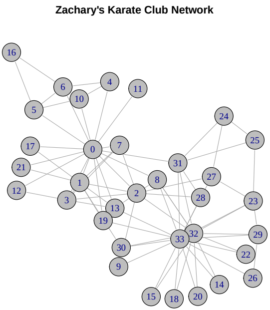
where

$$a_{i,j} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are connected,} \\ 0 & \textit{else.} \end{cases}$$

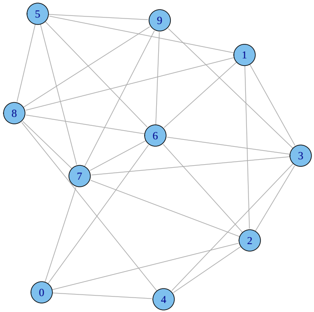
If G is a weighted graph, then $a_{i,j} = \omega(v_i, v_j)$ and then $A = W = (w_{i,j}) = (\omega(v_i, v_j))$.

The Graph Theory Goes Social

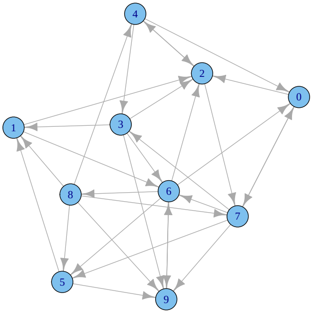
- Moreno (1933) 1st to use points and lines for social configurations,
- Cartwright and Harary (1956) link with the graph theory,
- individuals are represented using points, called *nodes* or *vertices*,
- and social relationships are represented using lines.



Directed or not?



An undirected graph (ex.: Facebook): $\{v, w\}$



A directed graph (ex.: Twitter): (v, w).

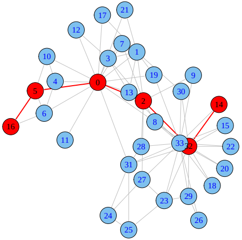
Graph Theory Basic Notions

- $|V| = n$ is called the *order* the graph.
- $|E| = m$ is called the *size* of the graph.
- If $|E| = n(n-1)/2$, i.e. (any pair of vertices are connected), the graph *complete*.
- v and u are *neighbours* if connected by an edge.
- The neighbourhood of a node v is denoted $\Gamma(v)$.
- A *subgraph* $G' = (V', E')$ of $G = (V, E)$ is such $V' \subset V$, $E' \subset E$ and $\{v, u\} \in E' \Rightarrow v, u \in V'$.
- A subset C of V can define an *induced subgraph* $G(C) = (C, E(C))$, where $E(C) = \{(v, u) \in E | v, u \in C\}$.

Graph Theory Basic Notions (ctd)

- A path P is a subgraph $P = (V(P), E(P))$ such $V(P) = \{v_{i_0}, \cdots, v_{i_k}\}$ and $E(P) = \{\{v_{i_0}, v_{i_1}\}, \{v_{i_1}, v_{i_2}\}, \cdots, \{v_{i_{k-1}}, v_{i_k}\}\}$.
- k is the length path.
- If no vertice is repeated, then the path is *simple*. length,
- If there exists a path between v and u, they are *connected*.
- The graph is a *connected graph* if $\forall v, u$, there is, at least, one path connecting v and u.
- A connected subgraph is called a *connected component*.

Diameter of the Zachary Karate Club network



Graph Theory Basic Notions (ctd)

- *Density* of a subgraph $C(V(C), E(C))$: ratio between $|E(C)|$ and the maximum possible number of edges:

$$\delta(G(C)) = \frac{|E(C)|}{|V(C)|(|V(C)|-1)/2} \tag{1}$$

- A partition of V in two subsets C and $V \setminus C$ is called a *cut*.
- The *cut size* is the number of edges joining vertices of C with vertices of $V \setminus C$:

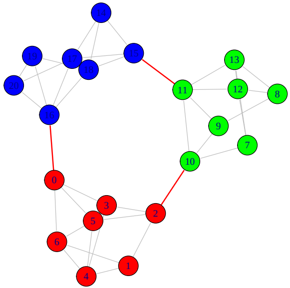
$$c(C, V \setminus C) = |\,\{\{u,v\} \in E | u \in C, v \in V \setminus C\}\,| \tag{2}$$

- For an unweighted graph, *degree* of a vertice v, $deg(v)$: number of incident edges,
- For weighted graph:

$$deg(v_i) = \sum_{j=1}^{n} w_{i,j}. \tag{3}$$

Plan

Detecting Communities



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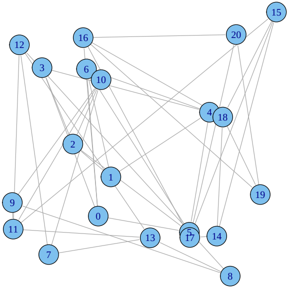
- In graph framework, clustering is dividing vertices such nodes of a community must be more connected with nodes of this community, than with nodes outside of the cluster ([Sha07], [For10]).
- It implies that it must exists at least a path between two nodes of a cluster, and this path must be internal to the cluster.

Then in a Community...

Connections must be minimum between groups and maximum within groups.



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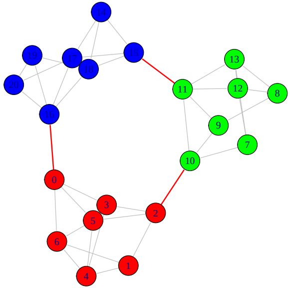

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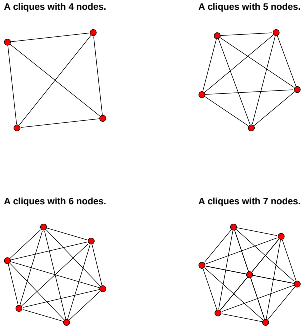
Wasserman Criteria

Four criteria:

- *Complete mutuality*: all member of a subgroup must be “linked” with all members of the subgroup,
- *Reachability*: existence (and length) of paths between vertices of a subgroup,
- *Nodal degree*: imposes a constraint on the number of adjacent vertices,
- *Internal versus external cohesion*: the former must be higher than the latter.

Complete mutuality

- In a very strict sense, in a community, all member of a subgroup must be “friends” with all members of the subgroup,
- In graph theory, it corresponds to a clique,
- But define of a community as a clique is very too strict (Alba 1973) calls it “a quite stingy”), that leads to relaxed definitions of the notion of clique...



Reachability

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- The classical clique is then a 1-clique.
- But a geodesic path of an *n*-clique could run outside of this latter, and then the diameter of the subgraph c an exceed *n*...
- That is why was defined *n-clan* which is an *n-clique* with diameter not larger than *n*.

Nodal degree

For a given graph G and a cluster C:

- A *k-plex* is a maximal subgraph such each vertice is adjacent to all other vertices of the subgraph except for k of them.
- Conversely a *k-core* is a maximal subgraph such each vertice is adjacent to, at least, k other vertices of the subgraph.

Internal versus external cohesion

For a given graph G and a cluster C:

- $deg_i(C)$: internal degree of C is the number of internal edges of C: $deg_i(v, C) = |\Gamma(v) \cap C|$,
- $deg_e(C)$: external degree of C is the number of edges with one vertice inside C, and the other outside of C: $deg_e(v, C) = |\Gamma(v) \cap (V \setminus C)|$,
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- Conversly if $deg_i(v, C) = 0$, then we must have $v \notin C$.
- A *LS-set*, or *strong community* is a subgraph C such for each node $v \in C$ we have $deg_i(v, C) > deg_e(v, C)$.

Internal versus external cohesion

For a given graph G and a cluster C:

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For a given partition $\{C_1, \cdots, C_k\}$ we want

$$\sum_{i=1}^{k} \delta_i(C_i) = \delta_i(G|C_1, \cdots, C_k) >> \delta(G).$$

Internal versus external cohesion

- Using the *cut size*, the number of edges joining vertices of C with vertices of $V \setminus C$,
- The conductance of a community C is defined to taking into account the order of the cluster and the outside of the cluster:

$$\Phi(C) = \frac{c(C, V \setminus C)}{\min\{deg(C), deg(V \setminus C)\}}$$

where $deg(C)$ and $deg(V \setminus C)$ are the total degrees of C and of the rest of the graph. The $min(\Phi(C))$ is obtained when C has a low cut size and when the total degree of the cluster and its complement are equal.

Internal versus external cohesion

- A community with strong inner ties must have a higher *Relative density*

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- The *edge connectivity* of a graph G is the minimal number of nodes to be removed so that G is disconnected, and is denoted $k(G)$.
- A community C can be defined as an Highly connected subgraph (HCS) such

$$k(C) > \frac{n}{2}.$$

Measures of Belonging to a Community

Dissimilarity Measures

A distance measure d between two objects must fulfill the following criteria:

1. separation: $d(u, u) = 0$,
2. symmetry: $d(u, v) = d(v, u)$,
3. triangle inequality: $d(u, v) \leq d(u, w) + d(w, v)$.

Similarity Measures

A similarity measure s must fulfill the following criteria:

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Link Between Similarities and Dissimilarity

A dissimilarity measure d can be converted into a similarity measure using a strictly decreasing functions ϕ with some boundary conditions :

$$d = \phi(s) \text{ and } s = \phi^{-1}(d).$$

Classical Distance Recall

Recall Distances for Classical Vectors

For two vectors in $\mathbb{R}^2$, $u = (u_1, \cdots, u_n)$ and $v = (v_1, \cdots, v_n)$:

- *Euclidean*:

$$d(u, v) = \sqrt{\sum_{k=1}^{n} (u_k - v_k)^2}$$

- *Manhattan distance*: $d^1(u, v) = \sum_{k=1}^{n} |u_k - v_k|$
- *Tchebychev's distance* $d^\infty(u, v) = \max_{k=1,\cdots,n} |u_k - v_k|$
- *Cosine*:

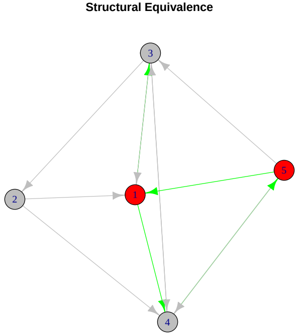
$$\theta(u, v) = \frac{\sum_{k=1}^{n} u_k \cdot v_k}{\sqrt{\sum_{k=1}^{n} (u_k)^2} \sqrt{\sum_{k=1}^{n} (v_k)^2}}.$$

Measures for Graphs: structural equivalence

Given the adjacency matrix $A = \{a_{i,j}\}$, u_i and u_j are structurally equivalent if they have the same neighbors, i.e. if $d_{i,j} = 0$:

$$d_{i,j} = \sqrt{\sum_{k \neq i,j} (a_{i,k} - a_{j,k})^2}$$

$$A = \begin{Bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{Bmatrix}$$



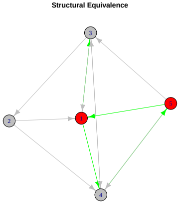
Measures for Graphs: Pearson Correlation

Another measure directly defined on A is the Pearson correlation matrix:

$$c_{i,j} = \frac{\sum_{k=1}^{n}(a_{i,k} - \mu_i)(a_{j,k} - \mu_j)}{n\sigma_j\sigma_j} \tag{4}$$

with $\mu_i = \sum_k a_{i,k}/n$ and $\sigma_i = \sqrt{\sum_k (a_{i,k} - \mu_k)^2/n}$.

$$A = \left\{ \begin{array}{ccccc} 1.00 & 0.16 & -0.16 & 0.16 & 0.66 \\ 0.16 & 1.00 & 0.66 & -0.66 & 0.66 \\ -0.16 & 0.66 & 1.00 & -1.00 & 0.16 \\ 0.16 & -0.66 & -1.00 & 1.00 & -0.16 \\ 0.66 & 0.66 & 0.16 & -0.16 & 1.00 \end{array} \right\}$$



Measures for Graphs: Jaccard Index

Another popular seed to build (dis)similarity measure is the *Jaccard index* which measures similarity between sets:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}. \quad (5)$$

A first use of the Jaccard index in the graph theory context is to measure the *overlap* of the neighborhoods of two nodes v and u:

$$\omega(v, u) = \frac{|\Gamma(v) \cap \Gamma(u)|}{|\Gamma(v) \cup \Gamma(u)|} \quad (6)$$

which is equal to zero when there is no common neighbors, and one when v and u are structurally equivalent. And, as $0 \leq J(A, b) \leq 1$, it is easy to define the Jaccard distance:

$$J_\delta(A, B) = 1 - J(A, B) = \frac{|A \cup B| - |A \cap B|}{|A \cup B|}. \quad (7)$$

Measures for Graphs: Tanimoto coefficient

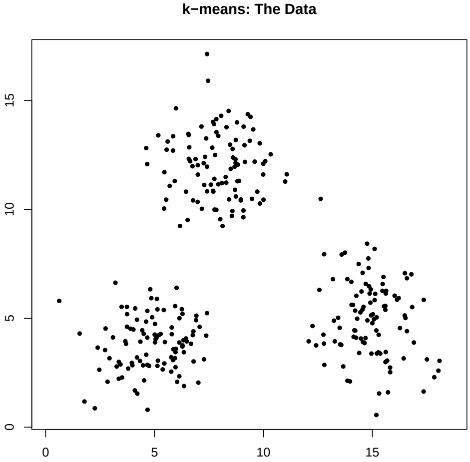
Tanimoto coefficient is an extension of the cosine similarity which coincide with the Jaccard index for binary vectors:

$$T(A,B) = \frac{\sum_{k=1}^{n} a_k \cdot b_k}{\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k - \sum_{k=1}^{n} a_k \cdot b_k}. \tag{8}$$

The Tanimoto coefficient gives the quotient between the number of shared features by A and B, divided by the whole number of features for A and B.

Plan

Partitional Algorithms - Introduction



Partitional Algorithms : k-means

k-means algorithms try to maximize the *intra-cluster dissimilarity*:

$$g_n(\mathcal{C}) = \sum_{i=1}^{k} \sum_{x \in C_i} d(x, c_i)$$

Algorithm:

Starting:Determine an initial vector (or centers) $(c_1^{(0)} \cdots, c_k^{(0)})$,

Repeat until stationarity: $t \leftarrow t + 1$

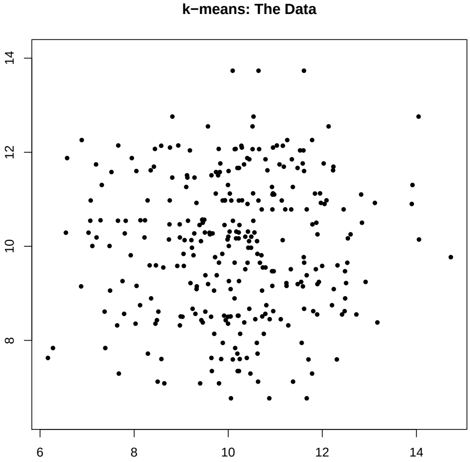
Assignment: observations go to the cluster with the closest centroid:

$$C_i^{(t+1)} = \left\{ x : d(x, c_i^{(t)}) \leq d(x, c_j^{(t)}) \forall j = 1, \ldots, k \right\}$$

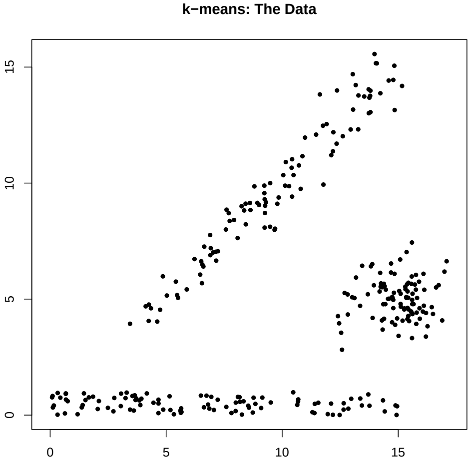
Update: Compute the new centroids $(c_1^{(t+1)}, \cdots, c_k^{(t+1)})$:

$$c_i^{(t+1)} = \frac{1}{|C_i^{(t)}|} \sum_{x \in C_i^{(t)}} x$$

Partitional Algorithms - Instability



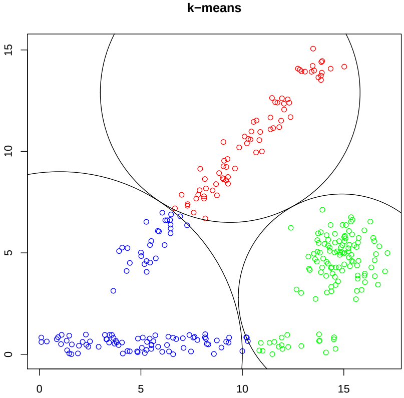
Partitional Algorithms - Spherical Clusters



Partitional Algorithms - Drawbacks

Drawbacks

- choice of k: an inappropriate choice leads too non significant results,
- spherical clusters: algorithms works better when spherical clusters are in data,
- instability: the random starting partition can leads to a local optimum for the criterion function.



Complexity: $O(m^2 \times n \times k)$ where m is the number of attributes, n the number of objects to cluster and k the number of clusters.

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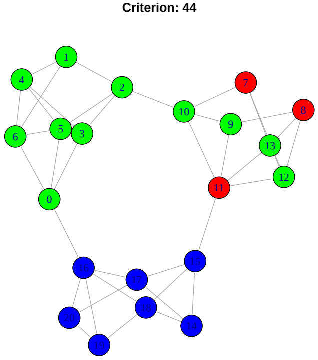

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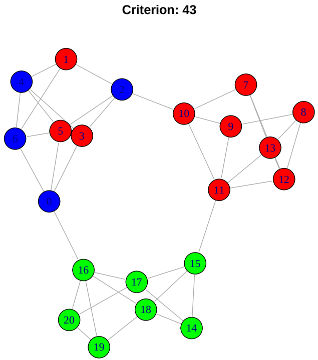

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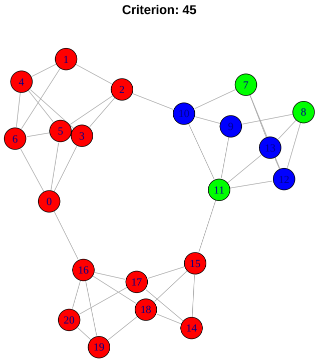

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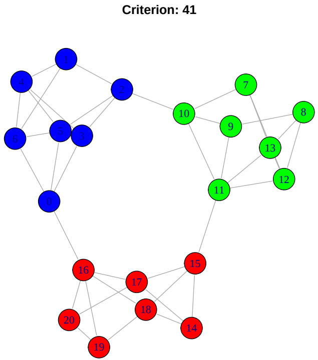

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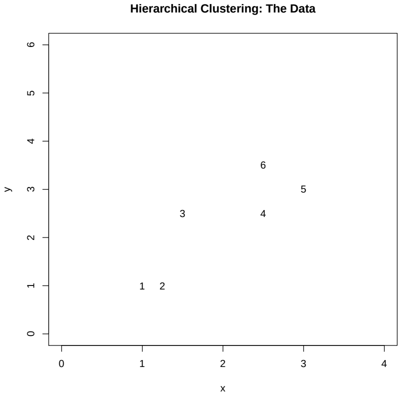
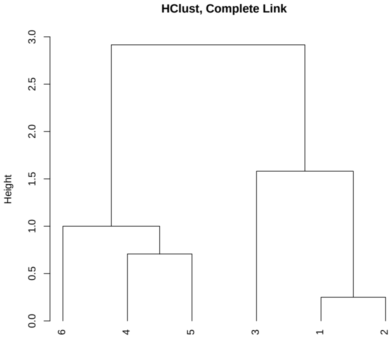
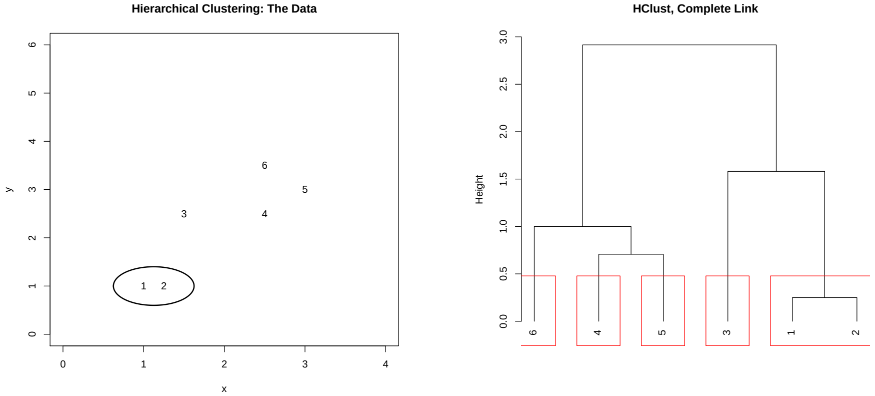
Agglomerative Hierarchical Algorithms - Intro





Agglomerative Hierarchical Algorithms - Intro



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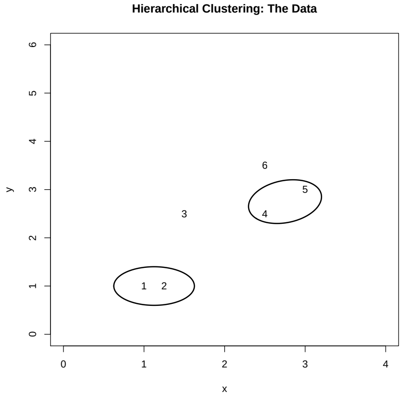
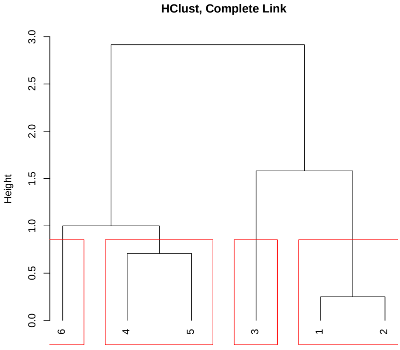
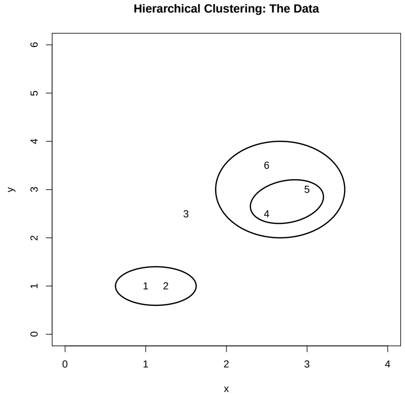
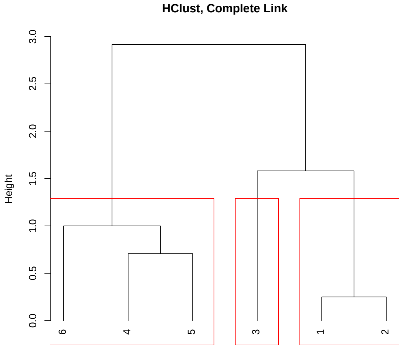




Agglomerative Hierarchical Algorithms - Intro





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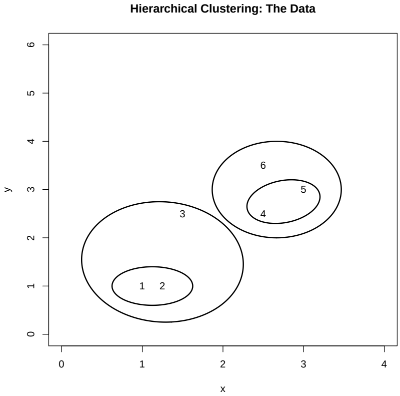

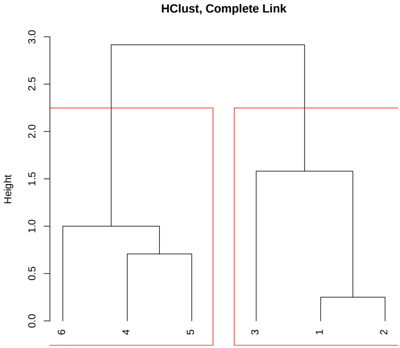

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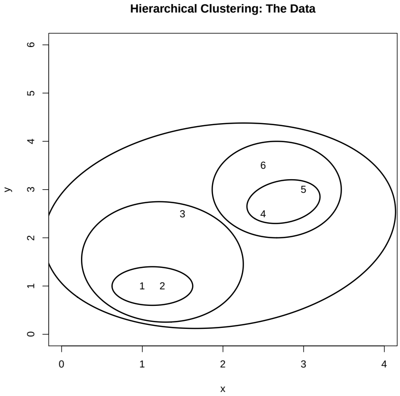

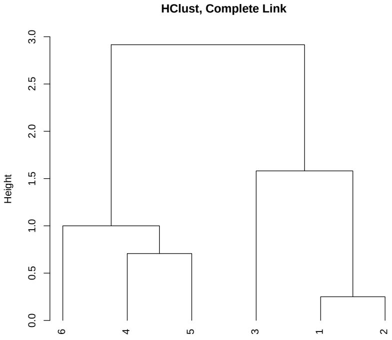

Agglomerative Hierarchical Algorithms - For SNA

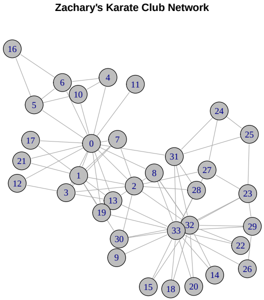

Figure: The Zachary Karate club network.

At the starting point, the n objects to cluster are their own classes: $\{\{x_1\}, \cdots, \{x_n\}\}$, then at each stage we merged the two more similar clusters.

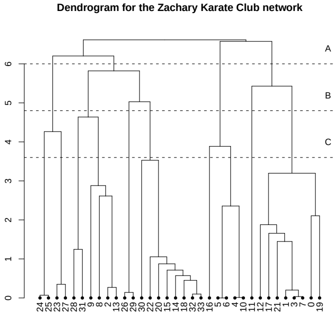

Figure: A dendrogram for the Zachary Karate club network.

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Agglomerative Hierarchical Algorithms - The Link Choice

For a given dissimilarity measure d between objects, several dissimilarities between clusters D exist:

- the single linkage:

$$D(A,B) = \min\{d(x,y) : x \in A,\ y \in B\},$$

- the complete linkage:

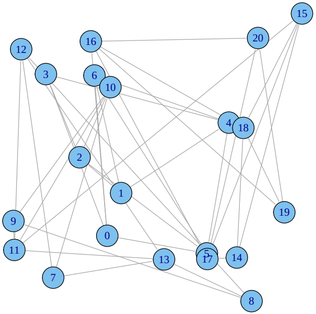
$$D(A,B) = \max\{d(x,y) : x \in A,\ y \in B\},$$

- the average linkage:

$$D(A,B) = \frac{1}{|A| \cdot |B|} \sum_{x \in A} \sum_{y \in B} d(x,y).$$

Drawbacks: vertices of a community may be not correctly classified.
Complexity: $O(n^2)$ for the single linkage and $O(n^2 \log n)$ for complete and average linkages.

Divisive Hierarchical Algorithms - Betweenness



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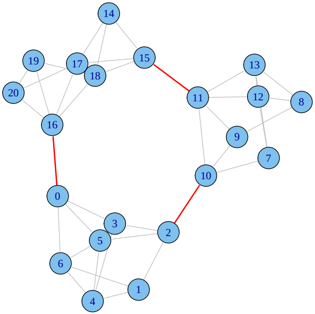

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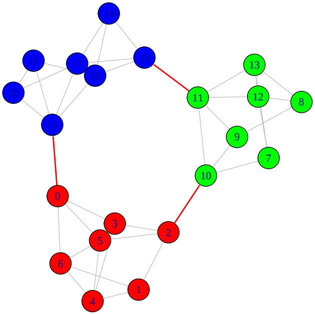

Divisive Hierarchical Algorithms - Betweenness

- Find the connecting edges to find the communities.
- *Edge betweenness* is the number of shortest paths between all vertex pairs that run along the edge.
- Divide the graph finding and "‘removing"’ edges connecting community, two communities, i.e. edges on the maximum of shortest paths between these Communities (max edge betweenness):
  1. compute the edge betweenness for all edges of the running graph,
  2. remove the edge with the largest value (which gives the new running graph).
- Stop when no improvement on a criterion like the *modularity*:

$$\mathcal{M}(C_1, \cdots, C_k) = \sum_i deg_e(C_i) - \sum_i deg_i(C_i)$$

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Complexity edge betwneenness on a graph can be computed in $O(n \cdot m)$ for unweighted graphs and in $O(n \cdot m + n^2 logn)$ for the weighted.

Spectral Methods - intro

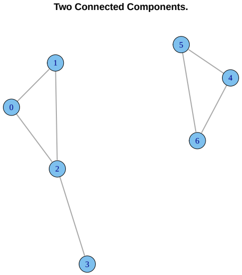
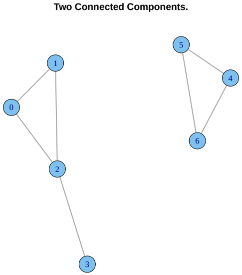


Figure: Two connected components.

Adjacency Matrix

$$W = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$



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Spectral Decomposition

$$W = Q \Lambda Q^{-1}$$

Spectral Methods - intro

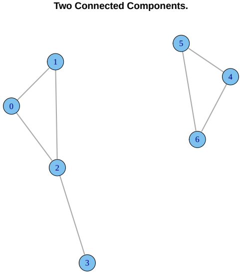

Figure: Two connected components.

Eigenvectors matrix

$$Q = \left(\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ -3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -2 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right).$$

spectrum=$\{4, 3, 1, 0\} \cup \{3, 3, 0\}$

Spectral Decomposition

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Spectral Methods - intro

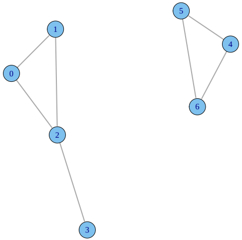

Figure: Two connected components.

Eigenvectors matrix

$$Q = \left(\begin{array}{cccc|ccc} 1 & 1 & 1 & \color{red}{1} & 0 & 0 & \color{red}{0} \\ 1 & -1 & 1 & \color{red}{1} & 0 & 0 & \color{red}{0} \\ -3 & 0 & 0 & \color{red}{1} & 0 & 0 & \color{red}{0} \\ 1 & 0 & -2 & \color{red}{1} & 0 & 0 & \color{red}{0} \\ \hline 0 & 0 & 0 & \color{red}{0} & 0 & -2 & \color{red}{1} \\ 0 & 0 & 0 & \color{red}{0} & 1 & 1 & \color{red}{1} \\ 0 & 0 & 0 & \color{red}{0} & -1 & 1 & \color{red}{1} \end{array}\right).$$

spectrum=$\{4, 3, 1, \color{red}{0}\} \cup \{3, 3, \color{red}{0}\}$

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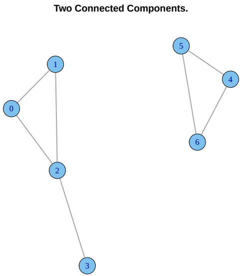
Spectral Methods - intro



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spectrum=$\{4, 3, 3, 3, 1, 0, 0\}$ Spectral Decomposition

$$W = Q \Lambda Q^{-1}$$

Spectral Methods - intro

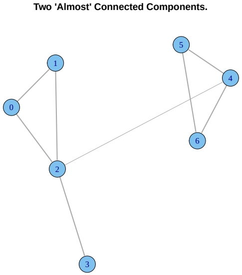


Figure: Two almost connected component.

Adjacency Matrix

$$W = \left(\begin{array}{cccc|ccc} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & \color{red}{1} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right).$$

Spectral Decomposition

$$W = Q \Lambda Q^{-1}$$

Spectral Methods - intro

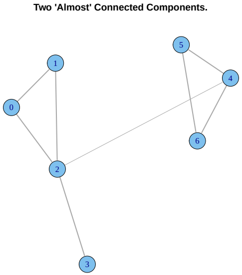


Figure: Two almost connected component.

Eigenvectors matrix

$$Q = \left(\begin{array}{ccccc|cc} . & . & . & . & . & -18 & 1 \\ . & . & . & . & . & -18 & 1 \\ . & . & . & . & . & -10 & 1 \\ . & . & . & . & . & -18 & 1 \\ \hline . & . & . & . & . & 15 & 1 \\ . & . & . & . & . & 25 & 1 \\ . & . & . & . & . & 25 & 1 \end{array}\right).$$

spectrum=$\{\cdots\}$ Spectral Decomposition

$$W = Q\Lambda Q^{-1}$$

Spectral Methods - Algorithm

For efficiency reason it is recommended to not work directly with the adjacency matrix W but with the Laplacian matrix:

$$L = D - W.$$

where D is such that we found the degrees $deg(v_i)$ on the diagonal, and then choose a normalization:

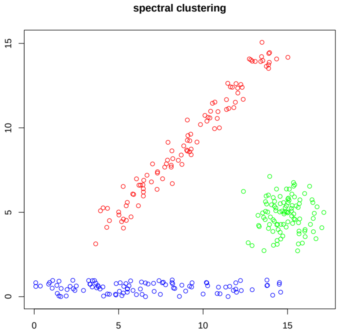
$$L_{\text{rw}} = D^{-1}L \text{ or } L_{\text{sym}} = D^{-1/2}LD^{-1/2}$$

Algorithm for spectral clustering is the following:

1. compute the eigenvalues and sort them such $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$,
2. compute the last k eigenvectors $\vec{u_{n-k}} \cdots, \vec{u_n}$,
3. form matrix $U \in \mathbb{R}^{n \times k}$ with $\vec{u_{n-k}} \cdots, \vec{u_n}$ as columns, and matrix $Y = U^t$,
4. cluster the points $(y_i)_{i=1,\cdots,n}$ using the k-means algorithm into clusters $A_1, \cdots, A_k$,
5. build the communities $C_1, \cdots, C_k$ such $C_i = \{v_j | y_j \in A_i\}$.

Spectral Methods - For More than Graphs

Efficient even for non "'graph"' data: it is sufficient to have a similarity measure $s(u, v)$ to build the weight/adjacency matrix W:



Complexity issue: the computation of the k eigenvectors of the Laplacian matrix require a time in $O(n^3)$.

Galois Lattices - Introduction

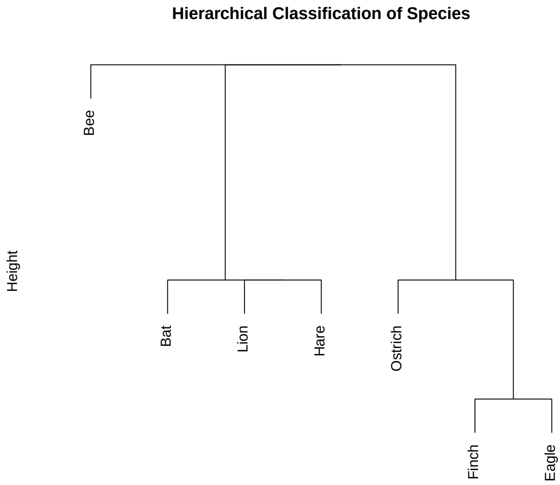
To cluster from a binary table cwith objects and attributes,

	Preying	Flying	Bird	Mammal
Lion	x			x
Finch		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		
Bat		x		x

we can compute a similarity table...

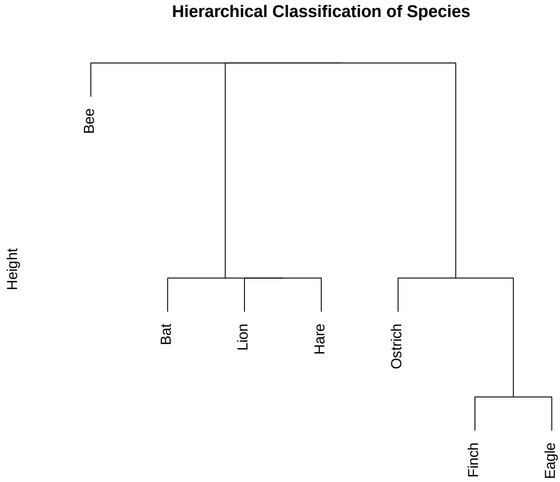
	Lion	Finch	Eagle	Hare	Ostrich	Bee	Bat
Lion	2	0	1	1	0	0	1
Finch	0	2	2	0	1	1	1
Eagle	1	2	3	0	1	1	1
Hare	1	0	0	1	0	0	1
Ostrich	0	1	1	0	1	0	0
Bee	0	1	1	0	0	1	1
Bat	1	1	1	1	0	1	2

do a hierarchical classification



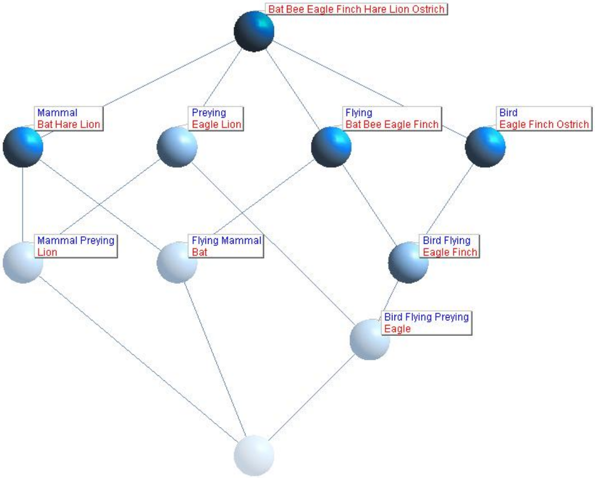
Galois Lattices - Introduction

do a hierarchical classification and choose one class per object!



Galois Lattices - Introduction

...or extract all the concepts and shared propeties!



Galois Lattices: Intension

Attributes A

Objects O

	Preying	Flying	Bird	Mammal
Lion	x			x
Finch		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		
Bat		x		x

For $X \in O$:

$$f(X) = \{a \in A | \forall o \in X, oIa\}.$$

Galois Lattices: Extension

Attributes A

Objects O

	Preying	Flying	Bird	MammifÃ¨re
Lion	x			x
Moineau		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		
Bat		x		x

For $Y \in A$:

$$g(Y) = \{o \in O | \forall a \in Y, oIa\}.$$

Attributes A

Objects O

	Preying	Flying	Bird	MammifÃ¨re
Lion	x			x
Moineau		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		
Bat		x		x

A concept is $(X, Y) \in O \times A$ such:

$$f(X) = Y \;\&\; g(Y) = X.$$

Galois Lattices: POSET

Attributes A

Objects O

	Preying	Flying	Bird	MammifÃ¨re
Lion	x			x
Moineau		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		
Bat		x		x

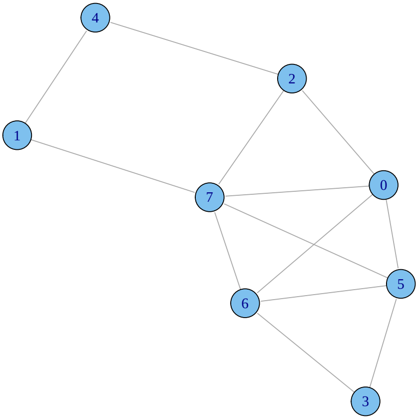
$(X_1, Y_1), (X_2, Y_2) \in O \times A : (X_1, Y_1) \leq (X_2, Y_2) \Leftrightarrow X_1 \subseteq X_2(\text{ or } Y_1 \supseteq Y_2)$.

$(\{\text{Moineau}, \text{Eagle}, \text{Ostrich}\}, \{\text{Birdx}\}) \supset$

$(\{\text{Moineau}, \text{Eagle}\}, \{\text{Flying}, \text{Birdx}\}) \supset$

$(\{\text{Eagle}\}, \{\text{Preying}, \text{Flying}, \text{Birdx}\})$

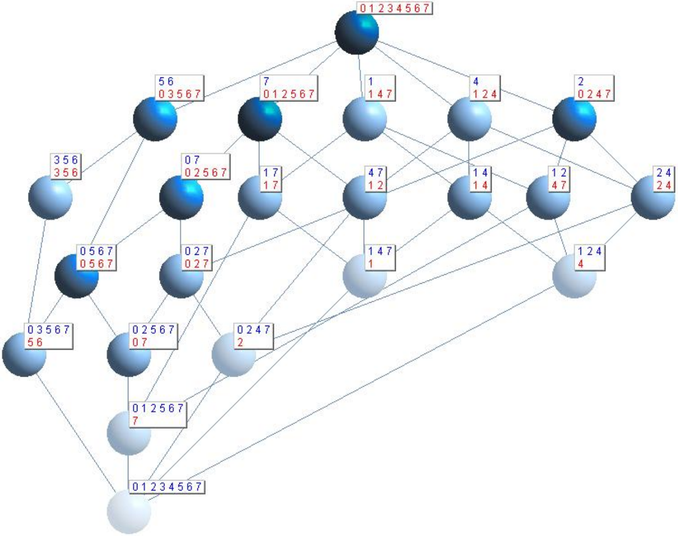
Galois Lattices for Social Networks



Galois Lattices for Social Networks

	1	2	3	4	5	6	7	8
1	×		×			×	×	×
2		×			×			×
3	×		×		×			×
4				×		×	×	
5		×	×		×			
6	×			×		×	×	×
7	×			×		×	×	×
8	×	×	×			×	×	×

Galois Lattices for Social Networks



Galois Lattices for Social Networks

Conceptual Metrics based on Galois Lattices.

Relatedness

Relatedness(O)= % of objects which share properties with O.

Closeness

Closeness(O)= % of shared properties of with its related objects.

Relatedness $\rightarrow$ $\downarrow$ Closeness	High	Low
High	Clustered	
Low		Marginal

Filter the p % of marginal objects for a chosen $p \in]0, 1[$.

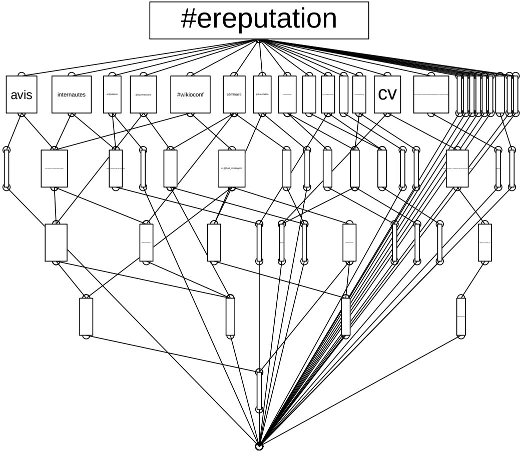
Galois Lattices vs Similarity Based Clusterings

Criteria	GL	SBC
Similarity	Equality	Proximity
Uniqueness of results	Y	N
Completeness of final structure	Y	N
Attribute Weight	N	Y
Continuous value management	Hard	Y

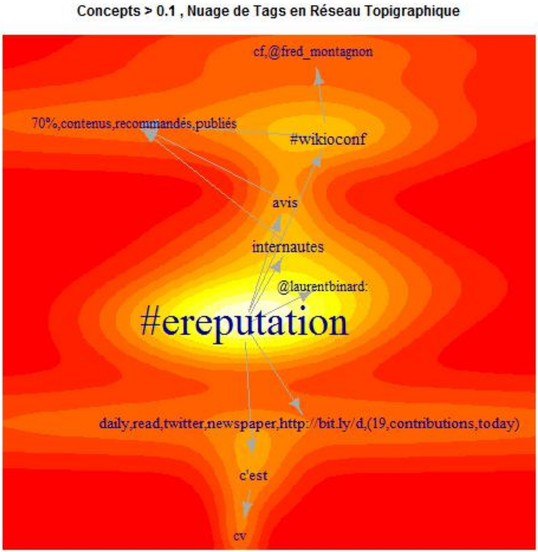
Complexity: if we denote $|O|$ the number of objects, $|A|$ the number of attributes and $|L|$ the size of the lattices (i.e. the number of concepts), then algorithms have a complexity time in $O(|O|^2|A||L|)$ or $O(|O||A|^2|L|)$.

EVARIST: eBuzz Monitoring on Twitter

Treillis Complet



EVARIST: eBuzz Monitoring on Twitter



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Pajek

Pajek runs on Windows and is free for noncommercial use.
It also runs on Unix or Mac.
Pajek manual. History.

Pajek for Windows 32 bit

Download the ESNA 2 version 2.04 / Book edition 2, (May 16, 2011) - installation pack. To install it run pajekBE2 and follov
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Pajek for Windows 64 bit

On Windows 64 bit a special version of Pajek can use up to 4GB of available computer memory.

Download the version 2.04 - 64bit / Book Edition 2, (May 16, 2011) of Pajek installation pack. To install it run pajek64-BE2 ar

Data sets

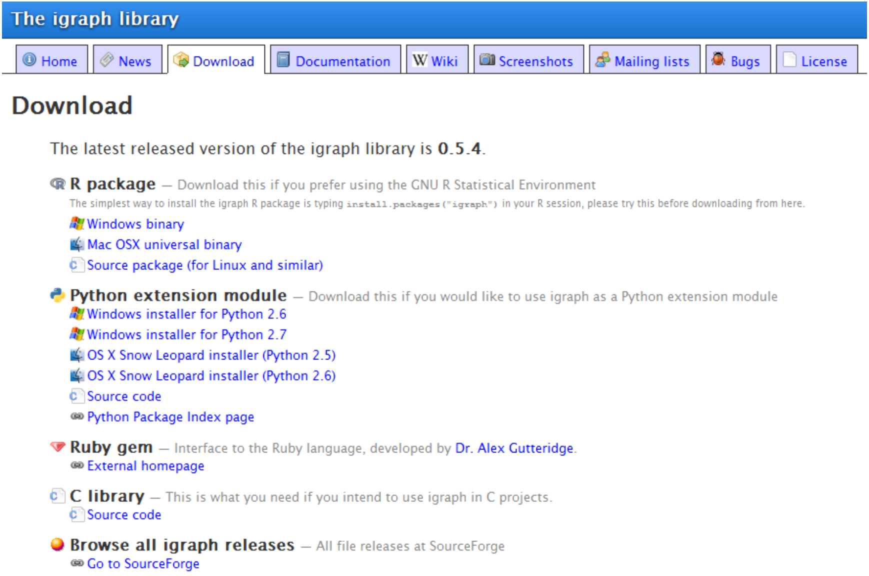
Pajek data sets.

- Data sets for experimenting with Pajek

Slides

San Diego Sunbelt XXIX workshop: slides 1, slides 2, data.

igraph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Windows binary
Mac OSX universal binary
Source package (for Linux and similar)
Python extension module — Download this if you would like to use igraph as a Python extension module
Windows installer for Python 2.6
Windows installer for Python 2.7
OS X Snow Leopard installer (Python 2.5)
OS X Snow Leopard installer (Python 2.6)
Source code
Python Package Index page
Ruby gem — Interface to the Ruby language, developed by Dr. Alex Gutteridge.
External homepage
C library — This is what you need if you intend to use igraph in C projects.
Source code
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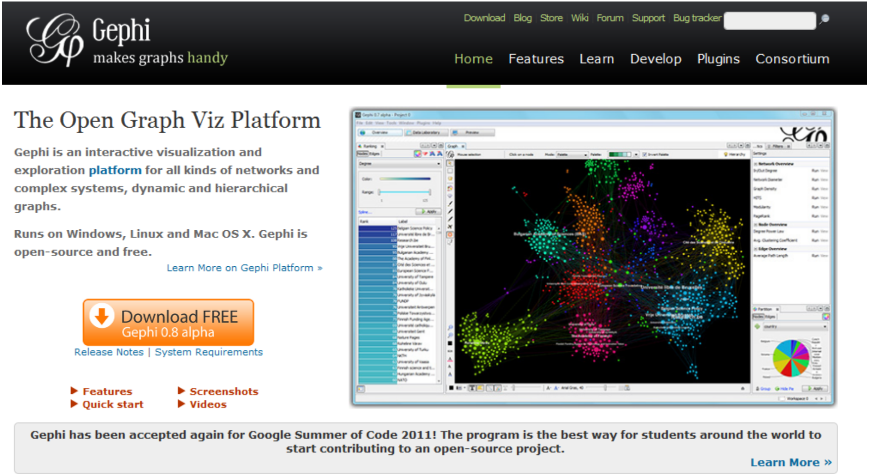
Gephi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Conclusions

- Definition of a community or cluster, is not an easy task.
- In fact, what is a cluster is in the eyes of the beholder,
- *One person's noise could be another person's signal*.
- *Cluster analysis is structure seeking although its operation is structure imposing*.
- In data clustering many choices must be done before any analysis (cluster definition , algorithms, measures, ...) which influence strongly the result.
- But, in spite of all these warnings, clustering algorithms allow us to retrieve valuable pieces of information in social networks, by finding communities.

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